Extensive Form Trembling-hand Perfect Eq.

Agent normal form of an extensive form game :

For each player, each of his information sets is a player.

Their payoffs are the same as the player's payoff.

EFTHPE in an extensive form

= NFTHPE in its agent normal form

NFTHPE in a normal form game $\Gamma = (N=\{0,1,\ldots,I\}, \{\Delta(S_i)\}, \{u_i\})$ is

a Nash eq. σ satisfying the following:

$\exists$ a sequence of perturbed games $\{\Gamma_{\varepsilon k}\}_{k=1}^{\infty}$ that converges to

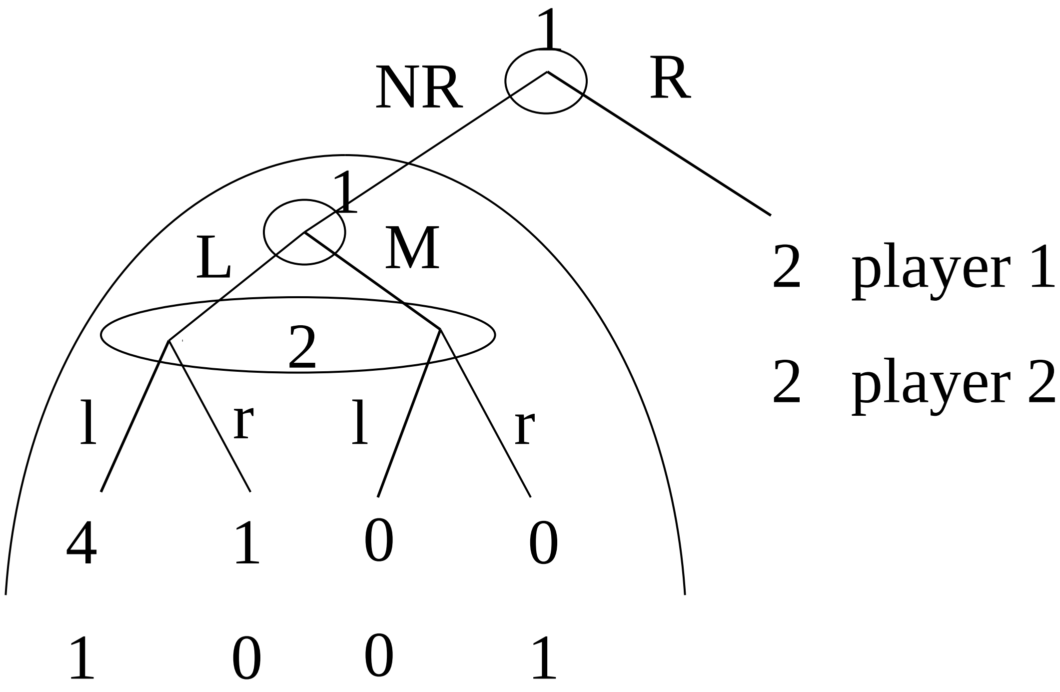
Γ, i.e., $\lim_{k\to\infty} \varepsilon^k{}_i(s_i) = 0 \ \forall s_i \in S_{i,}$, for which

$\exists$ a sequence of Nash eq. $\{\sigma^k\}_{k=1}^{\infty}$ such that $\lim_{k\to\infty} \sigma^k = \sigma$.

$$\Gamma_{\varepsilon k} = [N=\{0,1,\ldots,I\}, \{\Delta_{\varepsilon k}(S_i)\}, \{u_i\}]$$

$$\Delta_{\varepsilon k}(S_i) = \{\sigma_i \mid \sigma_i(s_i) \geq \varepsilon_i(s_i) \ \ \forall\ s_i \in S_i \ \text{ and } \Sigma_{si\ \in Si}\ \sigma_i(s_i) = 1\}$$

Extensive Form Trembling-hand Perfect Eq.



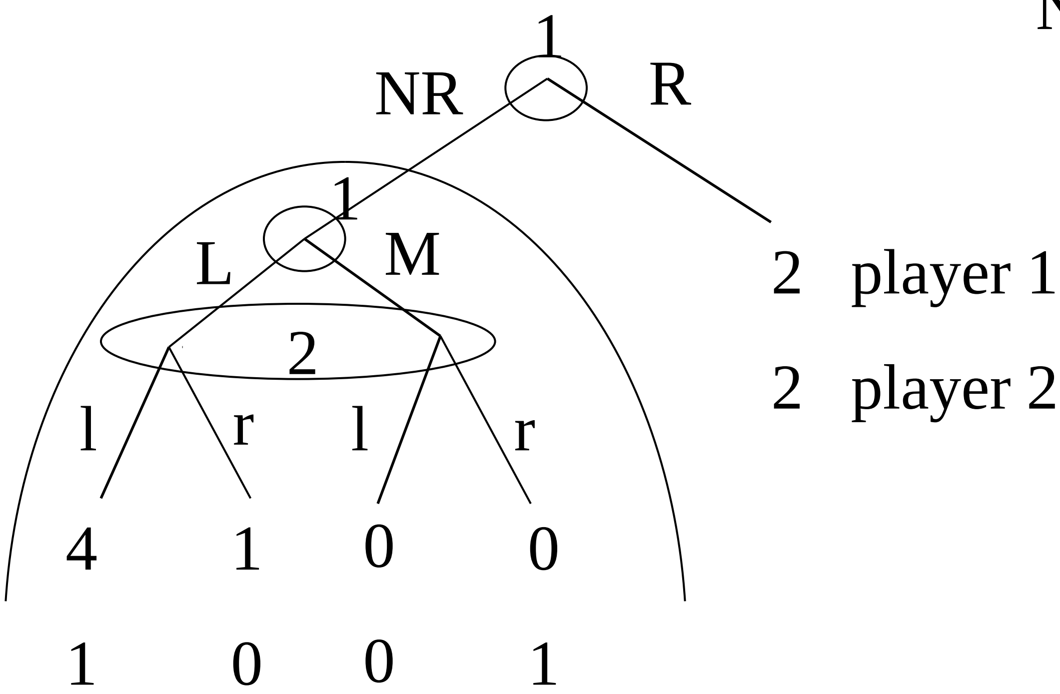
Subgame

2 \ 1	l	r
L	4, 1	1, 0
M	0, 0	0, 1

Nash eq. (L, l) (unique)

SPNE ((NR-L), l)

Extensive Form Trembling-hand Perfect Eq.



Normal form game

2 \ 1	l	r
NR-L	4, 1	1, 0
NR-M	0, 0	0, 1
R-L	2, 2	2, 2
R-M	2, 2	2, 2

Nash eq. (NR-L, l) (R-L, r), (R-M, r)

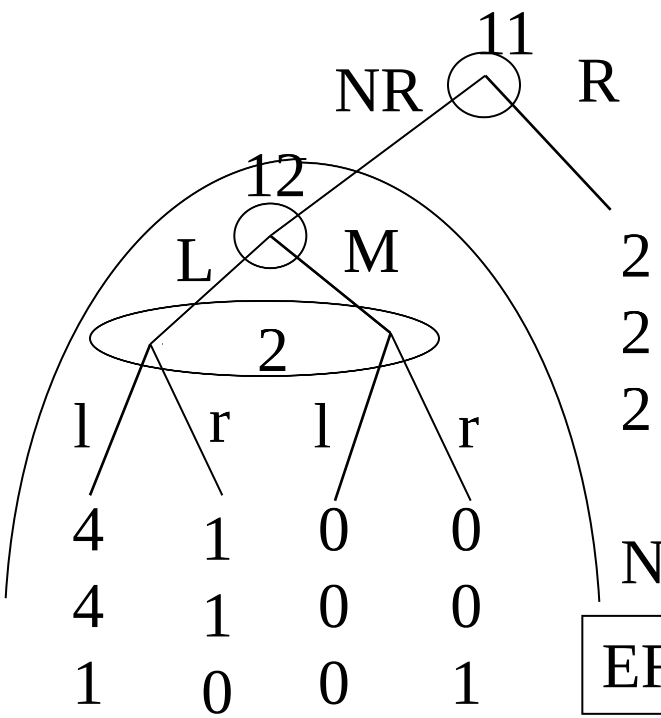
NFTHPE (NR-L, l) (R-L, r), (R-M, r)

$\varepsilon^k = (1/k^2, 1/k, 1/k, 1/k)$, $\sigma^k = (1/k^2, 1/k, 1-(1/k^2+2/k), 1/k)$

$\lim_{k\to\infty} \sigma^k$ = R-L and r is b.r. to σ^k $\rightarrow$ (R-L, r) NETHPE

similar for (R-M, r)

Extensive Form Trembling-hand Perfect Eq.



Agent normal form

11 plays NR

2 \ 12	l	r
L	4,4,1	1,1,0
M	0,0,0	0,0,1

11 plays R

2 \ 12	l	r
L	2,2,2	2,2,2
M	2,2,2	2,2,2

Nash eq. (NR,L, l), (R,L, r), (R,M, l), (R,M, r)

EFTHPE (NR,L, l)

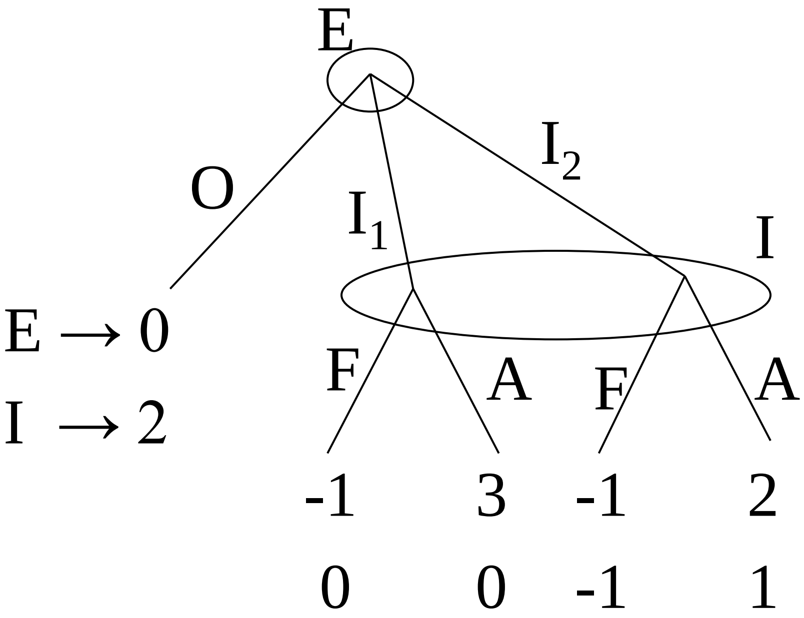
(R, L, r) : 2's payoff

(ε, 1-ε), (1-ε', ε'), (ε'', 1-ε'') → ε((1-ε')ε'' + ε'(1-ε'')) + 2(1-ε)

(1-ε'', ε'') → ε((1-ε')(1-ε'') + ε'ε'') + 2(1-ε) ↑

similar for (R, M, l) and (R, M, r)

Extensive Form Trembling-hand Perfect Eq.



Player E chooses the row; player I chooses the column.

	F	A
O	$\underline{0}$, $\underline{2}$	0, $\underline{2}$
I_1	-1, $\underline{0}$	$\underline{3}$, $\underline{0}$
I_2	-1, -1	2, $\underline{1}$

Nash eq (SPNE) →(O, F), (I_1, A)

<u>A weakly dominates F</u>

((O,F), (1,0)) SE

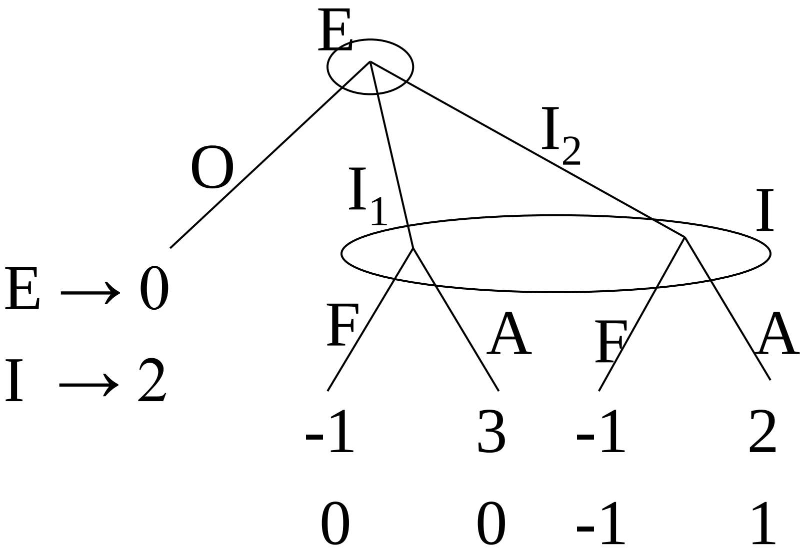
F is sequentially rational given (1,0)

Let $\sigma^k_E = (1-(1/k+1/k^2), 1/k, 1/k^2)$. Then $\lim_{k\to\infty} \sigma^k_E = (1,0,0) =$ O

Furthermore belief is $((1/k)/(1/k+1/k^2), (1/k^2)/(1/k+1/k^2))$

$= (k/(k+1), 1/(k+1)) \to (1,0)$ (as $k \to \infty$)

Extensive Form Trembling-hand Perfect Eq.



		I: F	I: A
E	O	0, 2	0, 2
	I_1	-1, 0	3, 0
	I_2	-1, -1	2, 1

Nash eq (SPNE) $\rightarrow$(O, F), (I_1, A)

A weakly dominates F

((O,F), (1,0)) SE but not EFTHPE

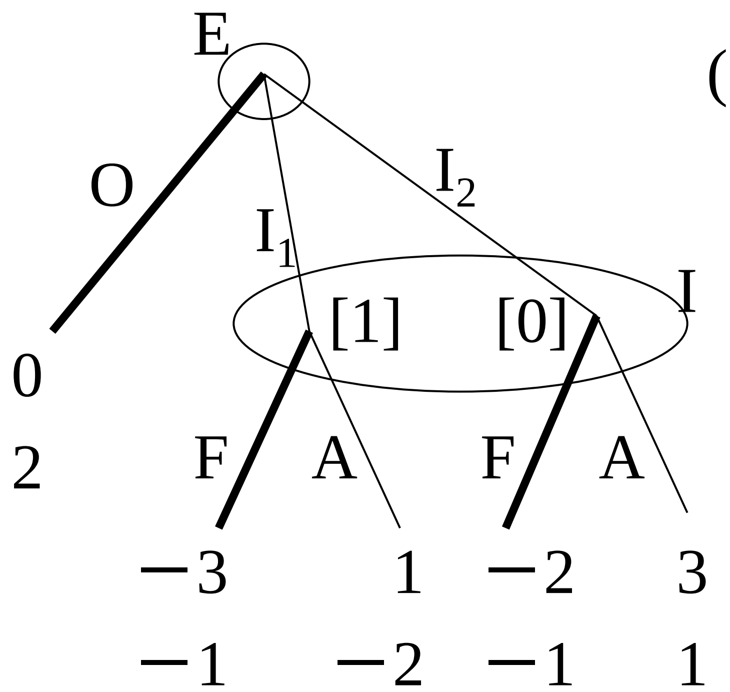
Let E's strategy (1-ε-ε', ε, ε'). Then

I's payoffs $\rightarrow$ F: 2(1-ε-ε')+0ε+(-1)ε'=2-2ε-3ε'

A: 2(1-ε-ε')+0ε+1ε'=2-2ε-1ε' > payoff under F

EFTHPE $\rightarrow$ SE

Forward Induction (motivation)



((O, F) , (1, 0)) → WPBE

SE, EFTHPE

I_2 strictly dominates I_1

$-3 < -2, \ 1 < 3$

E is rational

→ never choose I_1

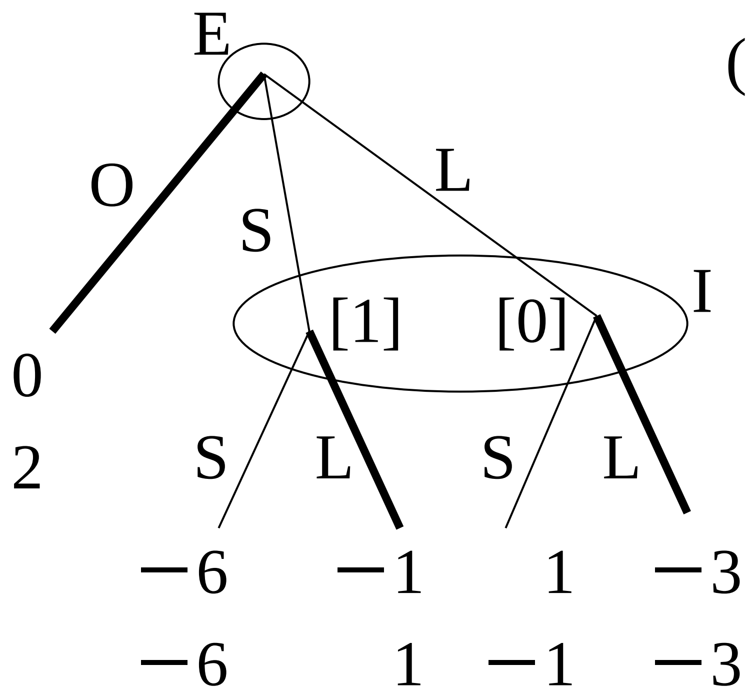
(choose I_2)

belief (1, 0) → ???

---

another ((I_2 , A), (0, 1)) OK !

Forward Induction (motivation)



((O, L) , (1, 0)) → WPBE

SE, EFTHPE

O strictly dominates S

$0 > -6, \ -1$

E is rational

→ never choose S

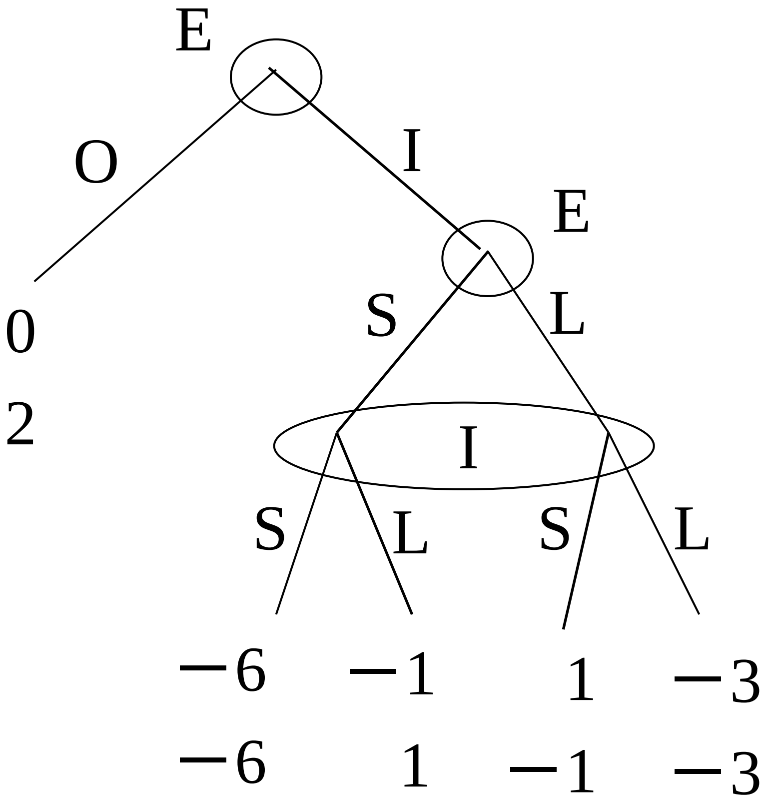
choose O

belief (1, 0) → ???

---

another ((L, S), (0, 1)) OK !

Forward Induction (motivation)



		I: S	I: L
E	S	−6, −6	−1, 1
	L	1, −1	−3, −3

Nash eq. (L, S), (S, L)

O strictly dominates S

$0 > -6, -1$

E never chooses I – S

Can eliminate a Nash eq. (S, L)

Bilateral Bargaining

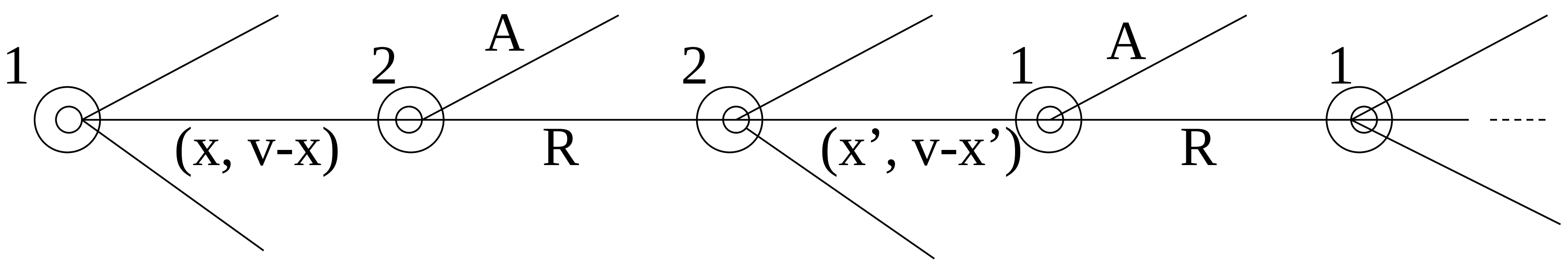
Players 1, 2 determine the split of v

Player 1 makes an offer of a split $(x,\ v-x)$ $(0 \leq x \leq v)$

Player 2 "accepts" $\rightarrow$ 1 gets x; 2 gets $v-x$

or "rejects" $\rightarrow$ 2 makes an offer of a split

....

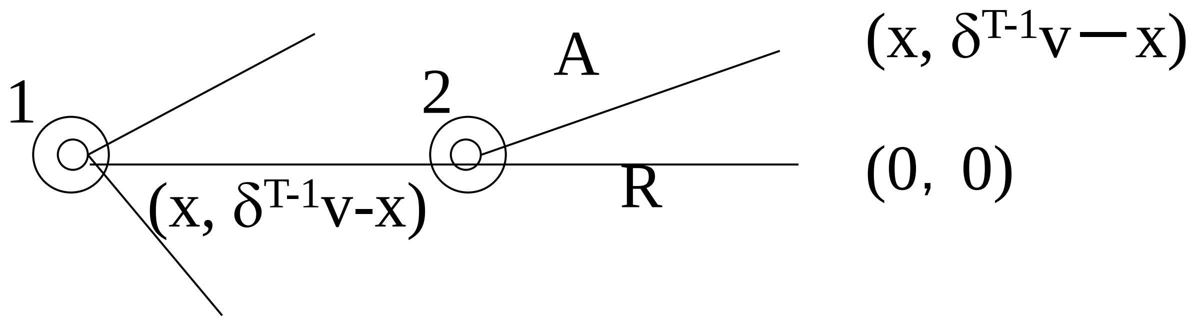


SPE ?

A : accept, R : reject

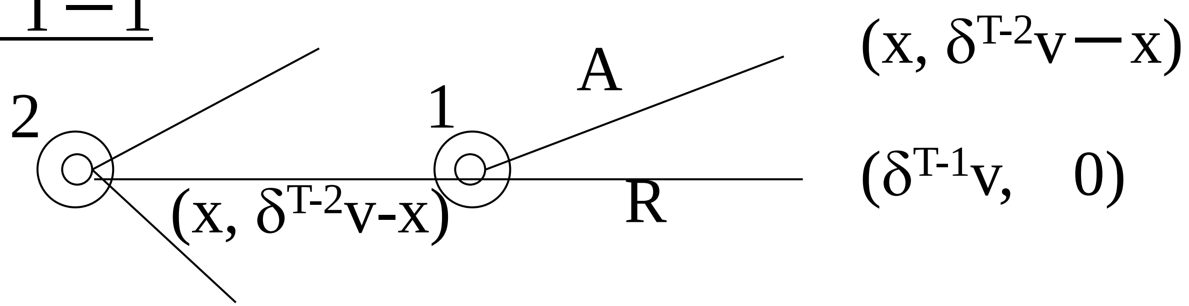
Finite Horizon (T (odd) periods)

Period T



Unique SPE → $((\delta^{T-1}v, 0), A)$ discounted payoffs $(\delta^{T-1}v, 0)$

Period T − 1



Unique SPE → $((\delta^{T-1}v, \delta^{T-2}v - \delta^{T-1}v), A)$

discounted payoffs $(\delta^{T-1}v, \delta^{T-2}v - \delta^{T-1}v)$

Finite Horizon (T (odd) periods)

Period T

Unique SPE → $((\delta^{T-1}v, 0), A)$ discounted payoffs $(\delta^{T-1}v, 0)$

Period T − 1

Unique SPE → $((\delta^{T-1}v, \delta^{T-2}v - \delta^{T-1}v), A)$

discounted payoffs $(\delta^{T-1}v, \delta^{T-2}v - \delta^{T-1}v)$

Period T − 2

Unique SPE → $((\delta^{T-3}v - \delta^{T-2}v + \delta^{T-1}v, \delta^{T-2}v - \delta^{T-1}v), A)$

discounted payoffs $((\delta^{T-3}v - \delta^{T-2}v + \delta^{T-1}v, \delta^{T-2}v - \delta^{T-1}v)$

Period 1

Unique SPE → $(v - \delta v + \delta^2 v - \cdots + \delta^{T-1}v, \delta v - \delta^2 v + \cdots - \delta^{T-1}v), A)$

Discounted payoffs $(v - \delta v + \delta^2 v - \cdots + \delta^{T-1}v, \delta v - \delta^2 v + \cdots - \delta^{T-1}v)$

Finite Horizon (T (odd) periods)

Period 1

Unique SPE → $(v - \delta v + \delta^2 v - \cdots + \delta^{T-1} v,\ \delta v - \delta^2 v + \cdots - \delta^{T-1} v),\ A)$

Discounted payoffs $(v - \delta v + \delta^2 v - \cdots + \delta^{T-1} v,\ \delta v - \delta^2 v + \cdots - \delta^{T-1} v)$

1's payoff $= v(1 - \delta + \delta^2 - \cdots + \delta^{T-1}) = v((1 - (-\delta)^T) / (1+\delta)$

$= v(1+\delta^T) / (1+\delta) = v^*_1(T)$

$\rightarrow\ v / (1+\delta)$ (as $T \rightarrow \infty$)

2's payoff $= v(1 - (1+\delta^T) / (1+\delta)) = v(\delta - \delta^T) / (1+\delta)$

$= v^*_2(T) = v - v^*_1(T)$

$\rightarrow\ v\,\delta / (1+\delta)$ (as $T \rightarrow \infty$)

Finite Horizon (T (<u>even</u>) periods)

<u>Period 1</u>

Unique SPE → $(v - \delta v^*_1(T\text{-}1),\ \delta v^*_1(T\text{-}1)),\ A)$

Discounted payoffs $(v - \delta v^*_1(T\text{-}1),\ \delta v^*_1(T\text{-}1))$

1's payoff

$$v - \delta v(1+\delta^{T-1}) / (1+\delta) = v(1-\delta^T) / (1+\delta)$$

$$\rightarrow v / (1+\delta) \quad (\text{as } T \rightarrow \infty)$$

2's payoff

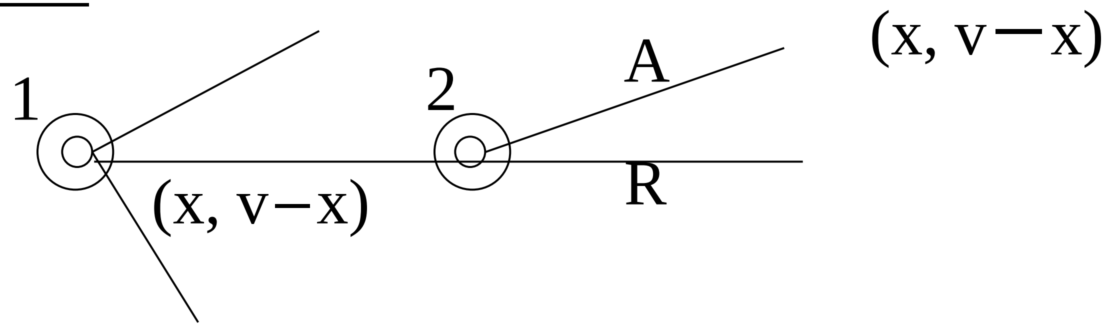
$$\delta v(1+\delta^{T-1}) / (1+\delta) = v(\delta+\delta^T) / (1+\delta)$$

$$\rightarrow \delta v / (1+\delta) \quad (\text{as } T \rightarrow \infty)$$

Infinite Horizon

Stationary SPNE

Period 1



v^+_1 = max payoff to 1 in any SPNE

2 can gain at most δv^+_1 if he rejects

2 will accept if he gets (more than or equal to) δv^+_1

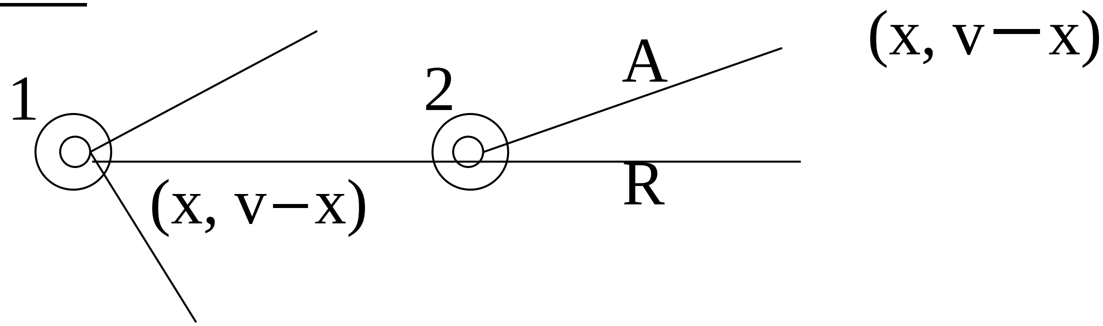
1gets at least $v - \delta v^+_1$

v^-_1= min payoff to 1 in any SPNE

$\rightarrow \; v^-_1 = v - \delta v^+_1$

Infinite Horizon

Period 1



$v^-_1 = v - \delta v^+_1$

Show $v^+_1 \le v - \delta v^-_1$

2 can gain at least δv^-_1 if he rejects

2 will reject if he gets less than δv^-_1

1 gets at most $\underline{v - \delta v^-_1}$ when 2 accepts his offer

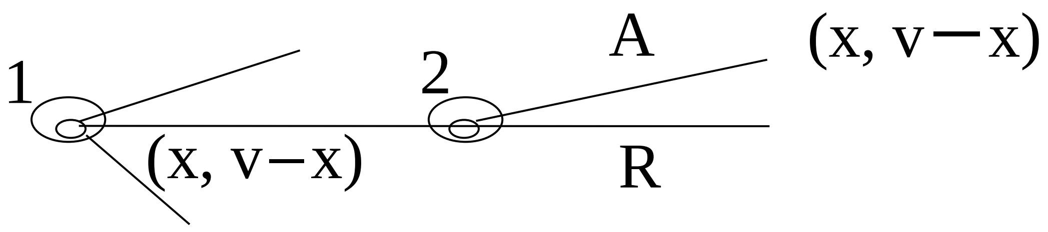
When 2 rejects, 2 gains at least δv^-_1 in period 2.

→ 1 can gain at most $\underline{\delta v - \delta v^-_1 \ (< v - \delta v^-_1)}$

Thus $v^+_1 \le v - \delta v^-_1$

Infinite Horizon

<u>Period 1</u>



$v^-_1 = v - \delta v^+_1 \qquad v^+_1 \leq v - \delta v^-_1$

$v^+_1 \leq v - \delta v^-_1 = v^-_1 + \delta v^+_1 - \delta v^-_1$

$(1-\delta)v^+_1 \leq (1-\delta)v^-_1 \;\rightarrow\; v^+_1 \leq v^-_1 \;\rightarrow\; v^+_1 = v^-_1 = v^0_1$

$v^0_1 = v - \delta v^0_1 \;\rightarrow\; v^0_1 = v/(1+\delta)$

$v^0_2 = v - v^0_1 = v - v/(1+\delta) = \delta v/(1+\delta)$

<u>SPNE</u> → a player making an offer offers $\delta v/(1+\delta)$

a player accepts an offer iff the offer $\geq \delta v/(1+\delta)$

(payoffs in finite horizon when $T \rightarrow \infty$)

Assignments

Problem Set 8 (due July 8)

Exercise 9.B.7 (p.302)

Reading Assignment:

Text, Chapter 6, pp.167-183