## Game (Matching Pennies)

## Players: Who is involved ? <br> Rules: Who moves when ? What do they know ? <br> What can they do ? <br> Outcomes: For each possible set of actions by the players, what is the outcome of the game? <br> Payoffs (利得): What are the players' preferences <br> (utility functions) over the possible outcomes ?

Example 7.B.1: Matching Pennies
Players: two players, denoted 1 and 2
Rules: Each player simultaneously puts a penny down, either heads up or tails up.
Outcomes: If the pennies match, 1 pays 1 dollar to 2 ; otherwise 2 pays 1dollar to 1 .

## Game (Tick-Tack-Toe)

Players, Rules, Outcomes, Payoffs
Example 7.B.2: Tick-Tack-Toe
Players: two players, denoted X and O
Rules: The players take turns putting their marks (an X or an O ) into as-yet-unmarked square. X moves first. All previous choices are observed.


Outcomes: The first player to have three of her marks in a row (horizontally, vertically, or diagonally) wins and receives 1 dollar from the other player. If no one succeeds in doing so after all nine boxes are marked, the game is a tie.
No payments are made.

## Preferences

Preference are described by a utility function（効用関数）which assigns a utility level for each outcome．

Von Neumann－Morgenstern utility function
$\rightarrow$ can take expected values（mixed strategy） $(\rightarrow$ Chapter 6）
payoff，payoff function
Examples 7．B． 1 and 7．B． $2 \rightarrow$ payoff $=$ amount of money
zero-sum games (ゼロ和ゲーム)

## Game (Meeting in New York)

Players, Rules, Outcomes, Payoffs
Example 7.B.3: Meeting in New York
Players: two players, Mr. T and Mr. S
Rules: The two players cannot communicate, and supposed to meet in N.Y. City. They have forgotten where.
Each must decide where to go.
Outcomes: If they meet each other, they get to enjoy each other's company. Otherwise, they are alone.
Payoffs: 100 dollars if they meet; 0 dollars otherwise

Each player's optimal action depends on what he thinks the other will do.

## Extensive Form Game (Matching Pennies B)

## Example 7.C.1: Matching Pennies Version B

Players: two players, denoted 1 and 2
Rules: Player 1 puts her penny down first; then after seeing
her choice (head or tail), player 2 puts her penny down.
Outcomes: match, $1 \rightarrow 2$ 1dollar; otherwise $2 \rightarrow 1$ 1dollar


## Extensive Form Game (Tick-Tack-Toe)

Example 7.C.2: The Extensive Form of Tick-Tack-Toe


## Perfect Information

Matching pennies version B，Tick－Tack－Toe
Each player knows whole history of the game when she moves
$\rightarrow$ perfect information（完全情報）

Information set（情報集合）
$\rightarrow$ subset of particular player＇s decision nodes
When play has reached one of the decision nodes in the information set of a player，she does not know which one of the nodes in the information set she is actually at．

Perfect information $\rightarrow$ every information set is a singleton．

## Extensive Form Game (Matching Pennies B)

Example 7.C.1: Matching Pennies Version B
Rules: Player 1 puts her penny down first; then after seeing her choice (head or tail), player 2 puts her penny down.


## Extensive Form Game (Matching Pennies C)

Example 7.C.3: Matching Pennies Version C Rules: Player 1 puts her penny down first; then player 2 puts her penny down without knowing 1's choice.


## Restrictions on Information Sets


information set
$\rightarrow$ every node has the same set of possible actions

## Restrictions on Information Sets－Perfect Recall

## Perfect Recall（完全記憶）

$\rightarrow$ A player does not forget what she knew，including her own actions


Not ：Player 2 forgets 1＇s first action which she knows before

## Restrictions on Information Sets - Perfect Recall

## Perfect Recall

$\rightarrow$ A player does not forget what she knew, including her own actions


Not : Player 1 forgets her first action.

## Extensive Form Game (Matching Pennies)

Example 7.C.4: Matching Pennies Players 1 and 2 move simultaneously.

Equivalent to Version C.


## Perfect Information

Perfect information:
Each information set contains a single decision node.
( $\Leftrightarrow$ imperfect information)

Perfect information:
Example 7.C. 1 (Matching Pennies Ver.B),
Example 7.C. 2 (Tick-Tack-Toe),

Imperfect information:
Example 7.B. 1 (Matching Pennies),
Example 7.C. 3 (Matching Pennies Ver.C),

## Chance Moves（偶然手番）

Example 7．C．5：Matching Pennies Version D
Players 1 and 2 flip a coin to decide who will play first．
Then play Version B．


## Common Knowledge

Basic postulate:
All players know the structures of the game
know that their rivals know it
know that their rivals know that their rivals know it and so on .......
$\rightarrow$ common knowledge

Game structure is common knowledge
$\rightarrow$ complete information game

## Assignments

## Problem Set 1 (due April 15):

Exercise (page 233) : 7.C. 1

Reading Assignment:
Text Chapter 7, pp.219-233

