Physics and Engineering of CMOS Devices

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Charge-Sheet Model

Charge-Sheet Approximation

- All the inversion-charges are located at the interface of Si and SiO₂.
- There is no potential drop and no band bending across the inversion layer.

Depletion Approximation & Charge-Sheet Approximation



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Charge-Sheet Model of MOSFETs



The channel is equivalent with the gated PN junction. At the position y, the quasi-Fermi energy level for electrons, $E_{Fn}(y)$, is given as

$$E_{Fn}(y) = V(y) + \phi_F$$

The threshold voltage at the position y is expressed as

$$V_{th}(y) = V_{FB} + 2\phi_F + V(y) + \frac{\sqrt{2qN_A\kappa_s\varepsilon_0(2\phi_F + V(y))}}{C_{ox}}$$

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Charge-Sheet Model –cont'd

Inversion-charge density $Q_i(y)$ at the position y is given by

$$Q_{i}(y) = C_{g} \left(V_{g} - V_{th}(y) \right)$$
$$= C_{g} \left(V_{g} - V_{FB} - 2\phi_{F} - V(y) - \sqrt{2qN_{A}\kappa_{s}\varepsilon_{0}\left(2\phi_{F} + V(y)\right)} \right)$$

The drain current is expressed as

$$I_{d} = WQ_{i}(y)\mu \frac{dV(y)}{dy}$$
$$= \mu C_{g}W \left(V_{g} - V_{FB} - 2\phi_{F} - V(y) - \sqrt{2qN_{A}\kappa_{s}\varepsilon_{0}\left(2\phi_{F} + V(y)\right)} \right) \frac{dV(y)}{dy}$$

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Charge-Sheet Model –cont'd

By integrating the previous equation, the drain current I_d is obtained as follows.

$$I_{d} = \mu C_{g} \frac{W}{L} \left[\left(V_{g} - V_{FB} - 2\phi_{F} \right) V_{d} - \frac{1}{2} V_{d}^{2} - \frac{2\sqrt{2\kappa_{s}\varepsilon_{0}qN_{A}}}{3C_{g}} \left(\left(2\phi_{F} + V_{d} \right)^{3/2} - \left(2\phi_{F} \right)^{3/2} \right) \right]$$
(15)

 I_d is expanded in the series of V_d .

$$I_{d} \approx \mu C_{g} \frac{W}{L} \left[\left(V_{g} - V_{FB} - 2\phi_{F} - \frac{\sqrt{4q\kappa_{s}\varepsilon_{0}N_{A}\phi_{F}}}{C_{g}} \right) V_{d} - \frac{1}{2} \left(1 + \sqrt{\frac{q\kappa_{s}\varepsilon_{0}N_{A}}{4\phi_{F}}} \left/ C_{g} \right) V_{d}^{2} \right]$$

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Charge-Sheet Model –cont'd

$$I_d \approx \mu C_g \frac{W}{L} \left[\left(V_g - V_{th0} \right) V_d - \frac{m}{2} V_d^2 \right]$$
(16)

Here,

 $V_{th0} =$

$$V_{FB} + 2\phi_F + \frac{\sqrt{4qN_A\kappa_s\varepsilon_0\phi_F}}{C_{ox}}$$

$$m = 1 + \frac{C_{dm}}{C_{ox}}$$
(17)

m is the body-effect coefficient.

Eq (16) reaches its maximum at the saturation drain voltage $V_{d,sat}$ given below.

$$V_{d,sat} = \frac{V_g - V_{th0}}{m}$$
(18)
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Charge-Sheet Model –cont'd

The saturation drain current, $I_{d,sat}$, is expressed as

$$I_{d,sat} = \frac{1}{2} \mu C_{ox} \frac{W}{L} \frac{\left(V_g - V_{th0}\right)^2}{m}$$
(18)

In charge-sheet model, the effect of substrate impurities (acceptor ions in the case of nMOSFETs) is included.

Because of the depletion charges, the saturation voltage and the saturation current are lower than those in the simple analytical MOSFET model discussed in the first lecture.

When the threshold voltage of MOSFETs is discussed, the source-to-gate voltage V_{gs} is usually considered. In other words, $V_{th}(0)$ in page 8 of this foil with V(y) = 0 is considered.

Substrate-Bias Effect

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Substrate-Bias Effect

The threshold voltage of nMOSFETs is increased if a negative substrate bias $(-V_b)$ is applied.

Let us consider the threshold voltage as a function of V_b .

$$V_{th}(0) = V_{FB} + 2\phi_F + \frac{\sqrt{4qN_A\kappa_s\varepsilon_0\phi_F}}{C_{ox}} = V_{FB} + 2\phi_F + \gamma\sqrt{2\phi_F}$$
$$V_{th}(V_b) = V_{FB} + 2\phi_F + \frac{\sqrt{2qN_A\kappa_s\varepsilon_0(\phi_F + V_b)}}{C_{ox}} = V_{FB} + 2\phi_F + \gamma\sqrt{2\phi_F} + V_b$$

Here, the body factor γ is defined as.

$$\gamma = \frac{\sqrt{2\kappa_s \varepsilon_0 q N_A}}{C_{ox}}$$

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Substrate-Bias Effect -cont'd

$$\Delta V_{th} = V_{th}(V_b) - V_{th}(0)$$

= $\gamma \left(\sqrt{2\phi_F + V_b} - \sqrt{2\phi_F} \right)$ (19)

$$\frac{\partial V_{th}}{\partial V_b} = \frac{\gamma}{2} \frac{1}{\sqrt{2\phi_F + V_b}}$$

$$\lim_{V_b \to 0} \frac{\partial V_{th}}{\partial V_b} = \frac{\gamma}{2} \frac{1}{\sqrt{2\phi_F}} = \frac{C_{dm}}{C_{ox}} = m - 1$$
(20)

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Summary

- Charge-sheet approximation is introduced.
- Charge-sheet model, which is one of the most widely used MOSFET model, is derived.
- The substrate-bias effect is discussed. The threshold voltage shift induced by the substrate bias is obtained.