# Physics and Engineering of CMOS Devices

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### Review of Last Lecture (1) Charge-Sheet Approximation

- All the inversion-charges are located at the interface of Si and SiO<sub>2</sub>.
- There is no potential drop and no band bending across the inversion layer.

**Depletion Approximation & Charge-Sheet Approximation** 



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### Review of Last Lecture (2) Charge-Sheet Model

Inversion-charge density  $Q_i(y)$  at the position y is expressed as



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# $V_g$ versus $\phi_s$

Surface charge = Depletion charge + Inversion charge

$$Q_{s} = -\kappa_{s}\varepsilon_{0}E_{s} = -\sqrt{2qN_{A}\kappa_{s}\varepsilon_{0}}\left[\phi + \frac{k_{B}T}{q}\left(\frac{n_{i}}{N_{A}}\right)^{2}\left[\exp\left(\frac{q\phi}{k_{B}T}\right) - 1\right]\right]^{1/2} (10)$$

Relationship between  $V_g$  and  $\varphi_s$ 

$$V_{g} = V_{FB} + \phi_{s} + \frac{\sqrt{2qN_{A}\kappa_{s}\varepsilon_{0}}}{C_{ox}} \left[\phi + \frac{k_{B}T}{q} \left(\frac{n_{i}}{N_{A}}\right)^{2} \left[\exp\left(\frac{q\phi_{s}}{k_{B}T}\right) - 1\right]\right]^{1/2}$$

By solving the above equation, we can express  $\phi_s$  as a function of V<sub>g</sub>. However, the equation cannot be solved analytically.

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# $V_g$ versus $\phi_s$ in a wide $V_g$ range



 $\phi_s$  increases linearly as  $V_g$  increases when  $V_g$ - $V_{FB}$  is small, and  $\phi_s$  is fixed at around  $2\phi_F$  when  $V_g$  is greater than  $V_{th}$ .

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 $V_{g} \text{ versus } \phi_{s} \text{ in Weak Inversion}$   $V_{g} = V_{FB} + \phi_{s} + \gamma \left[ \phi_{s} + \frac{k_{B}T}{q} \left( \frac{n_{i}}{N_{A}} \right)^{2} \left[ \exp \left( \frac{q\phi_{s}}{k_{B}T} \right) - 1 \right] \right]^{1/2}$   $\gamma \equiv \frac{\sqrt{2qN_{A}\kappa_{s}\varepsilon_{0}}}{C_{a}}$ 

In weak inversion,

When  $\phi_s = 2\phi_F$ 

$$V_g \approx V_{FB} + \phi_s + \gamma \sqrt{\phi_s}$$
$$\frac{dV_g}{d\phi_s} \approx 1 + \gamma \frac{1}{2\sqrt{\phi_s}}$$

 $V_g - V_{th} = m \left( \phi_s - 2 \phi_F \right)$ 

$$m = 1 + \gamma \frac{1}{2\sqrt{\phi_F}}$$

#### Subthreshold Current

In subthreshold region, diffusion current is greater than drift current.

Diffusion current:  $I_{d,diff} = WD \frac{\partial(qn)}{\partial y}$ Einstein's relationship:  $D = \frac{k_B T}{a} \mu$ 

$$I_{d,diff} = W \frac{k_B T}{q} \mu \frac{\partial Q_{inv}(y)}{\partial y}$$
$$= \mu \frac{k_B T}{q} \frac{W}{L} \left[ Q_{inv}(L) - Q_{inv}(0) \right]$$

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Subthreshold Current -cont'd

$$Q_{s} = -\sqrt{2qN_{A}\kappa_{s}\varepsilon_{0}}\left[\phi + \frac{k_{B}T}{q}\left(\frac{n_{i}}{N_{A}}\right)^{2}\exp\left(\frac{q\phi}{k_{B}T}\right)\right]^{1/2}$$

In weak inversion,  $Q_{depl} >> Q_{inv}$ . Since we have very small  $Q_{inv}$ , surface potential  $\phi_s(y)$  is almost constant from source to drain:  $\phi_s(0) = \phi_s(L) = \phi_{s0}$ 

$$Q_{s} = -\sqrt{2qN_{A}\kappa_{s}\varepsilon_{0}\phi_{s0}} \left[ 1 + \frac{k_{B}T}{q\phi_{s0}} \left( \frac{n_{i}}{N_{A}} \right)^{2} \exp\left( \frac{q\left(\phi_{s0} - V(y)\right)}{k_{B}T} \right) \right]^{1/2}$$
$$\approx -\sqrt{2qN_{A}\kappa_{s}\varepsilon_{0}\phi_{s0}} \left[ 1 + \frac{k_{B}T}{2q\phi_{s0}} \left( \frac{n_{i}}{N_{A}} \right)^{2} \exp\left( \frac{q\left(\phi_{s0} - V(y)\right)}{k_{B}T} \right) \right]$$

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#### Subthreshold Current -cont'd

$$Q_{inv} = -\sqrt{2qN_A\kappa_s\varepsilon_0\phi_{s0}} \left[ 1 + \frac{k_BT}{q\phi_{s0}} \left( \frac{n_i}{N_A} \right)^2 \exp\left(\frac{q\left(\phi_{s0} - V(y)\right)}{k_BT}\right) \right]^{1/2}$$

$$\approx -\sqrt{\frac{qN_A\kappa_s\varepsilon_0}{2\phi_{s0}}} \frac{k_BT}{q} \left( \frac{n_i}{N_A} \right)^2 \exp\left(\frac{q\left(\phi_{s0} - V(y)\right)}{k_BT}\right)$$

$$I_{d,diff} = \mu \frac{k_BT}{q} \frac{W}{L} \left[ Q_{inv} \left( L \right) - Q_{inv} \left( 0 \right) \right]$$

$$= \mu \frac{W}{L} \sqrt{\frac{qN_A\kappa_s\varepsilon_0}{2\phi_{s0}}} \left( \frac{k_BT}{q} \right)^2 \left( \frac{n_i}{N_A} \right)^2 \exp\left( \frac{q\phi_{s0}}{k_BT} \right) \left[ 1 - \exp\left( - \frac{qV_d}{k_BT} \right) \right]$$

$$\mu \frac{W}{L} \sqrt{\frac{qN_A\kappa_s\varepsilon_0}{3\phi_F}} \left( \frac{k_BT}{q} \right)^2 \left( \frac{n_i}{N_A} \right)^2 \exp\left( \frac{q\phi_{s0}}{k_BT} \right) \left[ 1 - \exp\left( - \frac{qV_d}{k_BT} \right) \right]$$
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#### **Drift Current versus Diffusion Current**



Below Vth, diffusion current is larger than drift current. Above Vth, drift current is larger than diffusion current. *Physics and Engineering of CMOS Devices, Ken Uchida, Tokyo Tech, June 9, 2010* 

#### Subthreshold Slope

The subthreshold slope, or S factor, is the gate voltage change (mV) necessary to have 10x drain current.

$$S = \frac{\partial V_g}{\partial \log I_d} = \ln(10) \frac{k_B T}{q} \frac{1}{\phi_{s0}} = \ln(10) \frac{m k_B T}{q}$$
$$= \ln(10) \frac{k_B T}{q} \left(1 + \frac{C_{dm}}{C_{ox}}\right)$$
$$= 59.5 \times \left(1 + \frac{C_{dm}}{C_{ox}}\right) \quad (\text{mV/decade @300K})$$

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#### Substrate-Bias Effect

The threshold voltage of nMOSFETs is increased if a negative substrate bias  $(-V_b)$  is applied.

Let us consider the threshold voltage as a function of  $V_b$ .

$$V_{th}(0) = V_{FB} + 2\phi_F + \frac{\sqrt{4qN_A\kappa_s\varepsilon_0\phi_F}}{C_{ox}} = V_{FB} + 2\phi_F + \gamma\sqrt{2\phi_F}$$
$$V_{th}(V_b) = V_{FB} + 2\phi_F + \frac{\sqrt{2qN_A\kappa_s\varepsilon_0(\phi_F + V_b)}}{C_{ox}} = V_{FB} + 2\phi_F + \gamma\sqrt{2\phi_F} + V_b$$

Here, the body factor  $\gamma$  is defined as.

$$\gamma = \frac{\sqrt{2\kappa_s \varepsilon_0 q N_A}}{C_{ox}}$$

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#### Substrate-Bias Effect -cont'd

$$\Delta V_{th} = V_{th}(V_b) - V_{th}(0)$$
  
=  $\gamma \left( \sqrt{2\phi_F + V_b} - \sqrt{2\phi_F} \right)$  (19)

$$\frac{\partial V_{th}}{\partial V_b} = \frac{\gamma}{2} \frac{1}{\sqrt{2\phi_F + V_b}}$$

$$\lim_{V_b \to 0} \frac{\partial V_{th}}{\partial V_b} = \frac{\gamma}{2} \frac{1}{\sqrt{2\phi_F}} = \frac{C_{dm}}{C_{ox}} = m - 1$$
(20)

#### Substrate-Bias Effect -cont'd



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### MOSFET Characteristics (5) Substrate bias dependence



## Summary

- Surface potential  $\phi_s$  is numerically calculated as a function of  $V_g$ .
- It is shown that  $V_g$  is proportional to  $\phi_s$  in weak inversion region.
- Subthreshold current as well as subthresoldd swing is obtained.
- Threshold voltage shift due to substrate bias is calculated.

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