## \＃14

## Protein Structure Prediction

## Topics：

－Needs for Protein Structure Prediction
－Preparation：Protein Structure Comparison
RMSD，RMSDd，double dynamic programming
－Structure Prediction：simple Lattice model
－Homology modeling
Modeller，Swiss－model，SCWRL
－Threading method
Sippl（Threading），Bowie－Eisenberg（3D－1D），Jones（Double DP）



## Protein Data Bank

- Tertiary (3-D) structure archive of proteins, DNAs, and complexes. 58,236 entries (as of $16^{\mathrm{h}}$ June, 2009)


## PDB Current Holdings Breakdown

|  | Proteins | Nucleic Acids | Protein/NA <br> Complexes | Other | Total |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| X-ray | 46626 | 1147 | 2163 | 17 | 49953 |
| NMR | 6886 | 856 | 146 | 6 | 7894 |
| Exp. | 168 | 16 | 59 | 0 | 243 |
| Method |  |  |  |  |  |
| Hybrid | 14 | 1 | 1 | 1 | 17 |
| Other | 112 | 4 | 4 | 9 | 129 |
| Total | 53806 | 2024 | 2373 | 33 | 58236 |

http://www.rcsb.org

- history

1971 Started at Brookhaven National Laboratory
1998 Moved to RCSB (Research Collaboratory for Structural Bioinformatics) 2006 wwPDB (The Worldwide Protein Data Bank) by US, Europe, and Japan.

Growth of Sequence and 3D Structure Databases


## Huge Cost Gaps among Data Types

- DNA Seq. information is inexpensively read by sequencers
- Protein 3-D structure information is extremely expensive



## Protein Structure

-Amino acid as a building block


## $C_{a}$ coordinates: a simplified representation of main chain



## RMSD

## RMSD（Root Mean Square Deviation）

$$
\begin{aligned}
& \operatorname{RMSD}(\mathrm{A}, \mathrm{~B}) \\
& =\operatorname{SQRT}\left\{\frac{1}{\mathrm{n}} \cdot \sum_{\mathrm{i}=1}^{\mathrm{n}}(\mathbf{A i}-\mathbf{B i})^{2}\right\} \\
& =\operatorname{SQRT}\left\{\frac{1}{\mathrm{n}} \cdot \sum_{\mathrm{i}=1}^{n}(\mathrm{xai}-\mathrm{xbi})^{2}+(\mathrm{yai}-\mathrm{ybi})^{2}+(\mathrm{zai}-\mathrm{zbi})^{2}\right\}
\end{aligned}
$$

In bioinformatics，＂RMSD＂usually means for the following ＂least＂RMSD between superposed（protein）coordinates A and B．

$$
\begin{aligned}
& \text { least-RMSD(A, B) } \\
& =\min _{\boldsymbol{U}} \operatorname{SQRT}\left\{\frac{1}{\mathrm{n}} \cdot \sum_{i=1}^{n}(U \mathbf{A i}-\mathbf{B} i)^{2}\right\}
\end{aligned}
$$

U：orthonormal（rotational）matrix正規直交（回転）行列
where coordinates A，B are centered beforehand．

## Best rotation matrix（1）

## Kabsch＇s method

W．Kabsch：＂A Solution for the Best Rotation to Relate Two Sets of Vectors＂， Acta Crystallographica，32，922－923（1976）．

Covariance matrix $(3 \times 3)$

$$
C=A B^{T}
$$

Perform Singular Value Decomposition

$$
C=V S W^{\top}
$$

where $S$ is composed of eigenvalues（固有値）$\lambda_{1}, \lambda_{2}$ ，and $\lambda_{3}$

The best rotation Matrix U is obtained as

$$
U=W V^{T}
$$

（if $\operatorname{det}(\mathrm{C})<0$ ，then mirror reflection is also needed）

## Best rotation matrix（2）

## Quaternion method

E．Coutsias，C．Seok，K．Dill：＂Using quaternions to calculate RMSD＂， Journal of Computational Chemistry，25，1849－1857（2004）．

Quaternion（四元数）

$$
r=w+x i+y \boldsymbol{j}+z \boldsymbol{k}
$$

$$
\begin{aligned}
& \boldsymbol{i}^{2}=\boldsymbol{j}^{2}=\boldsymbol{k}^{2}=-1 \\
& \boldsymbol{i} \cdot \boldsymbol{j}=\boldsymbol{k}, \quad \boldsymbol{j} \cdot \boldsymbol{k}=\mathbf{i}, \quad \boldsymbol{k} \cdot \mathbf{i}=\boldsymbol{j} \\
& \boldsymbol{j} \cdot \boldsymbol{i}=-\boldsymbol{k}, \quad k \cdot \boldsymbol{j}=-\mathbf{i}, \quad \boldsymbol{i} \cdot \boldsymbol{k}=-\mathbf{j}
\end{aligned}
$$

Rotation by unit quaternion

$$
\begin{aligned}
& \mathrm{r}=\mathrm{x} \boldsymbol{i}+\mathrm{y} \boldsymbol{j}+\mathrm{z} \boldsymbol{k} \\
& \mathrm{q}=\mathrm{a}+\mathrm{bi}+\mathrm{c} \boldsymbol{j}+\mathrm{d} \boldsymbol{k} \\
& \mathrm{q}^{*}=\mathrm{a}-\mathrm{b} i-\mathrm{c} \boldsymbol{j}-\mathrm{d} \boldsymbol{k}
\end{aligned} \mathrm{r}^{\prime}=\mathrm{q} \mathrm{r} \mathrm{q}^{*} \quad \begin{gathered}
\text { (rotation by } \\
\text { unit quaternion) }
\end{gathered}
$$

## distance based RMSD

## RMSDd

## RMSD(A, B)

$\left.=\operatorname{SQRT}\left\{\frac{1}{\mathrm{n}(\mathrm{n}-1)} \cdot \sum_{\mathrm{i}=1}^{\mathrm{n}} \sum_{\mathrm{j}=1}^{\mathrm{n}} \mathbf{A}_{\mathrm{ij}}-\mathbf{B} \mathrm{ij}\right)^{2}\right\}$

advantage:

1. Robust for outliers (while normal RMSD tends to be influenced)
2. Easy to calculate
disadvantage:
3. Mirror reflection images cannot be distinguished.

## Structure comparison based on RMSDd

Lisa Holm, Chris Sander: "Protein structure comparison by alignment of distance matrices", J Mol Biol, 233, 123-138 (199:

## DALI server (Distance matrix ALIgnment)

http://www2.ebi.ac.uk/dali/

- Not a direct calculation with whole protein length.
- Partial comparisons are done, and then combined.
- DALI system is used to define FSSP database (shown later).


## Structure Alignment



Two Structures (A, B)


RMSD is required to choose alignment ( P )
chicken and egg?
Equivalence (P)
between residues

## Easy

RMSD
calculation
A simple iterative approach

1) start from initial given alignment $P$.
2) calculate least RMSD rotation, based on the alignment $P$.
3) measure residue distances and make a score table Rij.
4) perform dynamic programming between (A,B) with Rij.
5) get new alignment $P^{\prime}$. (if $P^{\prime}$ is not enough, $P=P^{\prime}$ and go to 2)
S.T. Rao, M.G. Rossman: "Comparison of super-secondary structures
in proteins," J. Mol. Biol., 76, 211-256 (1973).

## Double Dynamic Programming


C. Orengo and W. Taylor: "SSAP: Sequential Structure Alignment Program for Protein Structure Comparison", Methods in Enzymology 266, 617-635 (1996).

## Algorithm for DDP

$$
R:=\{0\}
$$

for each pair ( $\mathrm{Ai}, \mathrm{Bj}$ ) do
force pairing between Ai and Bj compute the low level scoring matrix $\mathrm{R}^{(\mathrm{i}, \mathrm{j})}$ (score, P ) := DP (A, B) with $\mathrm{R}^{(\mathrm{i}, \mathrm{j})}$ forall ( $\mathrm{Ap}, \mathrm{Bq}$ ) in P do $R p q:=R p q+R^{(i, j)} p q$ end

$$
(s, p):=D P(A, B) \text { with } R \quad \text { (High level DP) }
$$

Cited and modified from "Protein structure comparison and structure patterns" by Ingvar Eidhammer and Inge Jonassen

## Geometric Hashing

- Suitable to quickly search similar sub-structures from a large-scale (protein) structure database. In order to do this, a large "hash table" is pre-calculated.
- Originally developed as a 3-D model comparison method in computer vision study.
- Does not care for residue number. Only compares vertex positions.

R. Nussinov and HJ Wolfson:

"Efficient Detection of Three Dimensional Structural Motifs in Biological Macromolecules by Computer Vision Techniques", Proc. Nat'l Acad. Sci., 88, 10495-10499 (1991).

## Geometric Hashing（example）



For example，
（1）as the origin，（1）－（4）as x－axis．
This is called＂$(1,4)$ transform＂．

$(0,0)$ hit！
$(1,1)$ hit！
$(4,2)$ hit ！
$(3,0)$ hit ！
$(4,-1)$ hit！
$(1,-1)$ hit ！
Hash Table：${ }^{0}+1+2+3+4 \quad X$－axis
For a figure with $n$ points， there are $z=n(n-1) / 2$ different＂transform＂ exist．

Figure A has 6 points．
Thus $Z=6(6-1) / 2=15$ 15 different＂transform＂ should be tested．

The number， $z=n(n-1) / 2$ ，is much smaller than that of possible smooth analogue transforms．

[^0]


All possible $z=m(m-1) / 2$ transform are sequentially tested．Now for example $(1,3)$ ．


Now $(1,3)$ map are compared with the pre－calculated hash table of figure $A$ ． $B(1,3)=A(1.4) \bigcirc$

## pre－calculated hash table

 can be an overlay of thousands different figures in a database． all figures are searched at once．
## Three approaches for structure prediction

- Homology modeling

- Fold recogniton

- New fold ("ab initio")



## Lattice model

- Extremely simplified model
- Easy to find out the minimum energy conformation


Lattice: cubic, tetrahedron, etc.
Node: single residue
Energy (only between neighboring nodes):

1) steric, 2) hydrophobic, 3) hydrogen-bond, etc.

HP model
Hydrophobic ( $\square$ ) vs. Polar ( $\square$ )

tetrahedron lattice model (green) vs. native BPTI structure (red)
D. Hinds and M. Levitt:"A lattice model for protein structure prediction at low resolution", PNAS, 89, 2536-2540 (1992).

## Homology modeling

- MODELLER by Andrej Sali

Main chain: template Side chain: modeling

Modeller

Program for Comparative Protein Structure Modelling by Satisfaction of Spatial Restraints
http://www.salilab.org/modeller/
A. Sali, T.L. Blundell: "Comparative protein modelling by satisfaction of spatial restraints", J. Mol. Biol. 234, 779-815 (1993).
http://swissmodel.expasy.org/
Schwede T, Kopp J, Guex N, and Peitsch MC: "SWISS-MODEL: an automated protein homology-modeling server", Nucleic Acids Research, 31, 3381-3385 (2003).

## Side chain prediction

## Input: Main chain structure

Output: Side chain structure (optimized rotamer arrangement)

## Rotamer Library:

Collection of frequently observed side chain dihedral angles (for each residue, with/without main-chain dependent situation)

## Combinatorial Optimization:

Maximizing total fitness of mutual relation among selected rotamers. Algorithms like DEE (Dead-End Elimination), etc.

- SCWRL (Library \& Software) by Roland Dunbrack

http://dunbrack.fccc.edu/SCWRL3.php
A. A. Canutescu, A. A. Shelenkov, and R. L. Dunbrack, Jr. : "A graph theory algorithm for protein sidechain prediction", Protein Science, 12, 2001-2014 (2003).


## mopplew How many protein folds exist?

## estimation of protein fold numbers

- G.E. Schulz: Angew. Chem. Int. Ed. Eng. (1981) ~ 500
- C. Chothia: Nature, 357, 543-544. (1992) ~1000
"One Thousand Families for the Molecular Biologist"

At 1992, there were 120 known unique folds in PDB.
At the same time, one quarter of known protein sequences were predicted to have one of 120 known folds, while other $3 / 4$ are unknown. Chothia estimated that only $1 / 3$ of existing proteins had been sequenced at the time. Thus, the first rough estimation for unique folds are given by $120 \times 4 \times 3=1440$
Sequence homology search method must have a limited sensitivity, thus, finally he estimated as: $120 \times(4 / 1.25) \times(3 / 1.25)=920$. (Recently researchers are believing $>3000$ folds? exist)

## Fold Recognition



## $\xrightarrow{\text { mppacer }}$ Potential-based Threading

Recognizing native structure by calculating pseudo energy potential among residues with "threading" the sequence into a 3D model.


M. Sippl:

"Calculation of Conformational Ensembles from Potentials of Mean Force,- An Approach to the Knowledge-based Prediction of Local Structures in Globular Proteins- ", J. Mol. Biol., 213, pp.859-883 (1990).
M. Hendlich, P. Lackner, S. Weitckus, .... and M. Sippl :
"Identification of Native Protein Folds Amongst a Large Number of Incorrect Models, - The Calculation of Low Energy Conformations from Potentials of Mean Force ", J. Mol. Biol., 216, pp.167-180 (1990).

## Inverse Folding

J. Bowie, R. Luthy, D. Eisenberg: "A Method to Identify Protein Sequences That Fold into a Known Three Dimensional Structure", Science, 253, 164-170 (1991).

## Inverse Folding problem

Find amino acid sequences that are compatible to a given 3-D structure.
Search the most compatible sequence with residue "environment" in a given protein 3-D structure. "Environment" includes:
(1) A: surface area which buried in the protein, and not exposed.
(2) f: fraction of side chain surface area covered by polar atoms ( $\mathrm{O}, \mathrm{N}$ )
(3) s: local secondary structure

Four examples are shown in this paper.

- globins
- cyclic AMP receptor-like proteins
- periplasmic binding proteins
- actins


David Eisenberg

## 3D-1D score

## 3D compatibility search

$$
\begin{array}{ccccl}
\mathrm{N}-\mathrm{X} 1- & \mathrm{X} 2- & \mathrm{X} 3- & \mathrm{X} 4-\ldots \mathrm{Xn}-\mathrm{C} & \text { (amino acid sequence) } \\
\mathrm{E}_{\alpha}- & \mathrm{P}_{2 \alpha}- & \mathrm{B}_{2 \alpha}-\mathrm{E}_{\alpha}-\ldots & & \text { (environment class) }
\end{array}
$$



## 3D-1D score:

$$
\text { score }=\sum_{i=1}^{n} f\left(\text { class }_{i}, \text { residue }_{i}\right)
$$

Best alignment between the 3-D sequence and another sequence can be efficiently calculated with Dynamic Programming technique

## 3D Profile

Gap
Amino acid type $=$ ponalty

| Postion in fold | Environment class | A | C | D | E | F | $G$ | H | 5 | 1 | V | W | $Y$ | Opn | Ext |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | E | 12 | 40 | 22 | 3 | -790 | $1+3$ | 32 | 32 | 12 | -91 | 214 | 4 | 2 | 0.02 |
| 2 | $\mathrm{B}_{2}$ | -60 | 5 | -126 | -135 | 105 | -180 | -60 | -117 | -76 | 60 | 102 | 112 | 2 | 0.02 |
| 3 | Ea | 46 | 4 | 44 | 59 | -280 | $\omega$ | $-84$ | 15 | -17 | 415 | - 545 | -270 | 200 | 000 |
| 4 | P24 | 0 | - | ${ }^{28}$ | 50 | -149 | - 50 | 50 | -t | 5 | . 48 | -114 | 79 | 200 | 200 |
| 5 | E | 46 | -4 | 44 | 59 | -220. | ${ }^{68}$ | 34 | 45 | -17 | - 10 | - 63 | 210 | 200 | 200 |
| ¢ | Pex | 6 | -90 | \% | 56 | -149 | 50 | 50 | -18 | -5 | -48 | . 114 | -79 | 200 | 200 |
| 7 | $\mathrm{B}_{2}{ }^{\text {a }}$ | -60 | -10 | -162 | . 74 | 00 | -140 | 6 | -147 | -150 | 68 | 60 | 日 5 | 200. | 200 |
| ${ }^{\prime}$ | E | 46 | -44 | 4 | 59 | -220 | ${ }^{68}$ | 3 | 15 | -7 7 | -710 | - k 5 | -210 | 209 | 200 |
| 9 | P20 | 6 | - ${ }^{\text {a }}$ | 9 | 68 | -149 | 60 | 50 | -18. | 5 | - | -174 | -79 | 200 | 200 |
| 10 | $\mathrm{B}_{4}$ | -80 | -73 | -997 | -174 | 132 | -285 | 967 | -73 | -129 | 66 | 100 | 18 | 200 | 2010 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| . |  |  | , | , | - | $\cdot$ | $\cdots$ |  |  | - |  |  |  |  |  |

-Fig. 3 3D profile example
3D profile of sperm whale myoglobin (original sequence length is 153)
The 3D-1D scores in this table are calculated (x 100) from probability in Fig. 5.
Heavy penalty applies for opening gap in a helix region at position 3-10.

## Double DP technique for Threading

D. Jones, W. Taylor, J. Thornton: "A new approach to protein fold recognition", Nature, 358, pp.86-89 (1992).

Difficulty of Sippl's approach:


David Jones

Gap (insertion/deletion) between model and query should be considered. Pairwise potential can be calculated only after determining residue positions.
Alignment and evaluation phases are like a "Chicken and Egg" problem.

- choice 1: Use sequence alignment between model and query sequence.
$\rightarrow$ usually difficult, because high homology cannot be expected.
-choice 2: Give up "pairwise" potential.
$\rightarrow$ use "environment" or such (Eisenberg's approach)
- choice 3: Give up real "pairwise" calculation, and use approximation.
$\rightarrow$ use "frozen approximation". (calculate potential between query residue and model residue)
- choice 4: Try to give good alignment between 3-D model and query.
$\rightarrow$ use "double DP" technique (This paper)


## Fold Recognition method: FORTE

## FORTE

Tomii, K. \& Akiyama, Y. :Bioinformatics, 20, (2004).


- Profile-Profile comparison based (not 3D-1D)
- Correlation coefficient for vector similarity score
- Frequent update of protein structure profiles
- Fully parallelized prediction (FORTE-SUITE)



MNI FEAI ENRHSVRDFLERKMPERVKDDI ENLLVKFI TKKLDUKI NLSSFPSYI YAKAEK HFDELVEYGFQGEQ VLFLTAQGFGTCWMARSPHPDVPYI I VFGYPRTRNFTRKRRPI TS FLENDLEELPPEI VKI VEMTI LAPSALNRQPWKI KYTGGELCI SSERPVDLG ALSHAYL TAREI FKREPVI QKRGEDTYCLI LNP

## T0223 206 AA

Putative Nitroreductase，T．maritima CM／hard and FR／H （hard for comparative modeling）

T0223TS272＿1．T0223＿2


CBRC－3D（1st $)$
 $3.68 \AA$ Native

MSALDNSI RVEVKTEYI EQQSSPEDEKYLFSYTI TI I NLGEQAAKLETRHW I TDANGKT SEVQGAGWGETPTI PPNTAYQYTSGTVLDTPFGI MYGTYGMNSESGEHFNAI I KPFRLA TPGLLH

## T0212 126 AA




[^0]:    detailed coordinates within mesh are ignored．

