Game (Matching Pennies)

Players:	Who is involved ?
<u>Rules</u> :	Who moves when ? What do they know ?
	What can they do?
Outcomes:	For each possible set of actions by the players,
	what is the outcome of the game?
Payoffs (利得	What are the players' preferences
	(utility functions) over the possible outcomes ?

Example 7.B.1: Matching Pennies

Players: two players, denoted 1 and 2
Rules: Each player simultaneously puts a penny down, either heads up or tails up.
Outcomes: If the pennies match, 1 pays 1 dollar to 2; otherwise 2 pays 1dollar to 1.

Game (Tick-Tack-Toe)

Players, Rules, Outcomes, Payoffs

Example 7.B.2: Tick-Tack-Toe

Players: two players, denoted X and O
Rules: The players take turns putting their marks (an X or an O) into as-yet-unmarked square. X moves first.
All previous choices are observed.



Outcomes: The first player to have three of her marks in a row (horizontally, vertically, or diagonally) wins and receives 1 dollar from the other player. If no one succeeds in doing so after all nine boxes are marked, the game is a tie. No payments are made. Preference are described by a <u>utility function</u> (効用関数)which assigns a utility level for each outcome.

Von Neumann-Morgenstern utility function \rightarrow can take expected values (mixed strategy) (\rightarrow Chapter 6)

payoff, payoff function Examples 7.B.1 and 7.B.2 \rightarrow payoff = amount of money

zero-sum games (ゼロ和ゲーム)

Game (Meeting in New York)

Players, Rules, Outcomes, Payoffs

<u>Example 7.B.3</u>: Meeting in New York
Players: two players, Mr. T and Mr. S
Rules: The two players cannot communicate, and supposed to meet in N.Y. City. They have forgotten where.
Each must decide where to go.
Outcomes: If they meet each other, they get to enjoy each other's company. Otherwise, they are alone.
Payoffs: 100 dollars if they meet; 0 dollars otherwise

Each player's optimal action depends on what he thinks the other will do.

Extensive Form Game (Matching Pennies B)

Example 7.C.1: Matching Pennies Version B
Players: two players, denoted 1 and 2
Rules: Player 1 puts her penny down first; then after seeing
her choice (head or tail), player 2 puts her penny down.
Outcomes: match, 1 → 2 1dollar; otherwise 2 → 1 1dollar



Extensive Form Game (Tick-Tack-Toe)

Example 7.C.2: The Extensive Form of Tick-Tack-Toe





Matching pennies version B, Tick-Tack-Tœ Each player knows whole history of the game when she moves → perfect information (完全情報)

Information set (情報集合)

→ subset of particular player's decision nodes
 When play has reached one of the decision nodes
 in the information set of a player, she does not
 know which one of the nodes in the information
 set she is actually at.

Perfect information \rightarrow every information set is a singleton.

Extensive Form Game (Matching Pennies B)

Example 7.C.1: Matching Pennies Version B Rules: Player 1 puts her penny down first; then after seeing her choice (head or tail), player 2 puts her penny down.



Extensive Form Game (Matching Pennies C)

Example 7.C.3: Matching Pennies Version C Rules: Player 1 puts her penny down first; then player 2 puts her penny down without knowing 1's choice.



Restrictions on Information Sets



information set

 \rightarrow every node has the same set of possible actions

Restrictions on Information Sets – Perfect Recall

<u>Perfect Recall</u>(完全記憶)

→ A player does not forget what she knew, including her own actions



<u>Not</u> : Player 2 forgets 1's first action which she knows before

Restrictions on Information Sets – Perfect Recall

Perfect Recall

→ A player does not forget what she knew, including her own actions



<u>Not</u> : Player 1 forgets her first action.

Extensive Form Game (Matching Pennies)

Example 7.C.4: Matching Pennies

Players 1 and 2 move simultaneously.

Equivalent to Version C.



Perfect Information

Perfect information:

Each information set contains a single decision node.

(\Leftrightarrow imperfect information)

Perfect information:

Example 7.C.1 (Matching Pennies Ver.B), Example 7.C.2 (Tick-Tack-Toe),

Imperfect information:

Example 7.B.1 (Matching Pennies), Example 7.C.3 (Matching Pennies Ver.C), Chance Moves (偶然手番)

Example 7.C.5: Matching Pennies Version DPlayers 1 and 2 flip a coin to decide who will play first.Then play Version B.



Common Knowledge

Basic postulate:

All players know the structures of the game

know that their rivals know it

know that their rivals know that their rivals know it

and so on

 \rightarrow <u>common knowledge</u>

Game structure is common knowledge

 \rightarrow complete information game

Assignments

Problem Set 1 (due April 15): Exercise (page 233) : 7.C.1

Reading Assignment: Text Chapter 7, pp.219-233