Physics and Engineering of CMOS Devices

Ken Uchida Department of Physical Electronics Tokyo Institute of Technology

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2D Electron System in MOS Structures



Scattering Rate



Mobility Limited by Various Scattering

Coulomb Scattering

$$\mu_{\rm Coulomb} = N_{sub}^{-1} \cdot N_{inv}^{+1}$$

Phonon Scattering

$$\mu_{\rm Phonon} = E_{eff}^{-1/3} \cdot T^{-1.25}$$

Roughness Scattering

$$\mu_{\text{Roughness}} = E_{eff}^{-m}$$

Scattering in 2D electron system

In 2D electron systems such as inversion-layer electrons in MOSFETs, 1) electrons are confined with potentials and 2) subbands are formed in each valley.

$$\Psi_{\mathbf{k}_{//},n} = \frac{1}{\sqrt{S}} \exp(i\mathbf{k}_{//} \cdot \mathbf{r}_{//}) \zeta_n(z)$$
$$= \zeta_n(z) |\mathbf{k}_{//}\rangle$$

$$\langle \mathbf{k}' | H_p(\mathbf{r}) | \mathbf{k} \rangle = \langle \mathbf{k}' | \sum_{\mathbf{q}} H_p(\mathbf{q}) \exp(i\mathbf{q} \cdot \mathbf{r}) | \mathbf{k} \rangle$$

= $\langle \mathbf{k}'_{\prime\prime} | \zeta_m^{*}(z) \sum_{\mathbf{q}} H_p(\mathbf{q}) \exp(i\mathbf{q} \cdot \mathbf{r}) \zeta_n(z) | \mathbf{k}_{\prime\prime} \rangle$
= $\sum_{\mathbf{q}} H_p(\mathbf{q}) \delta(\mathbf{k}_{\prime\prime} - \mathbf{k}'_{\prime\prime} + \mathbf{q}_{\prime\prime}) \int_0^W \zeta_m^{*}(z) \zeta_n(z) \exp(iq_z z) dz$

ct.
$$\langle \mathbf{k}' | H_p(\mathbf{r}) | \mathbf{k} \rangle = \sum_{\mathbf{q}} H_p(\mathbf{q}) \delta(\mathbf{k} - \mathbf{k}' + \mathbf{q})$$

P. J. Price, Annals of Physics, **133** (1981) p217. Physics and Engineering of CMOS Devices, Ken Uchida, Tokyo Tech, June 23, 2010

Form Factor

$$\delta(\mathbf{k} - \mathbf{k'} \pm \mathbf{q}) \rightarrow I_{mn}(q_z)\delta(\mathbf{k}_{\prime\prime} - \mathbf{k}_{\prime\prime}' \pm \mathbf{q})$$

where I_{mn} is the form factor. $I_{mn}(q_z) = \int_0^W \zeta_m(z)^* \zeta_n(z) \exp(iq_z z) dz$

$$\begin{split} \int_{-\infty}^{\infty} |I_{mn}(q_{z})|^{2} dq_{z} &= \int dq_{z} \int_{0}^{W} dz \int_{0}^{W} dz' \zeta_{m}(z)^{*} \zeta_{n}(z) \zeta_{m}(z') \zeta_{n}(z')^{*} \exp\left[iq_{z}(z-z')\right] \\ &= \int_{0}^{W} dz \int_{0}^{W} dz' \zeta_{m}(z)^{*} \zeta_{n}(z) \zeta_{m}(z') \zeta_{n}(z')^{*} \delta(z-z') \\ &= \int_{0}^{W} dz |\zeta_{m}(z)|^{2} |\zeta_{n}(z)|^{2} \\ &= \frac{\pi}{b_{mn}} \end{split}$$

 b_{mn} describes the effective extent of the interactions in the z-direction.

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Phonon Scattering in Bulk Si

- Lattice vibrations are an inevitable source of scattering
- Dominant scattering at high temperatures ~ RT

> Negligible contribution at low temperatures



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Phonon Dispersion Relationship



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Intra-Valley Acoustic Phonon Scattering - Mobility Calculation -

$$P(\mathbf{k}, \mathbf{k}') = \frac{2\pi}{\hbar} D_{ac}^{2} q^{2} \frac{\hbar}{2V\rho\omega_{q}} \left(n_{q} + \frac{1}{2} \mp \frac{1}{2} \right) \delta \left(E_{k} - E_{k'} \pm \hbar\omega_{q} \right)$$

$$\approx \frac{2\pi}{\hbar} \frac{D_{ac}^{2} k_{B} T}{2V\rho\nu_{s}^{2}} \delta \left(E_{k} - E_{k'} \pm \hbar\omega_{q} \right) \frac{k_{B} T}{\hbar\nu_{s} q}$$

$$\frac{1}{\tau(E)} = \frac{V}{8\pi^{3}} \int d^{3} \mathbf{k}' P(\mathbf{k}, \mathbf{k}') \left\{ 1 - \frac{\mathbf{e}_{ll'} \cdot \mathbf{k}'}{\mathbf{e}_{ll'} \cdot \mathbf{k}} \right\} = \frac{(2m^{*})^{3/2} D_{ac}^{2} k_{B} T}{2\pi \hbar^{4} \rho v_{s}^{2}} E^{1/2}$$

$$\langle \tau \rangle = \frac{2\pi \hbar^{4} \rho v_{s}^{2}}{(2m^{*})^{3/2} D_{ac}^{2} k_{B} T} (k_{B} T)^{-1/2} \Gamma \left(\frac{5}{2} - \frac{1}{2} \right) / \Gamma \left(\frac{5}{2} \right) = \frac{2^{3/2} \sqrt{\pi} \hbar^{4} \rho v_{s}^{2}}{3m^{*3/2} D_{ac}^{2} (k_{B} T)^{3/2}}$$

$$\mu_{ac} = \frac{2^{3/2} \sqrt{\pi} e \hbar^{4} \rho v_{s}^{2}}{3m^{*5/2} D_{ac}^{2} (k_{B} T)^{3/2}} \propto T^{-1.5}$$
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Inter-Valley Optical Phonon Scattering

Randomizing Collision Approximation

$$\mathbf{l} - \frac{\mathbf{e}_{\prime\prime\prime} \cdot \mathbf{k}'}{\mathbf{e}_{\prime\prime\prime} \cdot \mathbf{k}} = 0$$

Under this approximation, the inverse of relaxation time is equivalent to the scattering rate. Then, the momentum relaxation time is evaluated to be

$$\frac{1}{\tau_{\rm int}} = \frac{\left(2m^*\right)^{3/2}}{4\pi\hbar^3\rho} \frac{D_{op}^{2}}{\omega_0} \left[\left(n_q + 1\right)\sqrt{E - \hbar\omega_0} + n_q\sqrt{E + \hbar\omega_0} \right]$$

Subband Structure in Si MOS Interface



Crystal Structure Modification by Stress

Strain in one dimension



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Strain Tensor

The definition of the shear strain in the previous page includes the rotation without deformation.

We define the strain tensor as shown below. Based on this definition, the shear strain is zero, if the shear strain does not result in the deformation of the stuff.



In the case of three dimensional crystal, particularly the cubic crystal, the symmetry of the crystal result in $\varepsilon_{12}=\varepsilon_{21}$, $\varepsilon_{23}=\varepsilon_{32}$, $\varepsilon_{31}=\varepsilon_{13}$. As a result, the number of the independent elements of the strain tensor is 6 (3x3-3). By letting *xx*=1, *yy*=2, *zz*=3, *xy*=4, *yz*=5, *zx*=6, the strain tensor is expressed as (ε_1 , ε_2 , ε_3 , ε_4 , ε_5 , ε_6). The strain tensor of the strain tensor is expressed as (ε_1 , ε_2 , ε_3 , ε_4 , ε_5 , ε_6).

Stiffness Tensor



 $\sigma = c\epsilon$ Stress is proportional to strain.

Three-dimensional strain tensor is transformed to stress tensor by using stiffness tensor. In the case of cubic crystal, c_{iikl} is expressed by the simple formula as

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{12} & c_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{44} \end{pmatrix} \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{pmatrix}$$

例: Global Strain (Biaxial Stress)



Example : Local Strain (Uniaxial Stress) - <100> Uniaxial Strain -



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