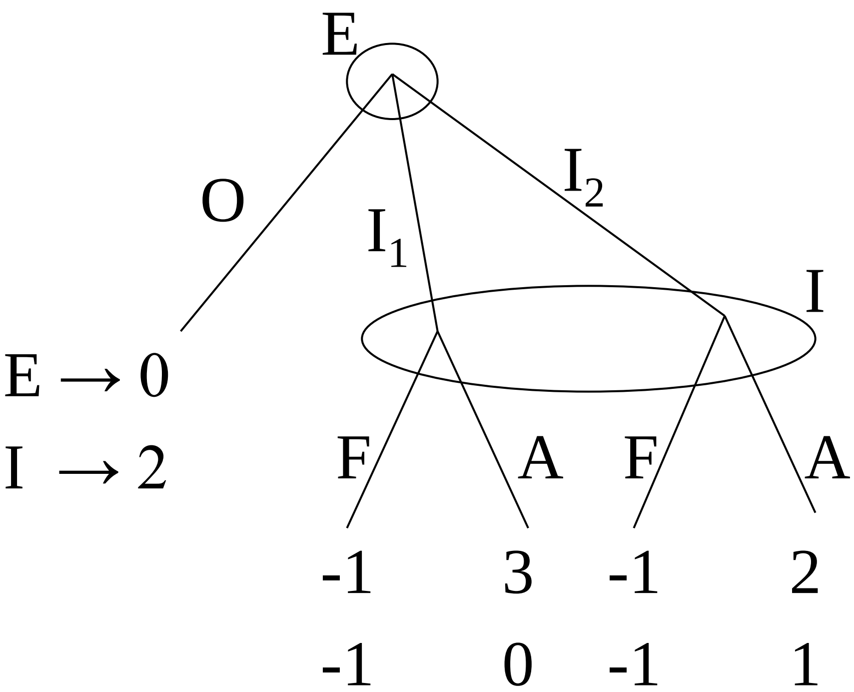
Weak Perfect Bayesian Nash Equilibrium (motivation)



E \ I	F	A
O	0, 2	0, 2
I_1	-1, -1	3, 0
I_2	-1, -1	2, 1

Nash eq (SPNE)

→ (O, F), (I_1, A)

For I : in either decision point, A > F ($-1 < 0$, $-1 < 1$)

→ I should play "A".

→ introduce "belief"

Weak Perfect Bayesian Nash Eq (definition)

Def. 9.C.1: $\mu = (\mu(x))_{x \in X}$ is a system of beliefs (X: set of all nodes)

if $\sum_{x \in H} \mu(x) = 1 \quad \forall$ information set H

Def. 9.C.2: $\sigma = (\sigma_1, \ldots, \sigma_I)$ is sequentially rational at H given μ

if $E(u_{i(H)} \mid H, \mu, \sigma_{i(H)}, \sigma_{-i(H)}) \geq E(u_{i(H)} \mid H, \mu, \hat{\sigma}_{i(H)}, \sigma_{-i(H)})$

$\forall \hat{\sigma}_{i(H)} \in \Delta(S_{i(H)})$ (i(H) : the player who moves at H)

$E(u_{i(H)} \mid H, \mu, \sigma_{i(H)}, \sigma_{-i(H)})$: expected payoff to i(H) from H

if he/she is in H according to the prob. given by μ

and he/she plays $\sigma_{i(H)}$, and rivals play $\sigma_{-i(H)}$.

$\sigma = (\sigma_1, \ldots, \sigma_I)$ is sequentially rational given μ

if $\forall$ H, $\sigma = (\sigma_1, \ldots, \sigma_I)$ is sequential rational at H given μ

Weak Perfect Bayesian Nash Eq (definition)

Def. 9.C.3.: (σ, μ) is a weak perfect Bayesian Eq (WPBE) if

(i) σ is sequential rational given μ

(ii) μ is derived from σ by Bayes' rule if possible, i.e.,

$\forall$ H such that Prob(H | σ) > 0

$\mu(x) = \text{Prob}(x \mid \sigma) / \text{Prob}(H \mid \sigma) \ \forall x \in H$

WPBE and Nash Equilibrium

Prop. 9.C.1: σ is a Nash Equilibrium

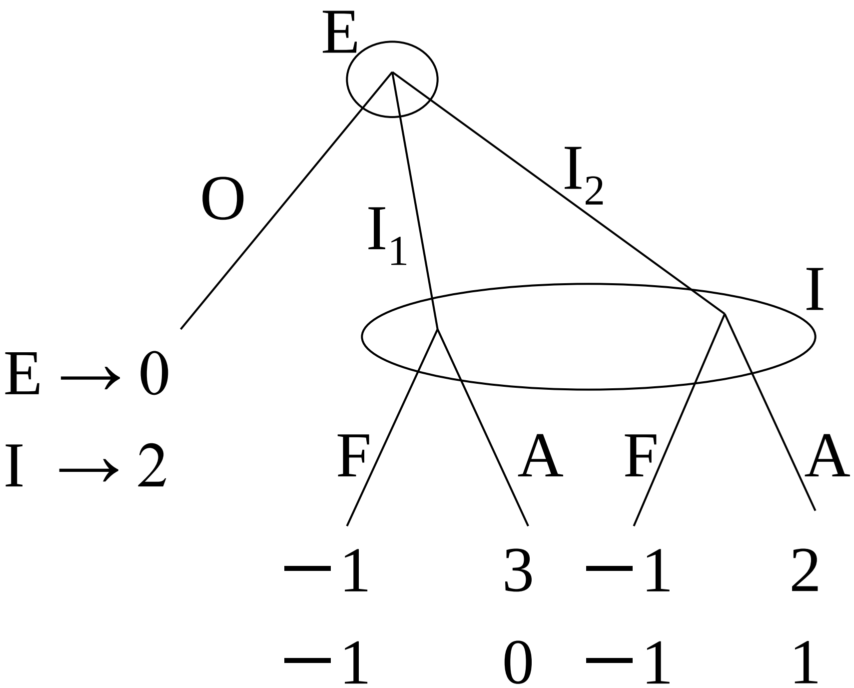
$\Leftrightarrow$ $\exists$ μ such that

(i) σ is sequentially rational given μ at H with $\text{Prob}(H \mid \sigma) > 0$.

(ii) μ is derived from σ by Bayes' rule whenever possible.

Cor.: (σ, μ) is a WPBE $\rightarrow$ σ is a Nash Equilibrium

WPBE in Ex.9.C.1



Nash eq (SPNE)

$\to$ (O, F), (I_1, A)

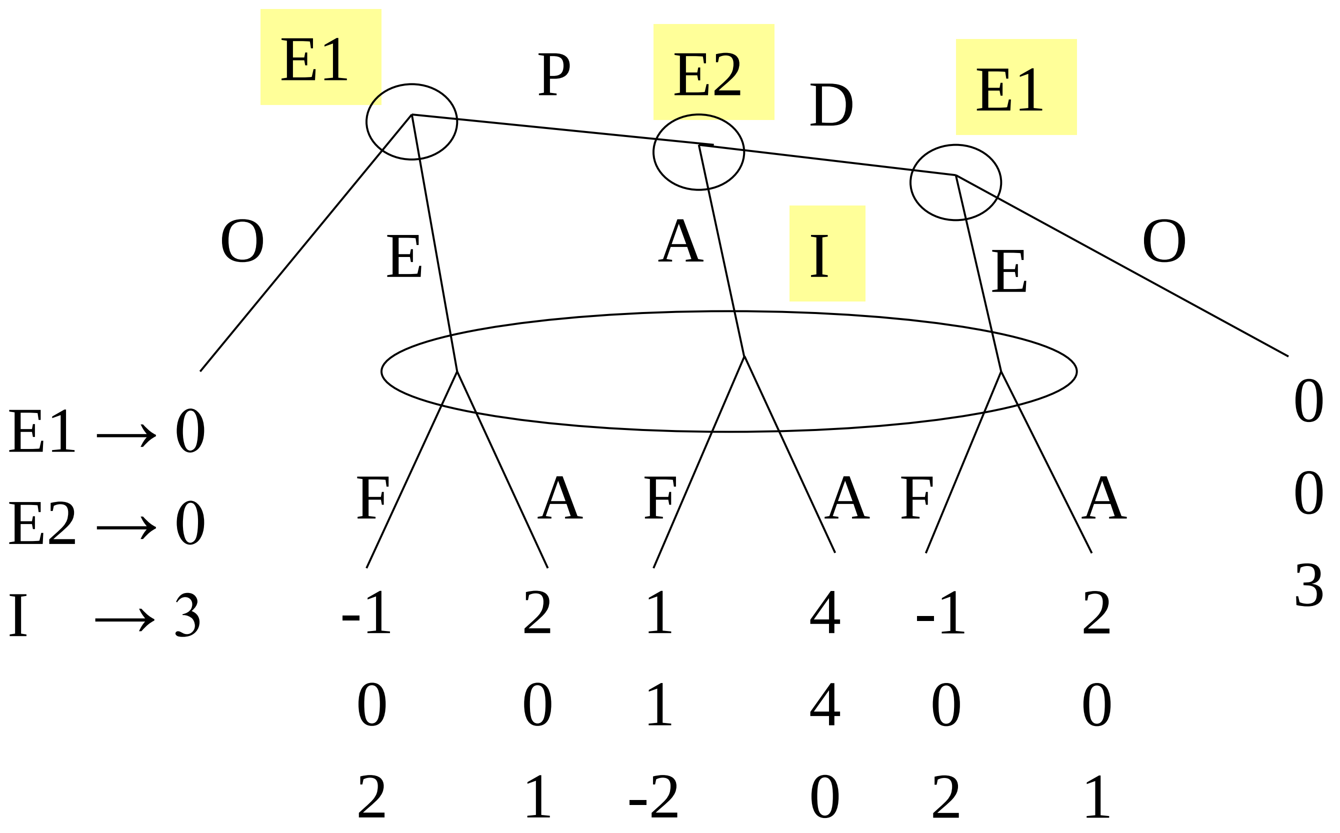
"F" is <u>not</u> sequentially rational

for any belief

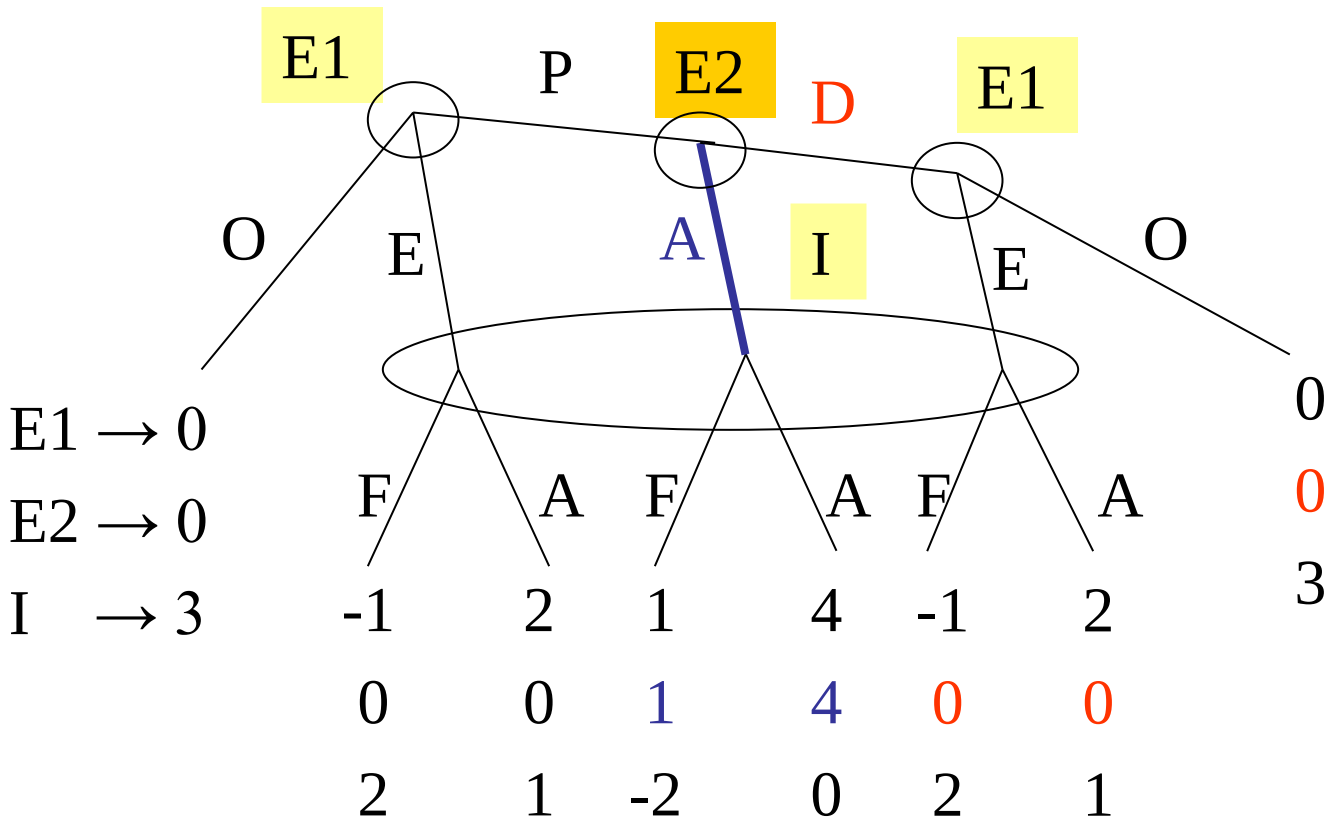
$-1 < 0$, $-1 < 1$

WPBE $\to$ ((I_1, A), $\mu = (1,0)$)

WPBE in Ex.9.C.2

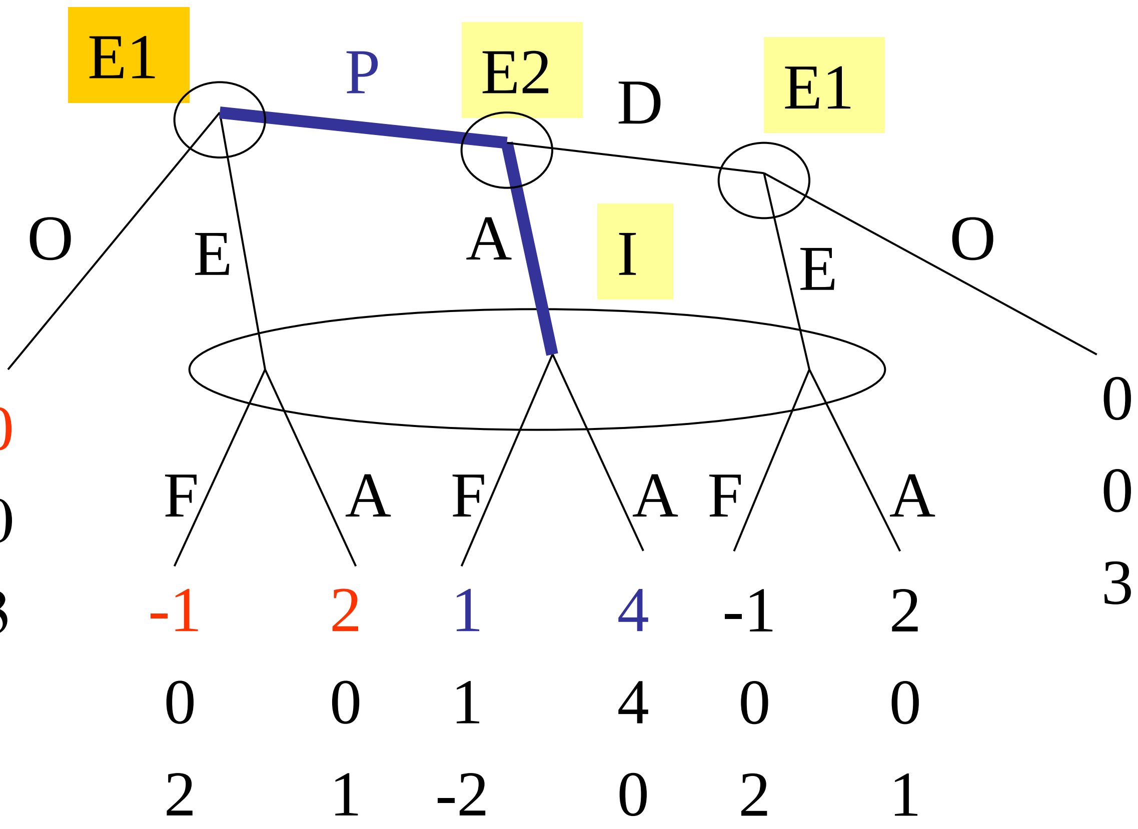


WPBE in Ex.9.C.2



E2 plays "A" since 1, 4 > 0

WPBE in Ex.9.C.2



E1 → 0

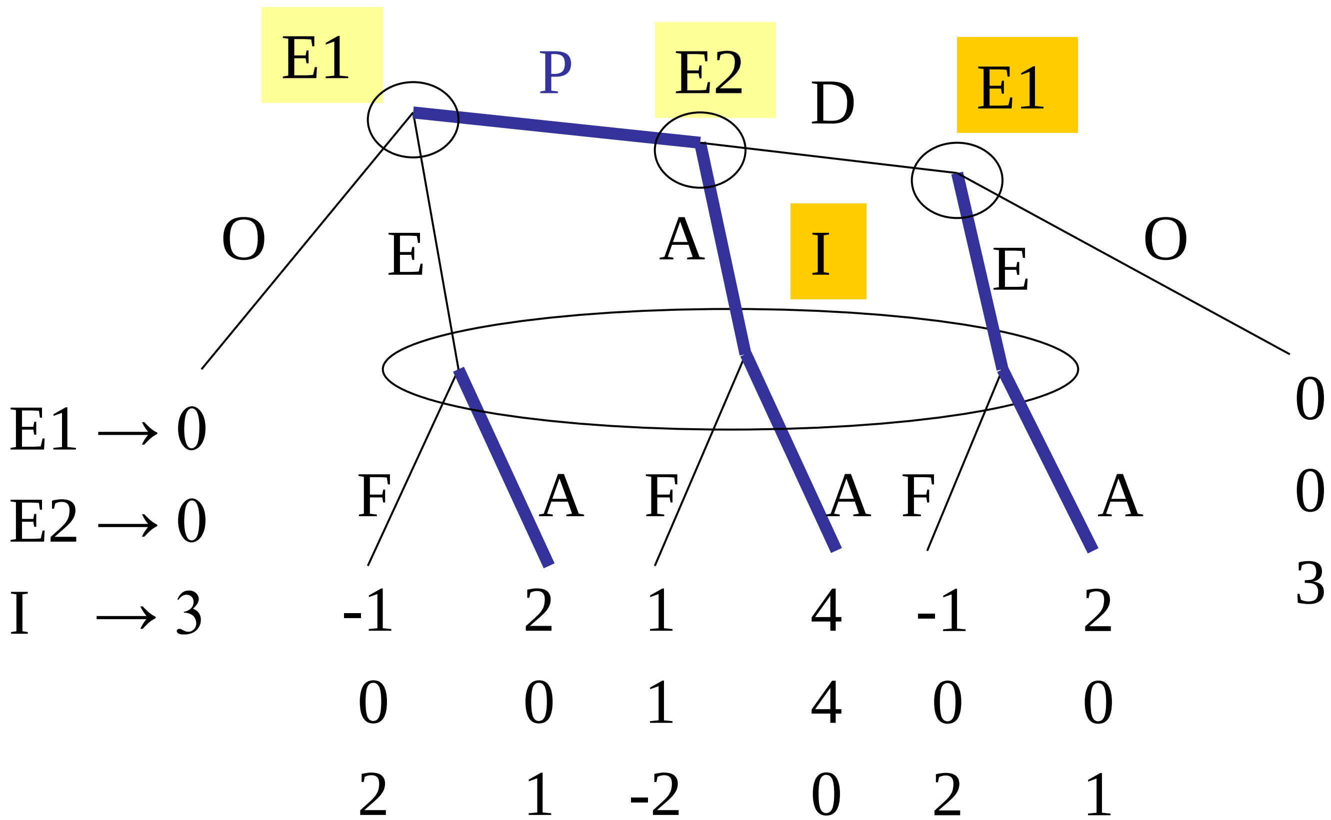
E2 → 0

I → 3

E1 plays "P" since 4 > 2, 1 > -1 → P > E

4, 1 > 0 → P > O

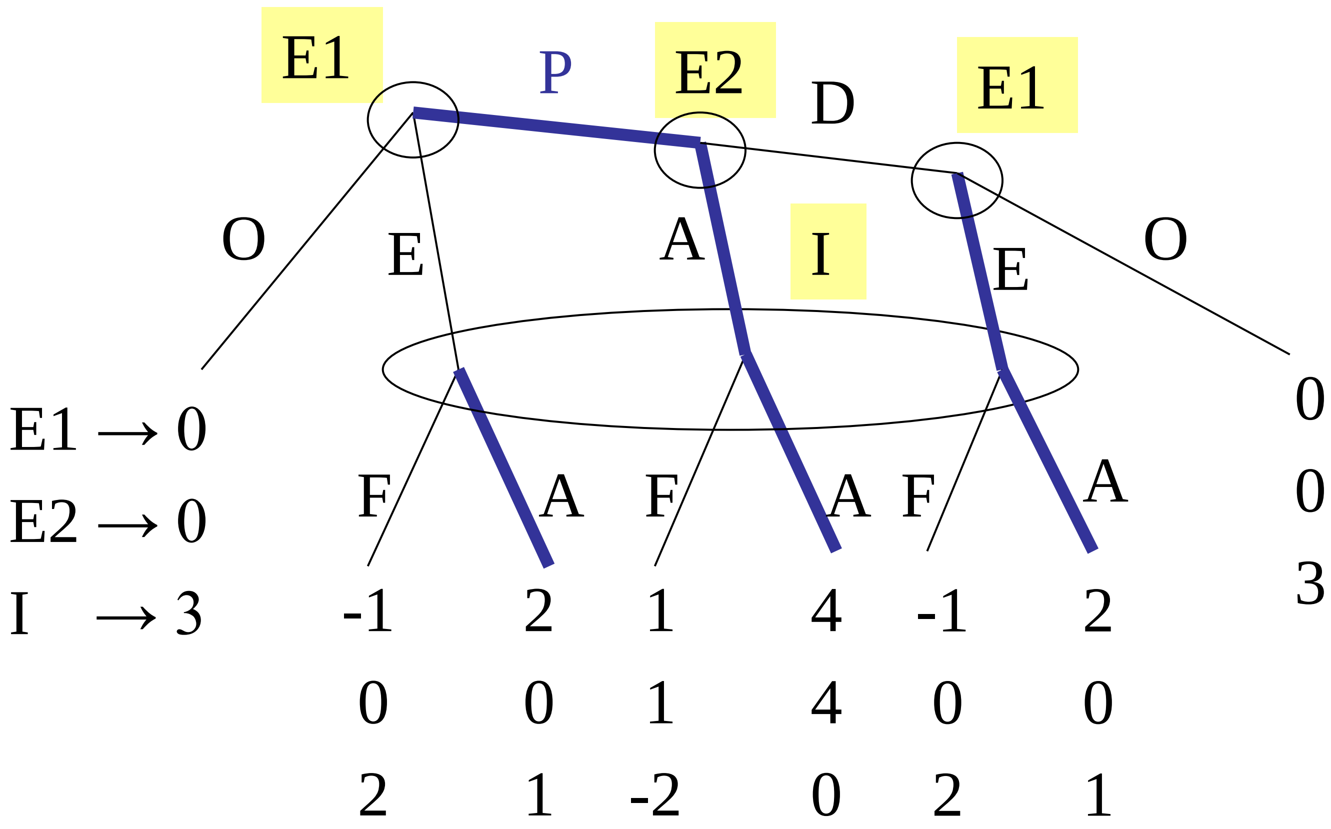
WPBE in Ex.9.C.2



I's belief (0, 1, 0) → I plays "A" since 0 > -2

Then E1 plays "E" since 2 > 0.

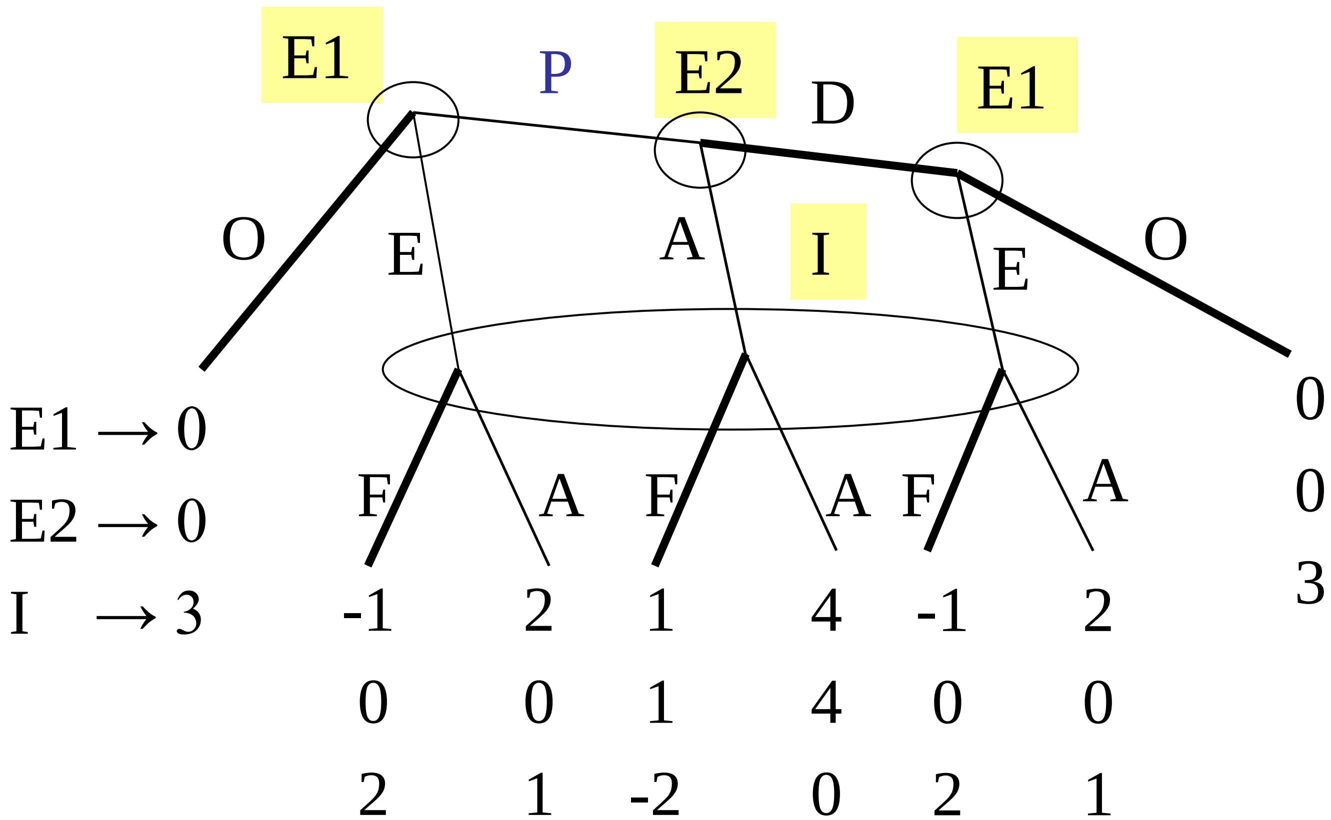
WPBE in Ex.9.C.2



WPBE : ((P, E), (A), (A), (0, 1, 0))

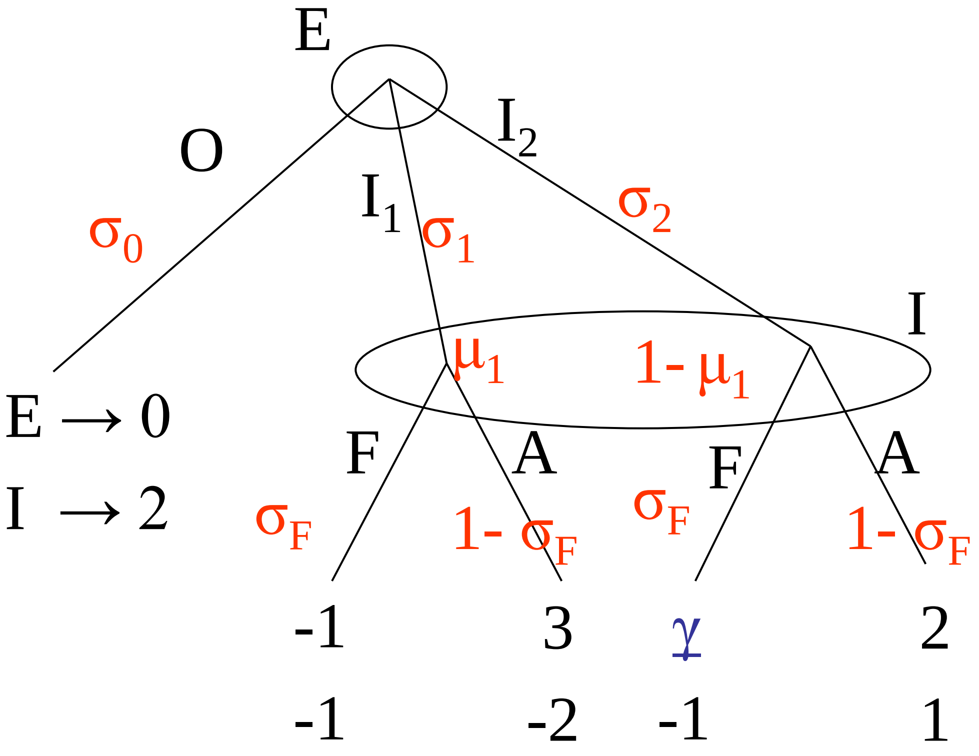
Note: ((O, O), (D), (F)) Nash eq. (SPNE)

WPBE in Ex.9.C.2



((O, O), (D), (F)) Nash eq. (SPNE)

WPBE in Ex.9.C.3



$\gamma \geq 0$

($-1 < \gamma < 0 \to$ Ex.9.C.2)

E's strategy: $(\sigma_0, \sigma_1, \sigma_2)$

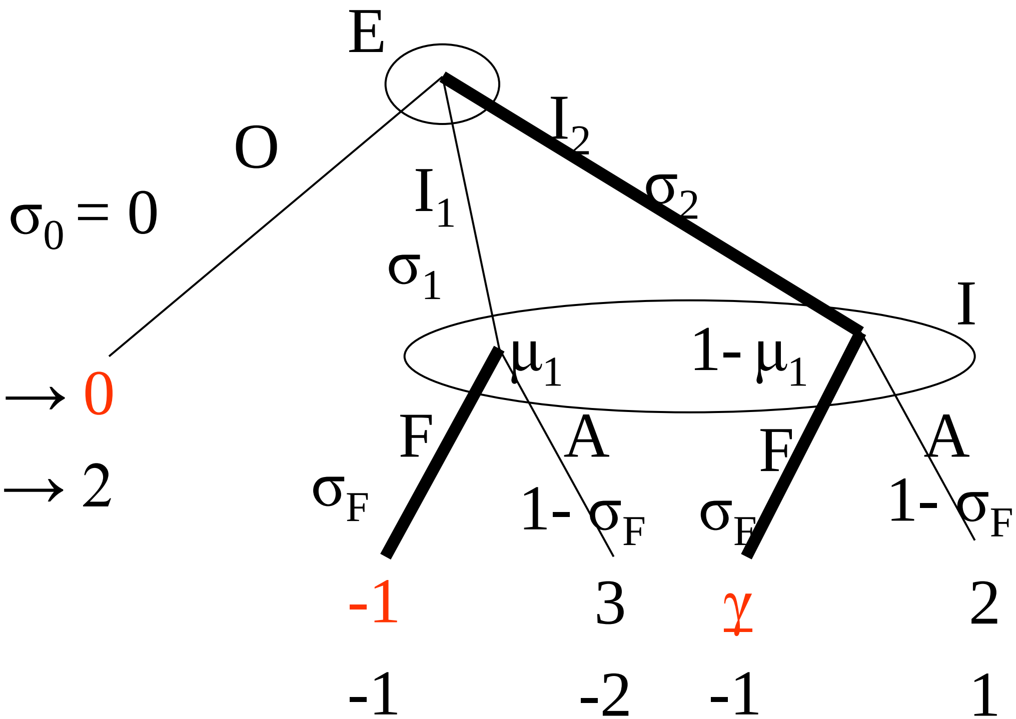
I's strategy: $(\sigma_F,\ 1- \sigma_F)$

I's belief: $(\mu_1,\ 1- \mu_1)$

I_2 dominates O $\to$ $\sigma_0 = 0$

	F	A
O	0, 2	0, 2
I_1	-1, -1	3, -2
I_2	γ, 1	2, 1

WPBE in Ex.9.C.3



E → 0

I → 2

<u>$\gamma \geq 0$</u>

$\mu_1 > 2/3 \rightarrow$ F
$\mu_1 < 2/3 \rightarrow$ A
$\mu_1 = 2/3 \rightarrow$ F or A

$F \rightarrow -1$

$A \rightarrow -2\mu_1 + (+1)(1-\mu_1)$

$= 1-3\mu_1$

$-1 > 1-3\mu_1 \leftrightarrow \mu_1 > 2/3$

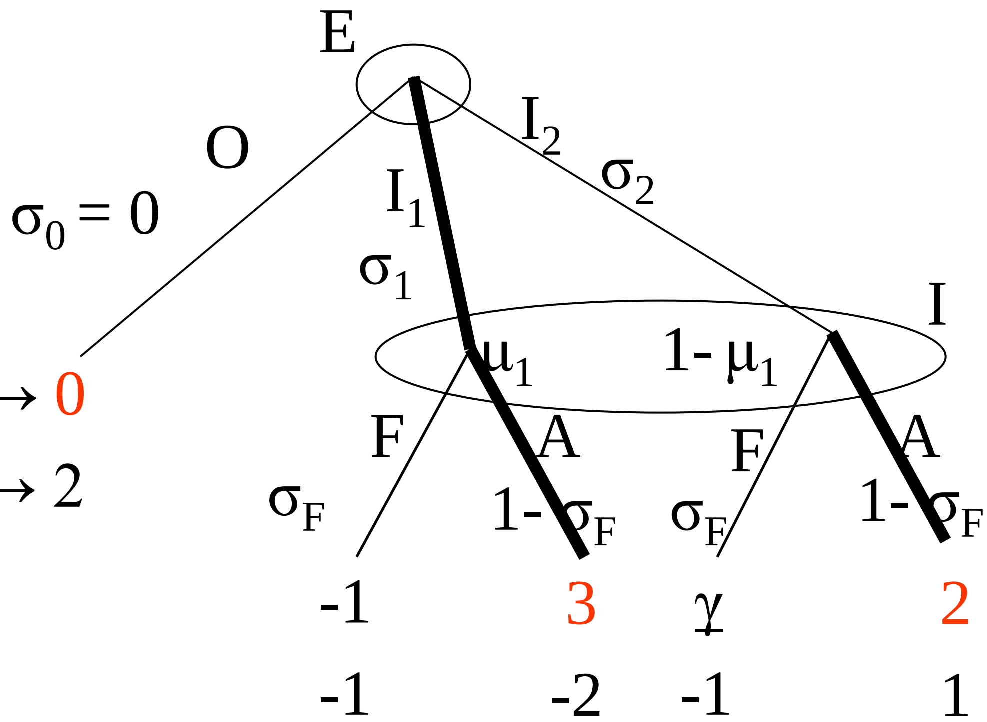
<u>$\mu_1 \geq 2/3$</u>

I plays F ($\sigma_F = 1$)

→ E plays I_2 since $\gamma > 0 > -1$

→ $\mu = (0, 1)$ C! to $\mu_1 > 2/3$

WPBE in Ex.9.C.3



E → 0

I → 2

$\underline{\gamma} \geq 0$

$\mu_1 > 2/3 \rightarrow$ F

$\mu_1 < 2/3 \rightarrow$ A

$\mu_1 = 2/3 \rightarrow$ F or A

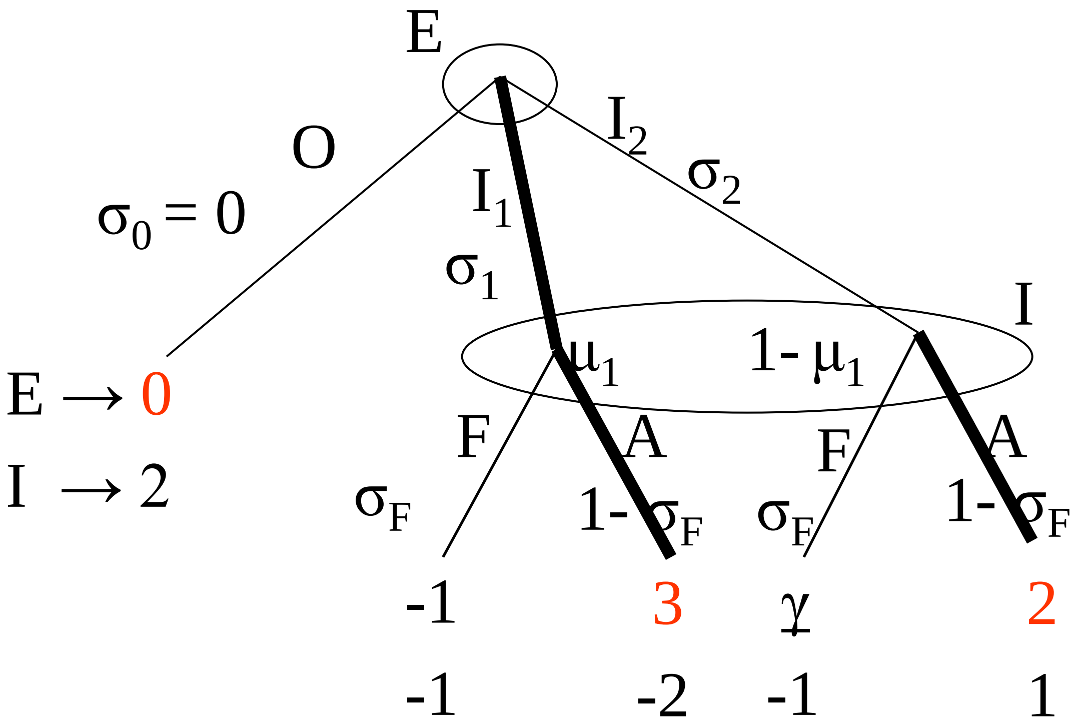
$\underline{\mu_1 < 2/3}$

I plays A ($\sigma_F = 0$)

→ E plays I_1 since 3 > 2 > 0

→ $\mu = (1, 0)$ C! to $\mu_1 < 2/3$

WPBE in Ex.9.C.3



$\underline{\gamma \geq 0}$

$\mu_1 > 2/3 \rightarrow$ F
$\mu_1 < 2/3 \rightarrow$ A
$\mu_1 = 2/3 \rightarrow$ F or A

$\underline{\mu_1 = 2/3}$

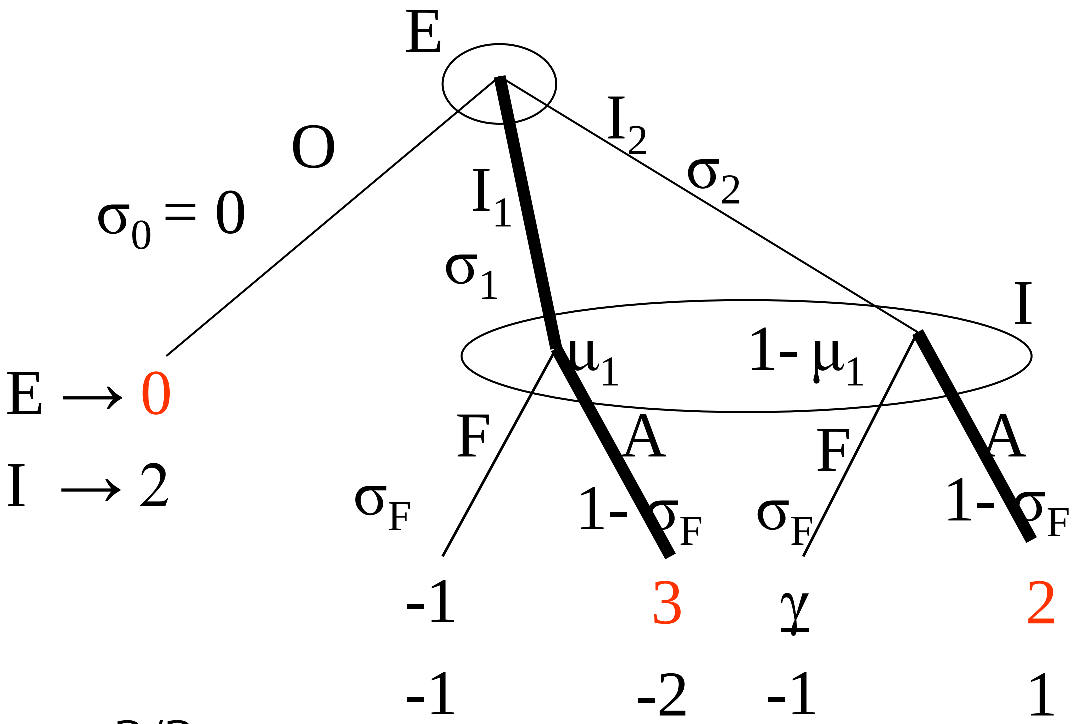
E : $\sigma_1 = 2/3$, $\sigma_2 = 1/3$

since $\sigma_0 = 0$, $\mu_1 = 2/3$ and $\mu_2 = 1/3$

→ E : I_1 and I_2 are indifferent under $(\sigma_F,\ 1 - \sigma_F)$

since $\sigma_1,\ \sigma_2 > 0$

WPBE in Ex.9.C.3



$\gamma \geq 0$

$\mu_1 > 2/3 \rightarrow$ F
$\mu_1 < 2/3 \rightarrow$ A
$\mu_1 = 2/3 \rightarrow$ F or A

$\mu_1 = 2/3$

E : I_1 and I_2 are indifferent under $(\sigma_F,\ 1 - \sigma_F)$ since $\sigma_1,\ \sigma_2 > 0$.

E's payoff : $I_1 \rightarrow -\sigma_F + 3(1-\sigma_F)$, $I_2 \rightarrow \gamma\sigma_F + 2(1-\sigma_F)$

$$-\sigma_F + 3(1-\sigma_F) = \gamma\sigma_F + 2(1-\sigma_F) \rightarrow \sigma_F = 1/(\gamma + 2)$$

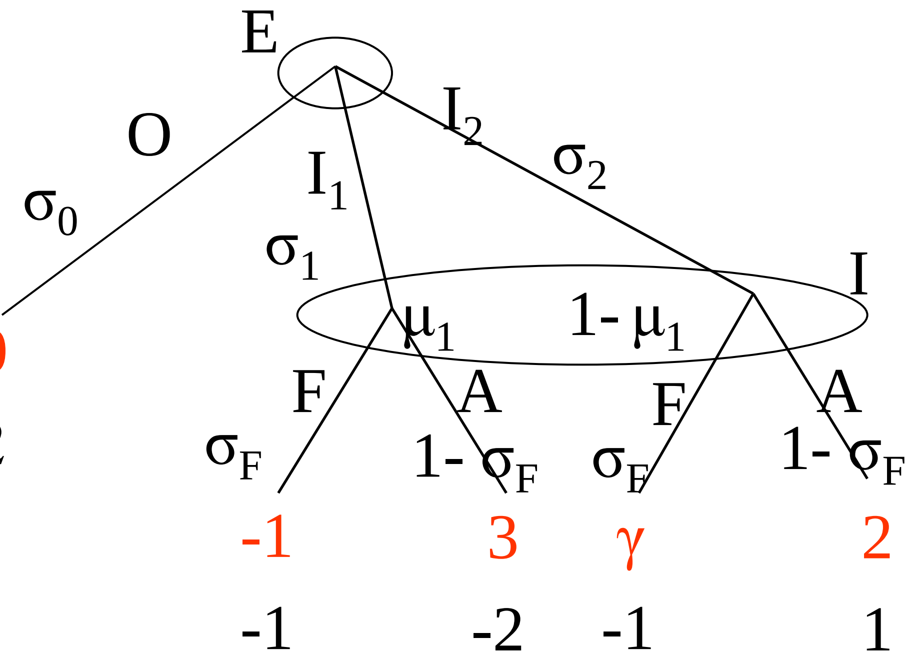
I's strategy : $(\ 1/(\gamma + 2),\ (\gamma + 1)/(\gamma + 2))$

WPBE in Ex.9.C.3

$\gamma \geq 0$

$E \rightarrow 0$

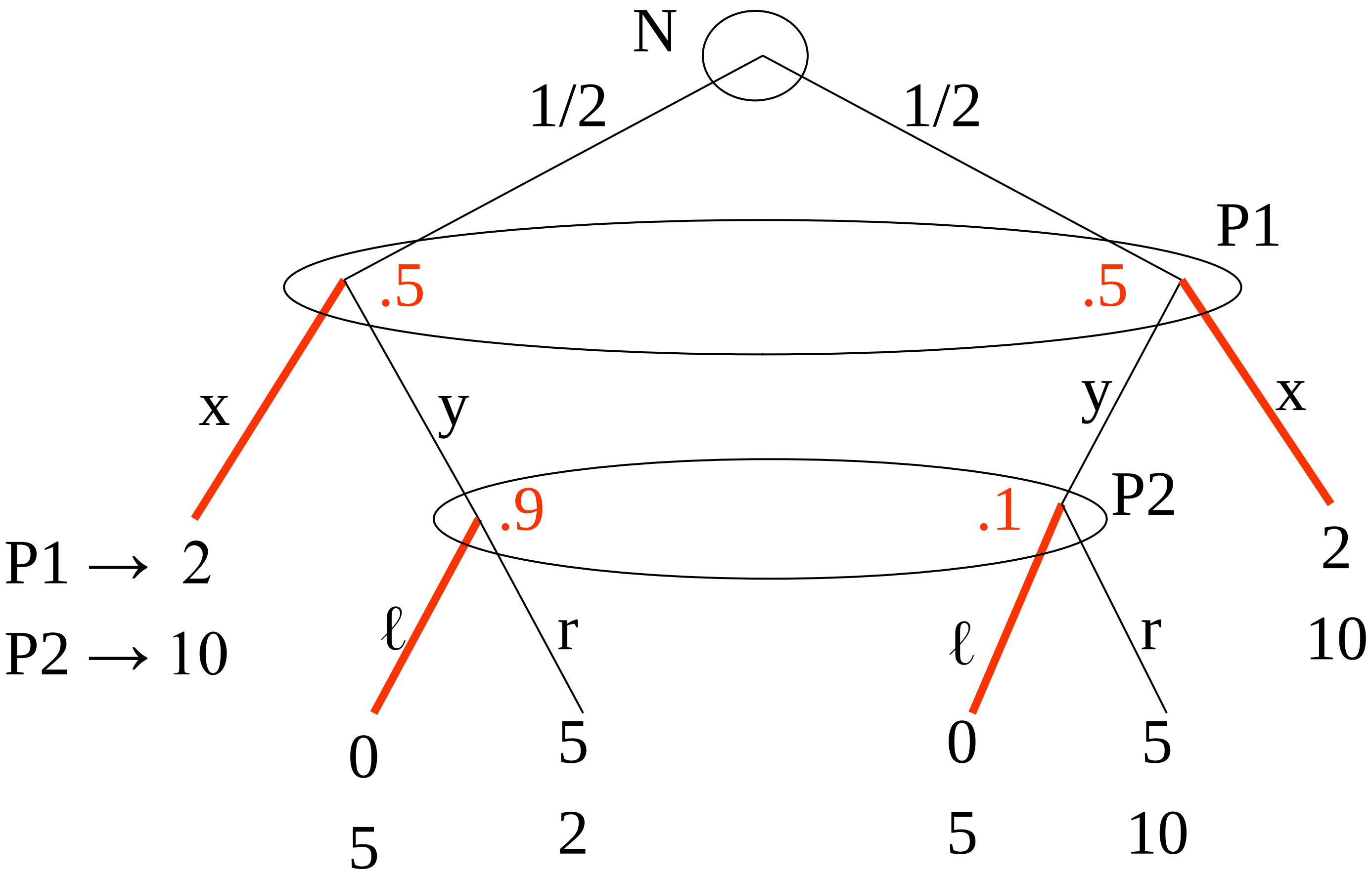
$I \rightarrow 2$



WPBE

$((0, 2/3, 1/3), (1/(\gamma+2), (\gamma+1)/(\gamma+2)), \ \mu = (2/3, 1/3))$

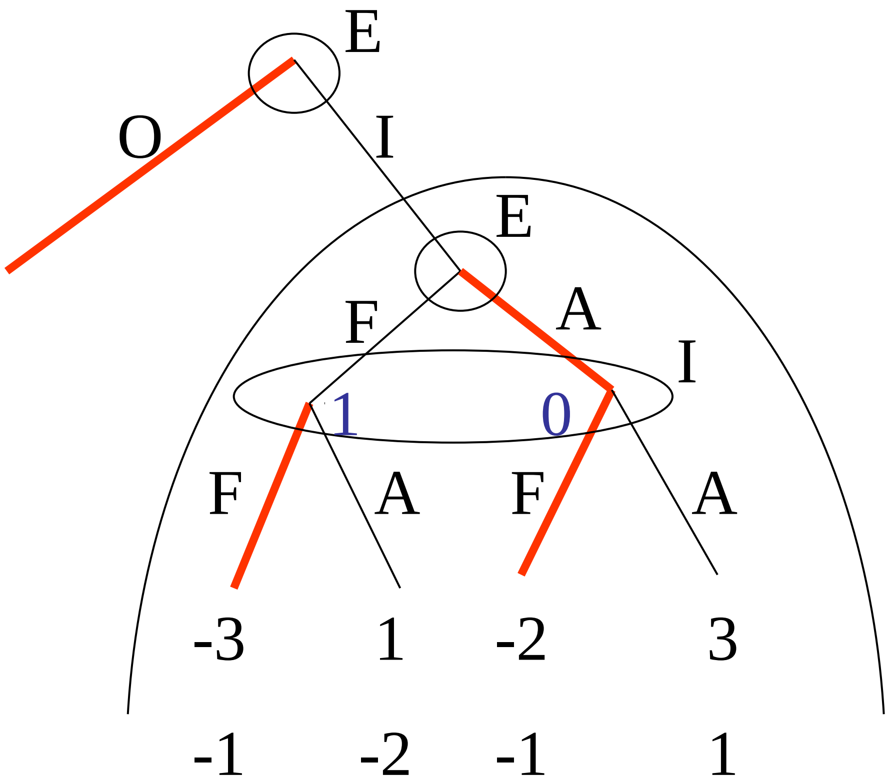
Sequential Equilibrium (motivation, Ex.9.C.4)



(x, ℓ, (.5, .5), (.9, .1)) → WPBE

P2 has an <u>arbitrary</u> belief since his information set is <u>not</u> reached in equilibrium. ? ? ?

Sequential Equilibrium (motivation, Ex.9.C.5)



E → 0

I → 2

((O,A), F, (1,0))

→ WPBE

E \ I	F	A
F	-3, -1	1,-2
A	-2, -1	3, 1

Nash eq → (A, A)

((O,A),F) is not SPNE

Sequential Equilibrium (definition)

Def. 9.C.4: (σ, μ) is a sequential equilibrium (SE) if

(i) σ is sequentially rational given μ ;

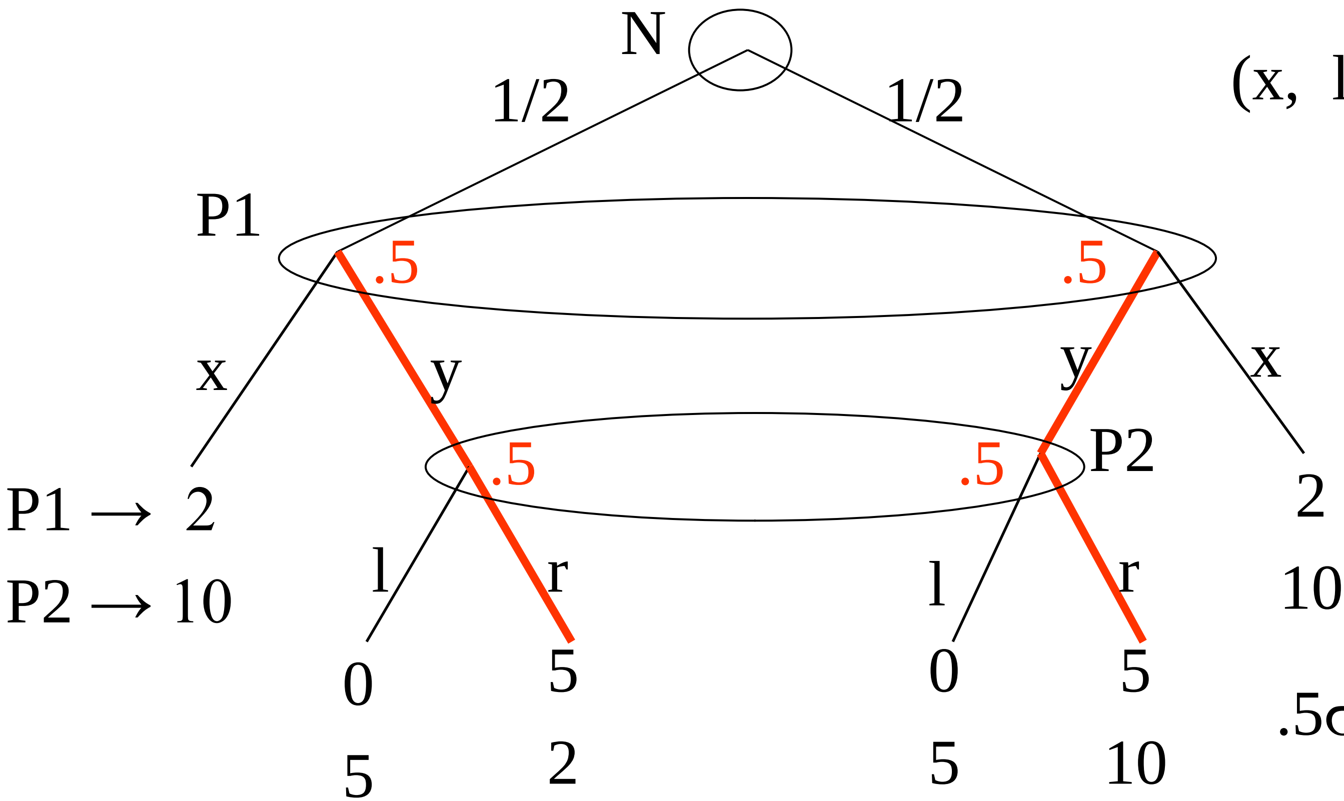
(ii) $\exists$ a sequence of completely mixed strategies $\{\sigma^k\}_{k=1}^{\infty}$

with $\lim_{k\to\infty}\sigma^k = \sigma$ such that $\mu = \lim_{k\to\infty} \mu^k$

where μ^k is the set of beliefs derived from σ^k

using Bayes' rule.

Sequential Equilibrium (Ex. 9.C.4)



(x, l, (.5, .5), (.9, .1))

→ WPBE

$.5\sigma_y/(.5\sigma_y+.5\sigma_y)=.5$

↓

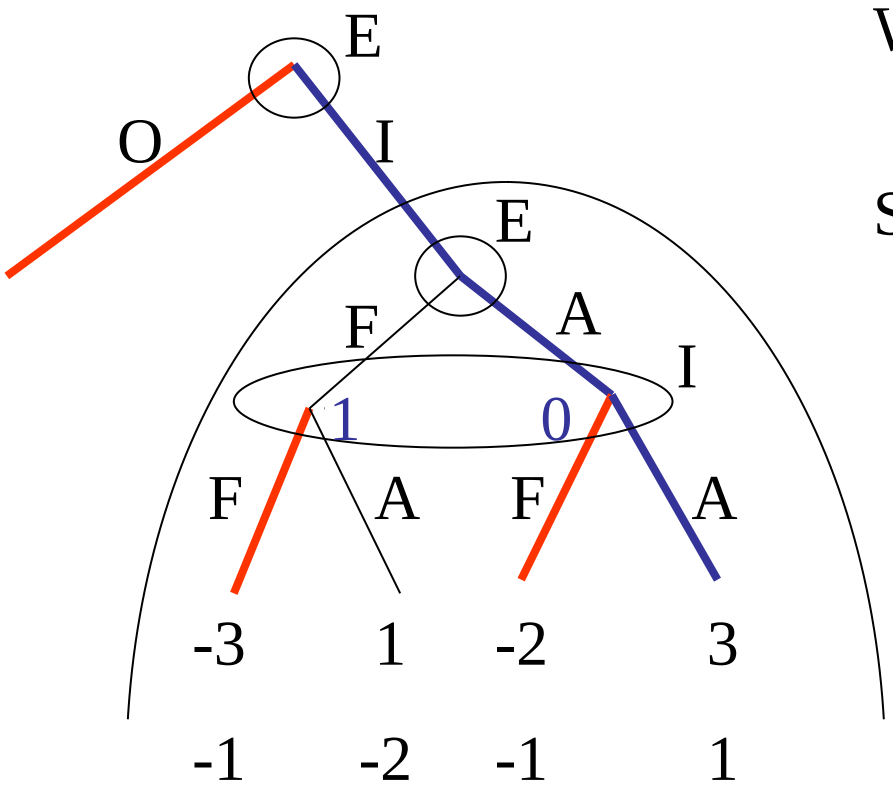
For any comp. mixed strategy (σ_x, σ_y), P2's belief = (.5, .5)

P2's choice must be "r" since $5 < 2\times.5 + 10\times.5 = 6$

P1's choice must be "y" since $2 < 5$

SE → (y, r, (.5, .5), (.5, .5))

Sequential Equilibrium (Ex. 9.C.5)



E → 0

I → 2

WPBE → ((O,A), F, (1,0))

((I, A), A, (0,1))

SPNE → ((I,A), A)

E \ I	F	A
F	-3, <u>-1</u>	1,-2
A	<u>-2</u>, -1	<u>3</u>, <u>1</u>

SE must contain (A, A). (→ next slide)

Sequential Equilibrium (Ex. 9.C.5)

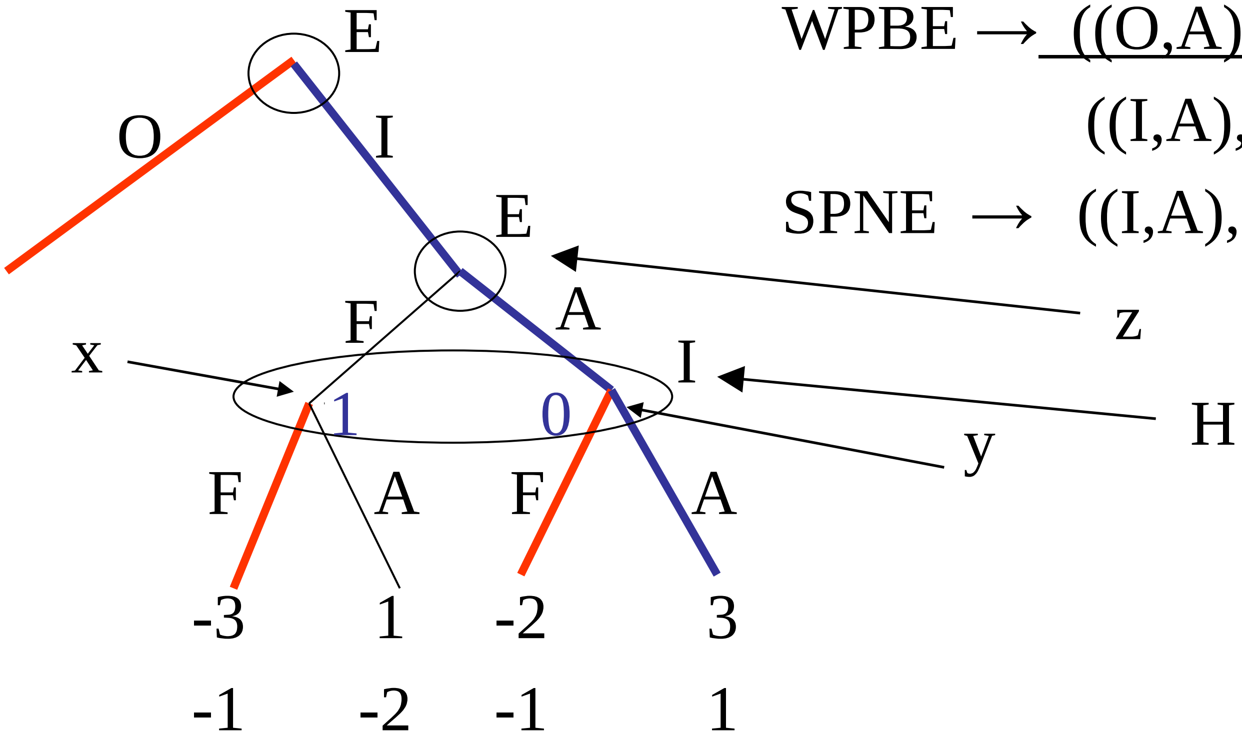
WPBE $\rightarrow$ ((O,A), F, (1,0))

((I,A),A,(0,1))

SPNE $\rightarrow$ ((I,A), A)

$E \rightarrow 0$

$I \rightarrow 2$



$\underline{\sigma_E(O) = 1,\ \sigma_E(I) = 0,\ \ \sigma_E(F) = 0,\ \sigma_E(A) = 1,\ \sigma_I(F) = 1,\ \sigma_I(A) = 0}$

$\rightarrow\ \ \sigma^k{}_E(O) = 1\text{-}\ \varepsilon,\ \sigma^k{}_E(I) = \varepsilon,\ \ \sigma^k{}_E(F) = \varepsilon',\ \sigma^k{}_E(A) = 1\text{-}\ \varepsilon',$

$\sigma^k{}_I(F) = 1\text{-}\ \varepsilon'',\ \sigma^k{}_I(A) = \varepsilon''$

$\text{Prob}(H \mid \sigma^k) = \sigma^k{}_E(I) = \varepsilon,\ \ \text{Prob}(x \mid \sigma^k) = \sigma^k{}_E(I) \times \sigma^k{}_E(F) = \varepsilon \times \varepsilon'$

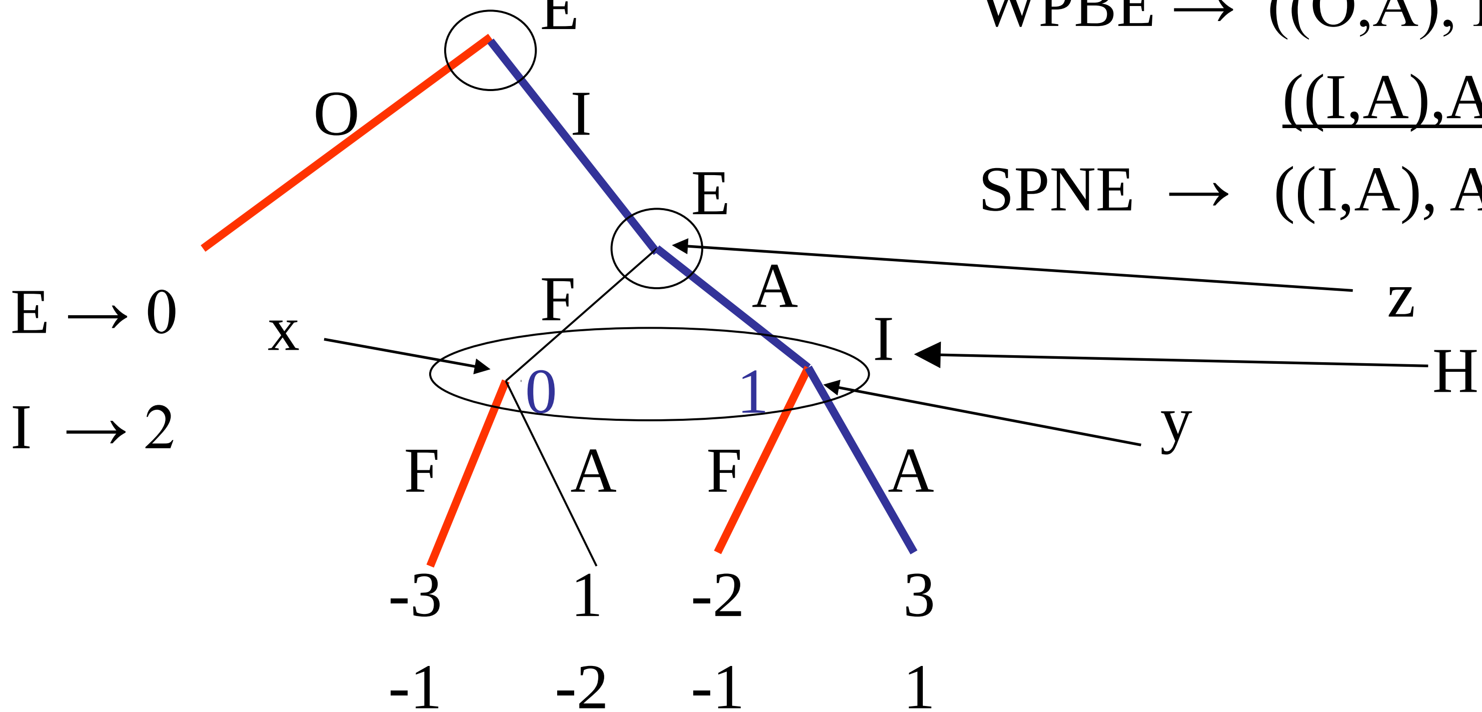
$\mu^k(x) = \varepsilon' \ \rightarrow\ \underline{\mu(x) = 0} \qquad \mu^k(y) = 1\text{-}\varepsilon' \ \rightarrow\ \underline{\mu(y) = 1}$

Sequential Equilibrium (Ex. 9.C.5)

WPBE $\rightarrow$ ((O,A), F, (1,0))

((I,A),A,(0,1))

SPNE $\rightarrow$ ((I,A), A)



$\sigma_E(O) = 0,\ \sigma_E(I) = 1,\ \sigma_E(F) = 0,\ \sigma_E(A) = 1,\ \sigma_I(F) = 0,\ \sigma_I(A) = 1$

$\rightarrow\ \sigma^k_E(O) = \varepsilon,\ \sigma^k_E(I) = 1-\varepsilon,\ \sigma^k_E(F) = \varepsilon',\ \sigma^k_E(A) = 1-\varepsilon',$

$\sigma^k_I(F) = \varepsilon'',\ \sigma^k_I(A) = 1-\varepsilon''$

$\text{Prob}(H \mid \sigma^k) = \sigma^k_E(I) = 1-\varepsilon,\ \ \text{Prob}(x \mid \sigma^k) = \sigma^k_E(I) \times \sigma^k_E(F) = (1-\varepsilon)\,\varepsilon'$

$\mu^k(x) = \varepsilon' \rightarrow \mu(x) = 0 \qquad \mu^k(y) = 1-\varepsilon' \rightarrow \mu(y) = 1$

Sequential Equilibrium and SPNE

Prop. 9.C.2: In every SE (σ, μ), σ is an SPNE.

Assignments

Problem Set 8 (due July 1)

Exercises (pp.301-305)

9.C.1, 9.C.2, 9.C.6（only 9.C.3 part)

Reading Assignment:

Text, Chapter 9, pp.292-300