Structural Dynamics 構造動力学 (11)

Kazuhiko Kawashima Department of Civil Engineering Tokyo institute of Technology 東京工業大学大学院理工学研究科土木工学専攻 川島一彦 Chapter 10 Formulation of the Nonlinear Multi-Degree-of-Freedom Equations of Motion (非線形多自由度系の運動方程式)

# 10.1 Incremental Equations of Motion of Nonlinear MDOF System (非線形多自由度系に対する増分系運動方程式)

•Equilibrium of the inertia force, damping force, restoring force and external force for SDOF system is given by Eq. (2.1). By extending Eq. (2.1) to MDOF system, one obtains

• Eq. (10.1) can be written in matrix form by  $\{F_I(t)\} + \{F_D(t)\} + \{F_R(t)\} = \{F(t)\}$  (10.2) in which,

 $\{F_I\}$ : inertia force vector (慣性力ベクトル)  $\{F_D\}$ : damping force vector (減衰力ベクトル)  $\{F_R\}$ : restoring force vector (復元力ベクトル)  $\{F\}$ : external force vector (外力ベクトル)

where,

 ${F_I} = [M]{\ddot{u}}$  ${F_D} = [C]{\dot{u}}$  • If the system is linear, the restoring force vector is given by

 $\{F_R\} = [K]\{u\}$  (10.3)

Then, Eq. (10.2) becomes completely the same with Eq. (9.45).

However, note in Eq. (10.2) that because the system is not linear elastic, the restoring force vector cannot be evaluated by Eq. (10.3).
Therefore, the equations of motion given by Eq. (10.2) cannot be solved by the mode superposition method whic we studied in Chapter 9.

 $\{F_{I}(t)\} + \{F_{D}(t)\} + \{F_{R}(t)\} = \{F(t)\}$ (10.2) $[M]{\ddot{u}} + [C]{\dot{u}} + [K]{u} = {P}$ (9.45)

• In linear system 
$$f_R$$
 Linear system  
 $f_R(u_{t+\Delta t}) = ku_{t+\Delta t}$   
• However in  $f_R(u_{t+\Delta t})$   
nonlinear system  $f_R(u_t)$   
 $f_R(u_{t+\Delta t}) \neq k(u)u_{t+\Delta t}$   
• However, if  $f_R(t)$  is  
the restoring force  
at  $u_t$ , the restoring  $f_R$   
force at  $u_{t+\Delta t}$  may be  
approximately  
evaluated using the  $f_R(u_{t+\Delta t})$   
 $tangential stiffness ( $\mathfrak{F} f_R(u_t)$   
 $k_R \mathfrak{M} \mathfrak{K}$ ) by  
 $\Delta f_{Rt} \equiv f_R(u_{t+\Delta t}) - f_R(u_t)$   
 $\approx k_t(u_t)\Delta u_t$  (a)  $u_t$   $u_{t+\Delta t}$   $\circ$   $u$$ 

In Eq. (a),  

$$\Delta u_t = u_{t+\Delta t} - u_t \quad (b)$$

is called incremental displacement (增分変  $f_R(u_{t+\Delta t})$ ) 位) at time t, and this  $f_R(u_t)$ represents an increase of displacement during a small time interval  $\Delta t$ . Tangential stiffness  $k_t$ 

 $u_t \quad u_{t+\Delta t}$ 

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•  $\Delta f_{Rt}$  is called incremental restoring force (増分復 元力) at time t (during  $\Delta u_t$ ).

 $f_R$ 

• Tangential stiffness is defined by  $k_t = \left(\frac{\partial f_R}{\partial f_R}\right)$  (c)

 $\Delta f_{Rt} \equiv f_R(u_{t+\Delta t}) - f_R(u_t) \approx k_t(u_t) \Delta u_t$ (a)

• If time interval  $\Delta t$  is small enough such as 1/100s-1/10,000s, the approximation of the incremental restoring force by Eq. (a) may be sufficient.

• Extending Eq. (a) to MDOF system, the incremental restoring force vector at time t is written by

$$\{\Delta F_{Rt}\} \equiv \{F_R(u_{t+\Delta t})\} - \{F_R(u_t)\}$$
$$= [K_t(u_t)]\{\Delta u_t\}$$
(d)

where,  $\{\Delta u_t\}$  is called the incremental displacement vector (增分変位ベクトル) during time  $t+\Delta t$  and t, and is expressed by

$$\{\Delta u_t\} \equiv \{u_{t+\Delta t}\} - \{u_t\}$$
 (e)

 $\Delta f_{Rt} \equiv f_R(u_{t+\Delta t}) - f_R(u_t) \approx k_t(u_t) \Delta u_t$ (a)

• Based on Eq. (10.2), the dynamic equilibrium at time t+ $\Delta t$  can be written by  $\{F_I(t + \Delta t)\} + \{F_D(t + \Delta t)\} + \{F_R(t + \Delta t)\} = \{F(t + \Delta t)\}$ (10.4)

• Subtracting Eq. (10.2) from Eq. (10.4), one obtains the dynamic equilibrium in an incremental form

 $\{\Delta F_{It}\} + \{\Delta F_{Dt}\} + \{\Delta F_{Rt}\} = \{\Delta F_t\}$ (10.5) where,

$$\{\Delta F_{It}\} \equiv \{F_{I}(t + \Delta t)\} - \{F_{I}(t)\}$$

$$= [M]\{\vec{u}_{t+\Delta t}\} - [M]\{\vec{u}_{t}\}$$

$$= [M]\{\Delta \vec{u}_{t}\}$$

$$= [M]\{\Delta \vec{u}_{t}\}$$

$$= \{\vec{u}_{t+\Delta t}\} - \{\vec{u}_{t}\}$$

$$(10.6)$$

$$= [M]\{\Delta \vec{u}_{t}\} = \{\vec{u}_{t+\Delta t}\} - \{\vec{u}_{t}\}$$

$$(10.7)$$

$$\{F_{I}(t)\} + \{F_{D}(t)\} + \{F_{R}(t)\} = \{F(t)\}$$

$$(10.2)$$

$$[M]\{\vec{u}\} + [K]\{u\} = -\vec{u}_{g}[M]\{I\}$$

$$(9.16)$$



$$\{\Delta F_{Dt}\} \equiv \{F_D(t + \Delta t)\} - \{F_D(t)\}$$

$$= [C]\{\Delta \dot{u}_t\}$$
(10.8)
$$\{\Delta F_{Rt}\} \equiv \{F_R(t + \Delta t)\} - \{F_R(t)\}$$

$$\approx [K_t]\{\Delta u_t\}$$
(10.9)
$$\{\Delta F_t\} = \{F(t + \Delta t)\} - \{F(t)\}$$
(10.10)

in which  

$$\{\Delta \dot{u}_t\} \equiv \{\dot{u}_{t+\Delta t}\} - \{\dot{u}_t\}$$
(10.11)  

$$\{\Delta u_t\} \equiv \{u_{t+\Delta t}\} - \{u_t\}$$
(10.12)

$$\{F_{I}(t + \Delta t)\} + \{F_{D}(t + \Delta t)\} + \{F_{R}(t + \Delta t)\} = \{F(t + \Delta t)\}$$
(10.4)  
$$\{F_{I}(t)\} + \{F_{D}(t)\} + \{F_{R}(t)\} = \{F(t)\}$$
(10.2)<sup>10</sup>

• Substitution of Eqs. (10.6), (10.8), (10.9) and (10.10) into Eq. (10.5) leads to  $[M] \{ \Delta \ddot{u}_t \} + [C] \{ \Delta \dot{u}_t \} + [K_t] \{ \Delta u_t \} = \{ \Delta F_t \}$ (10.12)

where,

$\{\Delta \ddot{u}_t\} \equiv \{\ddot{u}_{t+\Delta t}\} - \{\ddot{u}_t\}$	(10.7)
$\{\Delta \dot{u}_t\} = \{\dot{u}_{t+\Delta t}\} - \{\dot{u}_t\}$	(10.11)
$\{\Delta u_t\} \equiv \{u_{t+\Delta t}\} - \{u_t\}$	(10.12)

 $\{\Delta F_{It}\} + \{\Delta F_{Dt}\} + \{\Delta F_{Rt}\} = \{\Delta F_t\}$ (10.5)  $\{\Delta F_{It}\} \equiv \{F_I(t + \Delta t)\} - \{F_I(t)\} = [M]\{\Delta \ddot{u}_t\}$ (10.6)  $\{\Delta F_{Dt}\} \equiv \{F_D(t + \Delta t)\} - \{F_D(t)\} = [C]\{\Delta \dot{u}_t\}$ (10.8)  $\{\Delta F_{Rt}\} \equiv \{F_R(t + \Delta t)\} - \{F_R(t)\} = [K_t]\{\Delta u_t\}$ (10.9)  $\{\Delta F_t\} = \{F(t + \Delta t)\} - \{F(t)\}$ (10.10)

### 10.2 Direct Integration Method (直接積分法)

•Based on the Newmark's  $\beta$  method for SDOF system given by Eq. (7.27), the basic integration equations can be extended to MDOF system as

 $\{\Delta \ddot{u}_t\} = c_1 \{\Delta u_t\} - c_3 \{\dot{u}_t\} - c_4 \{\ddot{u}_t\}$ (10.13a)  $\{\Delta \dot{u}_t\} = c_2 \{\Delta u_t\} - c_4 \{\dot{u}_t\} - c_5 \{\ddot{u}_t\}$ (10.13b)

$$\Delta \ddot{u}_t = c_1 \Delta u_t - c_3 \dot{u}_t - c_4 \ddot{u}_t \qquad (7.27a)$$

$$\Delta \dot{u}_t = c_2 \Delta u_t - c_4 \dot{u}_t - c_5 \dot{u}_t$$
 (7.27b)  
n which

$$c_{1} = \frac{1}{\sigma\Delta t^{2}} \qquad c_{2} = \frac{\delta}{\sigma\Delta t} \qquad c_{3} = \frac{1}{\sigma\Delta t}$$

$$c_{4} = \frac{1}{2\sigma} \qquad c_{5} = \left(\frac{\delta}{2\sigma} - 1\right)\Delta t \qquad (7.28)$$

$$c_{4} = \frac{1}{2\sigma} \qquad c_{5} = \left(\frac{\delta}{2\sigma} - 1\right)\Delta t \qquad (7.28)$$

• In Eq. (10.13), combination of  $\delta = 1/2$  and  $\sigma = 1/4$ is the integration scheme corresponding to the constant acceleration method (**-cmibcb**), and combination of  $\sigma = 1/6$  and  $\delta = 1/2$  the linear acceleration method (**kmibcb**)

 Similar to Eq. (7.29), substitution of Eq. (10.13) into Eq. (10.12) leads to

$$\{\Delta \ddot{u}_t\} = c_1 \{\Delta u_t\} - c_2 \{\dot{u}_t\} - c_3 \{\ddot{u}_t\}$$
(10.13a)  
$$\{\Delta \dot{u}_t\} = c_2 \{\Delta u_t\} - c_4 \{\dot{u}_t\} - c_5 \{\ddot{u}_t\}$$
(10.13b)  
$$[M] \{\Delta \ddot{u}_t\} + [C] \{\Delta \dot{u}_t\} + [K_t] \{\Delta u_t\} = \{\Delta F_t\}$$
(10.12)

$$\left[\widetilde{K}_{t}\right]\!\left\{\Delta u_{t}\right\} = \left\{\Delta\widetilde{F}_{t}\right\}$$
(10.14)

in which,

$$\begin{split} & [\widetilde{K}_t] = [K_t] + c_1[M] + c_2[C] & (10.15) \\ & \{\Delta \widetilde{F}_t\} = \{\Delta F_t\} + [c_3[M] + c_4[C]] \{\dot{u}_t\} \\ & + [c_4[M] + c_5[C]] \{\ddot{u}_t\} & (10.16) \end{split}$$

$$\widetilde{k}_t \Delta u_t = \Delta \widetilde{p}_t \tag{7.29}$$

in which,

$$\widetilde{k}_{t} = k_{t} + c_{1}m + c_{2}c_{t} \quad (7.30)$$
  
$$\Delta \widetilde{p}_{t} = \Delta p_{t} + (c_{3}m + c_{4}c_{t})\dot{u}_{t} + (c_{4}m + c_{5}c_{t})\ddot{u}_{t} \quad (7.31)$$

• Once the incremental displacements  $\{u_t\}$  are computed by solving Eq. (10.14), the incremental accelerations  $\{\Delta \ddot{u}_t\}$  and incremental velocities  $\{\Delta \dot{u}_t\}$  during  $\Delta t$  can be obtained from Eq. (10.13).

•Then the response quantities at time  $t+\Delta t$  can be computed based on Eqs. (10.7), (10.11) and (10.12) by

$$\{ \ddot{u}_{t+\Delta t} \} = \{ \ddot{u}_t \} + \{ \Delta \ddot{u}_t \}$$
(10.17)  

$$\{ \dot{u}_{t+\Delta t} \} = \{ \dot{u}_t \} + \{ \Delta \dot{u}_t \}$$
(10.18)  

$$\{ u_{t+\Delta t} \} = \{ u_t \} + \{ \Delta u_t \}$$
(10.19)

$$\{\Delta \ddot{u}_t\} = c_1 \{\Delta u_t\} - c_2 \{\dot{u}_t\} - c_3 \{\ddot{u}_t\}$$
(10.13a)  

$$\{\Delta \dot{u}_t\} = c_2 \{\Delta u_t\} - c_4 \{\dot{u}_t\} - c_5 \{\ddot{u}_t\}$$
(10.13b)  

$$\{\Delta \ddot{u}_t\} \equiv \{\ddot{u}_{t+\Delta t}\} - \{\ddot{u}_t\}$$
(10.7)  

$$\{\Delta \dot{u}_t\} \equiv \{\dot{u}_{t+\Delta t}\} - \{\dot{u}_t\}$$
(10.11)  

$$\{\Delta u_t\} \equiv \{u_{t+\Delta t}\} - \{u_t\}$$
(10.12)

• As shown for SDOF system, the restoring force at time  $t+\Delta t$  cannot be directly computed by simply multiplying the displacement at time  $t+\Delta t$  by the tangential stiffness at time t, that is,

### 10.3 Idealization of Damping Matrix

In the analysis of linear MDOF system, damping ratio is assigned for each mode by Eqs. (9.60) and (9.61) after decoupling the equations of motion into n-sets of equation of motion of SDOF system. Hence, it is not needed to formulate damping matrix.
However in the analysis of nonlinear MDOF system, damping matrix has to be formulated because damping ratio cannot be assinged for each mode by Eqs. (9.60) and (9.61).

$$M_{r}^{*}\ddot{q}_{r} + C_{r}^{*}\dot{q}_{r} + K_{r}^{*}q_{r} = P_{r}^{*}$$
(9.60)  
$$\frac{C_{r}^{*}}{M_{r}^{*}} = 2\xi_{r}\omega_{r}$$
(9.61)

• Rayleigh damping given by Eq. (9.56) is generally assumed in nonlinear MDOS system. • Premultiplying  $[\Phi]^T$  and post-multiplying  $[\Phi]$  to Eq. (9.56), one obtains

$$[\Phi]^{T}[C][\Phi] = \alpha [\Phi]^{T}[M][\Phi] + \beta [\Phi]^{T}[K][\Phi] \quad (10.21)$$

 Based on the orthogonal condition by Eqs. (9.52) and (9.53), one obtains

 $C_i^* = \alpha M_i^* + \beta K_i^*$  (10.22)

• Dividing Eq. (10.22) by  $M_i^*$ ,

 $\frac{C_i^*}{M_i^*} = \alpha + \beta \frac{K_i^*}{M_i^*}$ (10.23)

 $[C] = \alpha[M] + \beta[K]$ (9.56)



 $\frac{C_i^*}{M_i^*} = 2\xi_i\omega_i$  $\frac{K_i^*}{M_i^*} = \omega_i^2$ 

Eq. (10.23) can be written as

 $\xi_i = \frac{1}{2} \left( \frac{\alpha}{\omega_i} + \beta \omega_i \right)$ 

(10.24)

 $\frac{C_i^*}{M_i^*} = \alpha + \beta \frac{K_i^*}{M_i^*}$ (10.23)





• Two parameters  $\alpha$  and  $\beta$  can be determined by assigning two pairs of  $(\omega_i, \xi_i)$  and  $(\omega_j, \xi_j)$ 

 Two modes i and j have to be determined so that the damping ratio at the predominant modes can be captured by Eq. (10.24)

$$\xi_i = \frac{1}{2} \left( \frac{\alpha}{\omega_i} + \beta \omega_i \right)$$
(10.24)

•  $\alpha$  and  $\beta$  are determined as

 $\begin{cases} \alpha \\ \beta \end{cases} = \frac{2\omega_i \omega_j}{\omega_j^2 - \omega_i^2} \begin{bmatrix} \omega_j & -\omega_i \\ -1/\omega_j & 1/\omega_i \end{bmatrix} \begin{cases} \xi_i \\ \xi_j \end{cases}$ (10.25)

### 10.4 Accuracy of Computed Responses (解析結 果の精度)

•Approximation of the restoring force by Eq. (10.20) is sometimes insufficient to compute reliable response of a structure.



 By extending the unbalance force of SDOF system to that of MDOF system, accuracy of the computed response at time t is generally represented in terms of unbalance force (不つり合い力)

$$\{\delta F_t\} = \{F_t\} - ([M]\{\ddot{u}_t\} + [C]\{\dot{u}_t\} + \{F_{Rt}\})$$
(10.27)

• Amount of the unbalance force can be represented in various ways. One of the expressions may be to define a ratio between the norm of the unbalance force and the norm of the external force as

$$\Delta_P = \frac{\|\delta F_t\|}{\|F_t\|} < \Delta_{PS} \tag{10.28}$$

where  $\Delta_{PS}$  is the threshold value (acceptable error)

$$\delta f_t = f_t - \tilde{f}_{Rt} = f_t - (m\ddot{u}_t + c\dot{u}_t + f_{Rt}) \quad (10.26)$$

10.4 improvement of Accuracy of Solutions

1) Use of smaller time interval for numerical integration

 It is always effective to use smaller time interval for numerical integration although computer time required increases.

●If strong nonlinearity exists only at several time intervals, it is useful to subdivide time interval only where the accuracy by Eq. (10.27) is insufficient.



2) Add unbalance force to incremental external force at the next time step

It is often used to add the unbalance force by Eq. (10.27) to the incremental load in the next time step. Rewriting Eq. (10.27) at time t, the unbalance force at time t is

 $\{\delta F_t\} = \{F_t\} - ([M]\{\ddot{u}_t\} + [C]\{\dot{u}_t\} + \{F_{Rt}\})$ (10.29)

and adding this unbalanced force to the incremental load in Eq. (10.12), one obtains  $[M] \{\Delta \ddot{u}_t\} + [C] \{\Delta \dot{u}_t\} + [K_t] \{\Delta u_t\} = \{\Delta F_t\} + \{\partial F_t\}$ (10.30)

 $\{\delta F_{t+\Lambda t}\} = \{F_{t+\Lambda t}\} - ([M]\{\ddot{u}_{t+\Lambda t}\} + [C]\{\dot{u}_{t+\Lambda t}\} + \{F_{Rt+\Lambda t}\})$ (2160.27)  $[M]\{\Delta \ddot{u}_t\} + [C]\{\Delta \dot{u}_t\} + [K_t]\{\Delta u_t\} = \{\Delta F_t\} \quad (10.12)$ 

• Based on Eq. (10.10), the right hand side of Eq. (10.30) becomes  $\{\Delta F_t\} + \{\partial F_t\} = \{F_{t+\Delta t}\} - \{F_t\} + \{\partial F_t\}$  $= \{F_{t+\Delta t}\} - \{F_t\} - \{\{F_t\} - ([M]\{\ddot{u}_t\} + [C]\{\dot{u}_t\} + \{F_{Rt}\})\}$  $= \{F_{t+\Delta t}\} + ([M]\{\ddot{u}_t\} + [C]\{\dot{u}_t\} + \{F_{Rt}\})$ • Thus, Eq. (10.30) becomes  $[M] \{\Delta \ddot{u}_t\} + [C] \{\Delta \dot{u}_t\} + [K_t] \{\Delta u_t\}$  $= \{F_{t+\Lambda t}\} + ([M]\{\ddot{u}_t\} + [C]\{\dot{u}_t\} + \{F_{Rt}\}) \quad (10.31)$  $\{\Delta F_t\} = \{F(t + \Delta t)\} - \{F(t)\}$ (10.10) $[M] \{\Delta \ddot{u}_t\} + [C] \{\Delta \dot{u}_t\} + [K_t] \{\Delta u_t\} = \{\Delta F_t\} + \{\partial F_t\} \quad (1Q.30)$ 

• By adding unbalanced force to the incremental force, accuracy of the solution of Eq. (10.12) is generally improved.



The equation of motion for the i-th equilibrium iteration  $m\delta \ddot{u}_t^{(i)} + c\delta \dot{u}_t^{(i)} + k_t \delta u_t^{(i)} = \delta f_t^{(i)}$  (10.32)<sup>28</sup>

Unbalance force for  
the i-th iteration at  

$$k_{t-\Delta t}\Delta u_{t-\Delta t} \lim_{t \to 0} it = f_t - f_{R_t}^{(i)}$$

$$f_{R_t}^{(1)} \xrightarrow{f_t} \int_{t-\Delta t} \int_{t-\Delta t} f_{t-\Delta t} = f_t - f_{t-\Delta t}$$

$$f_{R_t}^{(2)} \xrightarrow{f_{t-\Delta t}} \int_{t-\Delta t} \int_{t-\Delta t} f_{t-\Delta t} = k_{t-\Delta t}\Delta u_{t-\Delta t}$$

$$= k_{t-\Delta t}(u_t - u_{t-\Delta t})$$

$$= u_t^{(1)}$$

$$\delta u_t^{(i)} = u_t^{(i+1)} - u_t^{(i)}$$

$$\delta f_t^{(i)} = f_t - (m\ddot{u}_t^{(i)} + c\dot{u}_t^{(i)} + f_{R_t}^{(i)})$$

$$\delta \ddot{u}_t^{(i)} = \ddot{u}_t^{(i+1)} - \ddot{u}_t^{(i)}$$

$$\delta \dot{u}_t^{(i)} = \ddot{u}_t^{(i+1)} - \ddot{u}_t^{(i)}$$

$$\delta \dot{u}_t^{(i)} = \ddot{u}_t^{(i+1)} - \ddot{u}_t^{(i)}$$

$$\delta \ddot{u}_t^{(i)} = \ddot{u}_t^{(i+1)} - \ddot{u}_t^{(i)}$$

 $\left| m \delta \ddot{u}_t^{(i)} + c \delta \dot{u}_t^{(i)} + k_t \delta u_t^{(i)} = \delta f_t^{(i)} \qquad (10.32) \right|^{29}$ 

 Extending Eq. (10.32) to MDOF system, The equations of motion for the i-th equilibrium iteration at time t are expressed as

 $\begin{bmatrix} M \end{bmatrix} \left\{ \delta \ddot{u}_{t}^{(i)} \right\} + \begin{bmatrix} C \end{bmatrix} \left\{ \delta \dot{u}_{t}^{(i)} \right\} + \begin{bmatrix} K_{t} \end{bmatrix} \left\{ \delta u_{t}^{(i)} \right\} = \left\{ \delta F_{t}^{(i)} \right\} \quad (10.34)$ where,  $\left\{ \delta \ddot{u}_{t}^{(i)} \right\}$ ,  $\left\{ \delta \dot{u}_{t}^{(i)} \right\}$  and  $\left\{ \delta u_{t}^{(i)} \right\}$  are the corrective accelerations, velocities and displacements, respectively, for the i-th iteration as defined by

$$\begin{cases} \delta \ddot{u}_{t}^{(i)} \\ = \\ \ddot{u}_{t}^{(i+1)} \\ = \\ \dot{u}_{t}^{(i+1)} \\ = \\ \dot{u}_{t}^{(i+1)} \\ = \\ u_{t}^{(i+1)} \\ = \\ u_{t}^{(i+1)} \\ = \\ u_{t}^{(i)} \\ \end{cases}$$
(10.35)

 $m\delta \ddot{u}_{t}^{(i)} + c\delta \dot{u}_{t}^{(i)} + k_{t}\delta u_{t}^{(i)} = \delta f_{t}^{(i)}$ (10.32)30

and where  $\{\delta F_t^{(i)}\}$  represents the unbalance forces for the i-th iteration given as

$$\left\{ \delta F_t^{(i)} \right\} = \left\{ F_t \right\} - \left( [M] \left\{ \ddot{u}_t^{(i)} \right\} + [C] \left\{ \dot{u}_t^{(i)} \right\} + \left\{ F_{Rt}^{(i)} \right\} \right) \quad (10.36)$$

From Eq. (10.13), one obtains  

$$\left\{ \Delta \ddot{u}_{i}^{(i)} \right\} = c_{1} \left\{ \Delta u_{t}^{(i)} \right\} - c_{3} \left\{ \dot{u}_{t}^{(i)} \right\} - c_{4} \left\{ \ddot{u}_{t}^{(i)} \right\}$$
(10.37a  

$$\left\{ \Delta \dot{u}_{t}^{(i)} \right\} = c_{2} \left\{ \Delta u_{t}^{(i)} \right\} - c_{4} \left\{ \dot{u}_{t}^{(i)} \right\} - c_{5} \left\{ \ddot{u}_{t}^{(i)} \right\}$$
(10.37b)

$$\begin{bmatrix} M \end{bmatrix} \left\{ \delta \ddot{u}_{t}^{(i)} \right\} + \begin{bmatrix} C \end{bmatrix} \left\{ \delta \ddot{u}_{t}^{(i)} \right\} + \begin{bmatrix} K_{t} \end{bmatrix} \left\{ \delta u_{t}^{(i)} \right\} = \left\{ \delta F_{t}^{(i)} \right\}$$
(10.34)  
$$\left\{ \Delta \ddot{u}_{t} \right\} = c_{1} \left\{ \Delta u_{t} \right\} - c_{3} \left\{ \dot{u}_{t} \right\} - c_{4} \left\{ \ddot{u}_{t} \right\}$$
(10.13a)  
$$\left\{ \Delta \dot{u}_{t} \right\} = c_{2} \left\{ \Delta u_{t} \right\} - c_{4} \left\{ \dot{u}_{t} \right\} - c_{5} \left\{ \ddot{u}_{t} \right\}$$
(10.13b)  
$$31$$

•Similar to Eqs. (10.14), (10.15), and (10.16), substituting Eq. (10.35) into Eq. (10.34), we obtain  $\left| \widetilde{K}_{t}^{(i)} \right| \delta u_{t}^{(i)} \right| = \left| \delta F_{t}^{(i)} \right|$ (10.38)where,  $\left| \widetilde{K}_{t}^{(i)} \right| = \left| K_{t}^{(i)} \right| + c_{1}[M] + c_{2}[C]$ (10.39) $\left\{ \Delta \widetilde{F}_{t}^{(i)} \right\} = \left\{ \Delta F_{t}^{(i)} \right\} + \left[ c_{3}[M] + c_{4}[C] \right] \left\{ \dot{u}_{t}^{(i)} \right\}$  $+ [c_4[M] + c_5[C]] \{ \ddot{u}_t^{(i)} \}$ (10.40) $\left|\widetilde{K}_{t}\right|\left\{\Delta u_{t}\right\} = \left\{\Delta\widetilde{F}_{t}\right\}$ (10.14)(10.15) $|\widetilde{K}_t| = [K_t] + c_1[M] + c_2[C]$  $\{\Delta \tilde{F}_t\} = \{\Delta F_t\} + [c_3[M] + c_4[C]]\{\dot{u}_t\}$ 32  $+ [c_4[M] + c_5[C]] \{ \ddot{u}_t \}$ (10.16)

If convergence occurs, the iteration can be continued until the dynamic equilibrium of the motion is satisfied within the specified accuracy.
The improved solutions can be obtained from Eq. (10.35).

$$\begin{cases} \delta \ddot{u}_{t}^{(i)} \\ = \left\{ \ddot{u}_{t}^{(i+1)} \right\} - \left\{ \ddot{u}_{t}^{(i)} \right\} \\ \left\{ \delta \ddot{u}_{t}^{(i)} \\ = \left\{ \dot{u}_{t}^{(i+1)} \right\} - \left\{ \dot{u}_{t}^{(i)} \right\} \\ \left\{ \delta u_{t}^{(i)} \\ = \left\{ u_{t}^{(i+1)} \right\} - \left\{ u_{t}^{(i)} \right\} \end{cases}$$
(10.35)

Example 9.9 Compute nonlinear response of the 3 DOFS structure of Example 9.? under JMA Kobe ground acceleration which was recorded during the 1995 Kobe earthquake.

•Mass matrix  $m \ 0 \ 0$  $[M] = \begin{vmatrix} 0 & m & 0 \end{vmatrix}$  $0 \quad 0 \quad m$ •Stiffness matrix  $-k_{1}$ 0  $k_1$  $[K] = \begin{vmatrix} -k_1 & k_1 + k_2 & -k_2 \end{vmatrix}$  $0 - k_2 - k_2 + k_3$  Assume that m = 150 kN / g = 150 / 9.8

m m  $k_1$  m  $k_2$   $k_3$ 

•The restoring forces of three springs are assumed to be perfect elasto-plastic shown below



$$k_1 = k_2 = k_3 = k = 3050.9$$
 kN/m  
 $f_{y1} = f_{y2} = f_{y3} = f_y$ 

 $f_y = 500$ kN

• Eigen value analysis

$$\begin{vmatrix} k_1 - \omega^2 m & -k_1 & 0 \\ -k_1 & k_1 + k_2 - \omega^2 m & -k_2 \\ 0 & -k_2 & k_2 + k_3 - \omega^2 m \end{vmatrix} = 0$$

 $k_1 = k_2 = k_3 = k = 3050.9$  kN/m m = 150/9.8 ton

$$\left\{ \begin{array}{c} \omega_1 \\ \omega_2 \\ \omega_3 \end{array} \right\} = \left\{ \begin{array}{c} 6.28 \text{ rad/s} \\ 17.6 \text{ rad/s} \\ 25.4 \text{ rad/s} \end{array} \right\} \qquad \left\{ \begin{array}{c} T_1 \\ T_2 \\ T_3 \end{array} \right\} = \left\{ \begin{array}{c} 1.00 \text{ s} \\ 0.357 \text{ s} \\ 0.247 \text{ s} \end{array} \right\}$$

•Rayleigh damping is assumed. Based on Eq. (10.25),  $\alpha$  and  $\beta$  can be determined as

$$\begin{cases} \alpha \\ \beta \end{cases} = \frac{2\omega_i \omega_j}{\omega_j^2 - \omega_i^2} \begin{bmatrix} \omega_j & -\omega_i \\ -1/\omega_j & 1/\omega_i \end{bmatrix} \begin{cases} \xi_i \\ \xi_j \end{cases}$$
(10.25)



1st and 2nd mode is selected

$$\xi_1 = \xi_2 = \xi_3 = 0.05$$

 $\begin{cases} \alpha \\ \beta \end{cases} = \frac{2 \times 6.28 \times 17.6}{17.6^2 - 6.28^2} \begin{bmatrix} 17.6 & -6.28 \\ -1/17.6 & 1/6.28 \end{bmatrix} \begin{cases} 0.05 \\ 0.05 \end{cases} = \begin{cases} 0.463 \\ 4.19 \times 10^{-3} \end{cases}$ 

### Rayleigh damping is assumed







#### •response displacement





т  $k_1$ т  $m k_2$  $k_{3}$  $f_{y1} = 200 \text{kN}$  $f_{y2} = 400$ kN

 $f_{y3} = 500 \text{kN}$ 





#### •response displacement



Example 9.11

 $f_{y} = 500 \text{kN}$  $f_{y1} = \frac{2}{5}f_y$  $f_{y2} = \frac{4}{5}f_y$  $f_{y3} = \frac{3}{5}f_{y}$ 



$$f_{y1} = 200$$
kN  
 $f_{y2} = 400$ kN  
 $f_{y3} = 300$ kN



#### •response displacement



#### •response displacement



## 11. 動的解析を実際にやってみよう

## 1) DYMOとは

- ●「橋の動的耐震設計法マニュアル」((財)土木研究センター) に付属する動的解析体験版ソフトウェア
- ●1基の各部構造とそれが支持する上部構造部分に対して、簡単に動的解析を体験することができる。
- ●プログラムは土木研究センターのホームページ (http://www.pwrc.or.jp/)から無料でダウンロード可能

 ●参考資料として、「橋の動的耐震設計法マニュアルー動的解 析および耐震設計の基礎と応用」が土木研究センター(電話:3 835 - 3609, e-mail: <u>kikaku@pwrc.or.jp</u>)から販売されてい るので、適宜、購入すると参考になる。



#### お知らせ (最終更新日:2008年5月1日)

2007.11.27「橋梁の免震設計に関する講習会」(2007.11.27 終了)「Q&Aコーナー」 2007.10.17「建設技術審査証明 第6回技術報告会」(2007.10.17 終了) 2007.9.5 「技術の紹介」

・鋼橋の長寿命化を支える塗替え工法(ブラストを用いた局部補修)

🕝 インターネット



)土木研究センター/フリーソフト&マニュアルのご紹介





🥑 インターネット



り土木研究センター/動的解析体験版ソフトウエア"DYMO"のご紹介

#### 動的解析体験版ソフトウェア DYMO のご紹介

川島一彦東京工業大学教授を委員長とする「橋の動的耐震設計法マ ニュアル検討委員会」において、簡単な鉄筋コンクリート橋脚を対象とし て動的解析が可能な「動的解析体験版ソフウェア"DYMO"」を作成致しま した。「橋の動的耐震設計法マニュアル」に連動した内容となっておりま すので、ソフトウェアをダウンロードの上起動して、マニュアルを片手に橋 の動的解析を体験してみてください。

今後、DYMOおよび「橋の動的耐震設計法マニュアル」が橋梁の耐震 設計に関係する技術者に有効に活用されることを期待致します。



ージが表示されました



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2) 動的解析結果のディジタルデータを得るためには
2) How can we get digital data of the response?

●DYMOをインストールしたフォルダに"SpecialDYMO.ini" というファイルを作る。空ファイルで良い。

●この空ファイルが作られていれば、「非線形動的解析」ー >「安全性の照査」画面の「結果のファイル出力」ボタンが 出てくる。

●「結果のファイル出力」ボタンをクリックすると、今開いて いるデータファイルのあるフォルダーに"OutType1.csv"(タ イプ 地震動の結果)と"OutType2.csv"(タイプ 地震動 の結果)というファイルが出力される。



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3) DYMOを使った解析
 3) Analysis using DYMO

●計算例2を読み込む

●構造条件の入力ーほとんどはディフォルトデータでよい。ただし、支承条件では、"固定支承"にする

●固有振動解析の中では、以下の点を試してみる。

✓固有周期、固有振動モード、刺激係数の理解

✓減衰の設定では、どの固有振動モードに着目するかを 変えてみる。、の値もチェックしてみる。

●入力地震動として、道路橋示方書の参考資料に示されている標準波形がすぐ読み込めるようになっているので、このタイプとタイプ地震動を使用する。

●タイプ 地震動とは、加速度応答スペクトルで約1gの地震 動、タイプ 地震動とは加速度応答スペクトルで約2gの加速 度である。ただし、地盤種別で異なる。

●"安全性の照査"の中に"結果のファイル出力"ボタンを押す と、応答値に関するエクセルデータが得られる。これをもとに、 自分で、応答波形をプロットしてみる。ただし、動的解析結果の ディジタルデータを得るためには、p.8の対応が必要である。

●応答波形をまとめて、課題として提出する(提出:6月4日)。