

Basic Mathematical Notations

Norimasa Kobayashi @ Tokyo Institute of Technology

1 Logic

\wedge denotes “and”

\vee denotes “or”

\neg denotes “not”

$p \Rightarrow q$ denotes “if p then q ”

$p \Leftarrow q$ denotes “if q then p ”

$p \Leftrightarrow q$ is defined by $(p \Rightarrow q) \wedge (p \Leftarrow q)$ and is read “ p if and only if (iff) q ”

\forall denotes “for all”

\exists denotes “exists”

2 Real Number and Vectors

Definition 2.1 Denote \mathbb{R} the set of real numbers and \mathbb{R}_+ the set of nonnegative real numbers.

Definition 2.2 (Vectors) Vectors are denoted with normal fonts.

- $x = (x_i)_{i \in N} = (x_1, \dots, x_N) \in \times_{i \in N} X_i$
- $x_{-i} := (x_j)_{j \in N \setminus \{i\}} = (x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_N) \in X_{-i} := \times_{j \in N \setminus \{i\}} X_j$

Definition 2.3 (Pareto Order) For $x, y \in \mathbb{R}^N$:

- $x \geq y \Leftrightarrow (\forall i \in N)(x_i \geq y_i)$
- $x > y \Leftrightarrow x \geq y \wedge x \neq y$
- $x \gg y \Leftrightarrow (\forall i \in N)(x_i > y_i)$

Definition 2.4 (Pareto Efficiency (Optimality)) $x \in S \subset \mathbb{R}^N$ is

- (weakly) Pareto efficient iff not $(\exists y \in S)(y \gg x)$
- strongly Pareto efficient iff not $(\exists y \in S)(y > x)$

3 Probability

Definition 3.1 Denote $\Delta(X)$ a set of probability measures over set X .

If X is finite, $\Delta(X) = \{\phi(x) \in \mathbb{R}^X \mid \sum_{x \in X} \phi(x) = 1 \wedge \forall x \in X [\phi(x) \geq 0]\}$ is a simplex.

Definition 3.2 (Restriction) Probability $\phi \in \Delta(X)$ is restricted to $Y \subset X$ iff $\text{supp } \phi \subset Y$.