宿題の解答例

座標 A(x, y)
座標 D$(x+dx, y+dy)$
座標 A'$(x+u, y+v)$
座標 D'$\left(x+dx+u+\frac{\partial u}{\partial x}dx+\frac{\partial u}{\partial y}dy, y+dy+v+\frac{\partial v}{\partial x}dx+\frac{\partial v}{\partial y}dy \right)$
⇒座標 D'$(x+dx+u+\varepsilon_x dx+\beta dy, y+dy+v+\alpha dx+\varepsilon_y dy)$

$$\left|\overrightarrow{AD}\right| = \sqrt{dx^2+dy^2}$$

$$\left|\overrightarrow{A'D'}\right| = \sqrt{(dx+\varepsilon_x dx+\beta dy)^2+(dy+\alpha dx+\varepsilon_y dy)^2}$$

高次の微小項を無視すると

$$\left|\overrightarrow{A'D'}\right| = \sqrt{dx^2+2\varepsilon_x dx^2+2\beta dxdy+dy^2+2\varepsilon_y dy^2+2\alpha dxdy}$$

上式に $dx = \left|\overrightarrow{AD}\right|\cos\theta$, $dy = \left|\overrightarrow{AD}\right|\sin\theta$ を代入，$\alpha+\beta=\gamma_{xy}$ （工学せん断ひずみ）

$$\left|\overrightarrow{A'D'}\right| = \left|\overrightarrow{AD}\right|\sqrt{1+2\varepsilon_x\cos^2\theta+2\varepsilon_y\sin^2\theta+2\gamma_{xy}\sin\theta\cos\theta}$$

上式に下記の関係（高次の微小項を無視）を代入

$$(1+\varepsilon_x\cos^2\theta+\varepsilon_y\sin^2\theta+\gamma_{xy}\sin\theta\cos\theta)^2 \approx 1+2\varepsilon_x\cos^2\theta+2\varepsilon_y\sin^2\theta+2\gamma_{xy}\sin\theta\cos\theta$$

$$\left|\overrightarrow{A'D'}\right| = \left|\overrightarrow{AD}\right|\sqrt{(1+\varepsilon_x\cos^2\theta+\varepsilon_y\sin^2\theta+\gamma_{xy}\sin\theta\cos\theta)^2}$$

$$= \left|\overrightarrow{AD}\right|(1+\varepsilon_x\cos^2\theta+\varepsilon_y\sin^2\theta+\gamma_{xy}\sin\theta\cos\theta)$$

$$\therefore \varepsilon_\xi = \frac{\left|\overrightarrow{A'D'}\right|-\left|\overrightarrow{AD}\right|}{\left|\overrightarrow{AD}\right|} = \varepsilon_x\cos^2\theta+\varepsilon_y\sin^2\theta+\gamma_{xy}\sin\theta\cos\theta$$

教科書(1.51)と一致