# Speech Recognition (Language Modeling) 

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## Language model is crucial !

- Rudolph the red nose reindeer.
- Rudolph the Red knows rain, dear.
- Rudolph the Red Nose reigned here.

- This new display can recognize speech.
- This nudist play can wreck a nice beach.


## Language model is crucial !

- It was a nice hockey game.
- It was an ice-hockey game.
- Did U2 have a nice trip?
- Did you two have a nice trip?


## An example of FSN (Finite State Network) grammar



| 1. | l | 5. ONE | 9. BOOKS | 13. OLD |
| :--- | :--- | :--- | :--- | :--- |
| 2. | WANT | 6. A | 10. COAT |  |
| 4. | NEED | 7. AN | 11. COATS |  |

A simple, unconstrained finite state network containing $N$ keywords $W_{k 1}, \ldots, W_{k N}$ and $M$ fillers $W_{f 1}, \ldots, W_{f M}$. Associated with each keyword and filler are word transition penalties $C_{k i}$ and $C_{j}$ respectively.


## Complete Hidden Markov Model of a simple grammar



## Statistical language modeling

Probability of the word sequence $w_{1} \boldsymbol{k}=w_{1} w_{2} \ldots w_{k}$ :
$P\left(w_{1}{ }^{k}\right)=\prod_{i=1}^{k} P\left(w_{i} \mid w_{1} w_{2} \ldots w_{i-1}\right)=\prod_{i=1}^{k} P\left(w_{i} \mid w_{1}^{i-1}\right)$
$P\left(w_{i} \mid w_{1}{ }^{i-1}\right)=N\left(w_{1}{ }^{i}\right) / N\left(w_{1}{ }^{i-1}\right)$
where $N\left(w_{1}{ }^{i}\right)$ is the number of occurrences of the string $w_{1}{ }^{i}$ in the given training corpus.
Approximation by Markov processes:
Bigram model $\quad \boldsymbol{P}\left(\boldsymbol{w}_{\boldsymbol{i}} \mid \boldsymbol{w}_{\mathbf{1}}{ }^{\boldsymbol{i - 1}}\right)=\boldsymbol{P}\left(w_{i} \mid w_{i-1}\right)$
Trigram model $\quad \boldsymbol{P}\left(w_{i} \mid w_{1}{ }^{i-1}\right)=\boldsymbol{P}\left(w_{i} \mid w_{i-2} w_{i-1}\right)$
Smoothing of trigram by the deleted interpolation method:

$$
P\left(w_{i} \mid w_{i-2} w_{i-1}\right)=\lambda_{1} P\left(w_{i} \mid w_{i-2} w_{i-1}\right)+\lambda_{2} P\left(w_{i} \mid w_{i-1}\right)+\lambda_{3} P\left(w_{i}\right)
$$

## Good-Turing estimate

For any $\boldsymbol{n}$-gram that occurs $r$ times, we should pretend that it occurs $r^{*}$ times as follows:

$$
r^{*}=(r+1) \frac{n_{r+1}}{n_{r}}
$$

where $\boldsymbol{n}_{r}$ is the number of $\boldsymbol{n}$-grams that occur exactly $r$ times in the training data.

## Katz smoothing algorithm

Katz smoothing extends the intuitions of the Good-Turing estimate by adding the combination of higher-order models with lower-order models.

$$
P_{\text {Kats }}\left(w_{i} \mid w_{i-1}\right)= \begin{cases}C\left(w_{i-1} w_{i}\right) / C\left(w_{i-1}\right) & \text { if } r>k \\ d_{r} C\left(w_{i-1} w_{i}\right) / C\left(w_{i-1}\right) & \text { if } k \geq r>0 \\ \alpha\left(w_{i-1}\right) P\left(w_{i}\right) & \text { if } r=0\end{cases}
$$

where $d_{r}=\frac{\frac{r^{*}}{r}-\frac{(k+1) n_{k+1}}{n_{1}}}{1-\frac{(k+1) n_{k+1}}{n_{1}}}$ and $\alpha\left(w_{i-1}\right)=\frac{1-\sum_{w_{i}: r>0} P_{\text {Kats }}\left(w_{i} \mid w_{i-1}\right)}{1-\sum_{w_{i}: r>0} P\left(w_{i}\right)}$

## Task difficulty (Entropy)

- Yes, No

1 bit (- $\log 0.5)$

- Digits

$$
\begin{aligned}
& * 0 \sim 9: 0.1 \\
& *\left\{\begin{array}{cc}
\mathbf{0}: 0.91 \\
\mathbf{1} \sim 9 & \mathbf{9} \boldsymbol{0 . 0 1} \\
& \mathbf{( - 0 . 9 1} \log \mathbf{0 . 9 1} \mathbf{- 0 . 0 9} \operatorname{0.09} \log \mathbf{0 . 0 1})
\end{array}\right.
\end{aligned}
$$

## Test-set perplexity

## Vocabulary: A, B Test sentence: ABB



Test sentence entropy: $-\log 1 / 8-\log 1 / 4-\log 1 / 4=7$ bits (ABB)

Entropy per word: $7 / 3$ = 2.33 bits
Test-set perplexity: $\mathbf{2}^{2.33}=5.01$

A unigram grammar network where the unigram probability is attached as the transition probability from starting state S to the first state of each word HMM.


A bigram grammar network where the bigram probability $P\left(w_{j} \mid w_{i}\right)$ is attached as the transition probability from word $w_{i}$ to $w_{j}$.


A trigram grammar network where the trigram probability $P\left(w_{k} \mid w_{i}, w_{j}\right)$ is attached to transition from grammar state $w_{i}, w_{j}$ to the next word $w_{k}$. Illustrated here is a two-word vocabulary, so there are four grammar states in the trigram network.


A highway distance map for cities S, A, B, C, D, E, F, and G. The salesman needs to find a path to travel from city $S$ to city $G$.


The search tree (graph) for the salesman problem illustrated in the previous figure. The number next to each node is the accumulated distance from start city to end city.


The node-expanding procedure of the depth-first search for the path search problem. When it fails to find the goal city in node C , it backtracks to the parent and continues the search until it finds the goal city. The green nodes are those that are explored. The doted nodes are not visited during the search.


The node-expanding procedure of a breath-first search for the path search problem. It searches through each level until the goal is identified. The green nodes are those that are explored. The dotted nodes are not visited during the search.


Beam search for the city-travel problem. The nodes with green color are the ones kept in the beam. The transparent nodes were explored but pruned because of higher cost. The dotted nodes indicate all the savings because of pruning.


System diagram of a generic speech recognizer based on statistical models, including training and decoding processes and the main knowledge sources.


Example word lattice generated by a speech recognizer using a bigram language model for a 2.1 s utterance. Each graph edge corresponds to a word hypothesis and a time interval (as specified by the time information on the nodes).


