# Physics and Engineering of CMOS Devices

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### **Depletion Approximation**

Effect of substrate impurity

# Brief Review of Semiconductor Physics Bulk Bandstructure



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# Brief Review of Semiconductor Physics

#### **MOS Structure**

Flat-band condition



 $q\phi_m = q\chi + \frac{E_g}{2} + q\phi_F$ 

The band in Si is bent even under zerobias condition, because of the work function difference.



Flat band voltage,  $V_{FB}$  is the gate-tosubstrate voltage necessary to realize flatband condition.

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# **MOS Capacitor: Operation**



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# MOS Capacitor: $\phi_s$ versus $Q_{depl}$



#### **Depletion Approximation**

- The carrier concentrations are assumed to be negligibly small compared to the net doping concentration.
- The charge density outside the depletion region is assumed to be zero.

# MOS Capacitor: $\phi_s$ versus $Q_{depl}$



# MOS Capacitor: $\phi_s$ versus $Q_{depl}$ –cont'd

Electric Field at the surface,  $F_s$ .  $F_s = \sqrt{\frac{2qN_A\phi_s}{\kappa_s\varepsilon_0}}$ Depletion region has the capacitance.  $C_d = \frac{\partial |Q_{depl}|}{\partial \phi_s}$   $C_d = \sqrt{\frac{qN_A\kappa_s\varepsilon_0}{2\phi_s}}$ (5)

As  $N_A$  increases, depletion capacitance,  $C_d$ , increases. This is due to the reduction of depletion layer width.

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### Solving Poisson's Equation

#### Accumulation and Inversion Conditions

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Suppose that substrate is p-type with acceptor concentration of  $N_A$ .

$$n_{p0}p_{p0} = n_i^2 \qquad N_A \approx p_{p0} = n_i \exp\left(\frac{q\phi_F}{k_B T}\right) \qquad (6)$$

$$n_{p0} \approx \frac{n_i^2}{N} \qquad \frac{n_i^2}{N} \approx n_{p0} = n_i \exp\left(-\frac{q\phi_F}{k_B T}\right) \qquad (7)$$

### Brief Review of Semiconductor Physics Bandstructure at Semiconductor Surface



MOS Capacitor:  $\phi_s$  versus  $Q_s$ 

We will solve Poisson's equation.

$$\frac{d^2\phi}{dx^2} = -\frac{q}{\kappa_s \varepsilon_0} \left( N_D - N_A + p_p - n_p \right)$$
$$= -\frac{q}{\kappa_s \varepsilon_0} \left[ -N_A - \frac{n_i^2}{N_A} \exp\left(\frac{q\phi}{k_B T}\right) \right] \qquad N_D \approx 0$$
$$p_p \approx 0$$

Multiplying both sides by  $(d\phi/dx)dx$ , we will obtain

$$\frac{1}{2}\left(\frac{d\phi}{dx}\right)^{2} = \frac{qN_{A}}{\kappa_{s}\varepsilon_{0}}\left[\phi + \frac{k_{B}T}{q}\left(\frac{n_{i}}{N_{A}}\right)^{2}\left[\exp\left(\frac{q\phi}{k_{B}T}\right) - 1\right]\right]$$

$$Q_{s} = -\kappa_{s}\varepsilon_{0}E_{s} = -\sqrt{2qN_{A}}\kappa_{s}\varepsilon_{0}}\left[\phi + \frac{k_{B}T}{q}\left(\frac{n_{i}}{N_{A}}\right)^{2}\left[\exp\left(\frac{q\phi}{k_{B}T}\right) - 1\right]\right]^{1/2} (10)$$
electrons + substrate impurity

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# MOS Capacitor: $\phi_s$ versus $Q_s$ –cont'd

Of course, we can include  $p_p$ .

$$\frac{d^2\phi}{dx^2} = -\frac{q}{\kappa_s \varepsilon_0} \left[ -N_A + N_A \exp\left(-\frac{q\phi}{k_B T}\right) - \frac{n_i^2}{N_A} \exp\left(\frac{q\phi}{k_B T}\right) \right]$$

$$\frac{1}{2} \left(\frac{d\phi}{dx}\right)^2 = \frac{qN_A}{\kappa_s \varepsilon_0} \left[\phi - \frac{k_B T}{q} \left[\exp\left(-\frac{q\phi}{k_B T}\right) - 1\right] + \frac{k_B T}{q} \left(\frac{n_i}{N_A}\right)^2 \left[\exp\left(\frac{q\phi}{k_B T}\right) - 1\right]\right]$$

$$Q_{s} = \mp \sqrt{2qN_{A}\kappa_{s}\varepsilon_{0}} \left[ \phi - \frac{k_{B}T}{q} \left[ \exp\left(-\frac{q\phi}{k_{B}T}\right) - 1 \right] + \frac{k_{B}T}{q} \left(\frac{n_{i}}{N_{A}}\right)^{2} \left[ \exp\left(\frac{q\phi}{k_{B}T}\right) - 1 \right] \right]^{1/2}$$
  
electrons + holes + substrate impurity (11)

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When  $\phi_s = 2\phi_F$ , the minority carrier density at the surface is comparable to the depletion charge density. Therefore,  $\phi_s$  of  $2\phi_F$ is defined as the threshold. When  $\phi_s$  is in the range from  $\phi_F$  to  $2\phi_F$ , the condition is called the weak inversion. When  $\phi_s$  is greater than  $2\phi_F$ , the condition is called the strong inversion.

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# MOS Capacitor: $\phi_s$ versus $Q_s$

 $\phi_s = 2\phi_F$ 

At  $\phi_s = 2\phi_F$ , the minority carrier density at the surface is equal to the majority carrier density in the substrate;  $n_s = N_A$ 

The threshold voltage, where the surface condition changes from the weak inversion to the strong inversion, is defined as,



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# Non-equilibrium condition

### Quasi-Fermi Level in non-equilibrium Semiconductor

In equilibrium semiconductors, Fermi energy level ( $E_F$ ) is defined either by the electron density and the hole density. In other words, both electron and hole densities can calculated using the Fermi energy level.

However, in non-equilibrium semiconductors, the single  $E_F$  is not enough to describe carrier numbers. For example, in light-illuminated semiconductors, many electrons and holes are populated. In those non-equilibrium semiconductors,  $E_F$  for electrons should be closer to the conduction band than to the valence band in order to generate appropriate number of electrons with  $n_i \exp(E_i - E_F/k_B T)$ , whereas  $E_F$  for holes should be closer to the valence band. Therefore, under non-equilibrium condition, Fermi energy level for electrons and that for holes should be defined separately, based on the number of electrons and holes respectively.

These Fermi energy levels are called **qusi-Fermi energy levels**. For electrons and holes, they are written as  $E_{Fn}$  and  $E_{Fn}$ , respectively.

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A biased PN junction is another example of the non-equilibrium semiconductor.

In the depletion layer, we have  $E_{Fn}$  for electrons and  $E_{Fn}$  for holes.

## **Gated PN Junction**



In gated PN junctions, we have  $E_{Fn}$  for electrons and  $E_{Fp}$  for holes when the junction is biased.

The point is that in the depletion region  $E_{Fn}$  should be used to calculate the electron densities.

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### Gated PN Junction -cont'd



$$V_{th} = V_{FB} + 2\phi_F + V_R + \frac{\sqrt{2qN_A\kappa_s\varepsilon_0\left(2\phi_F + V_R\right)}}{C_{ox}}$$
(13)

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# Summary

- Depletion approximation is introduced to derive  $\phi_s$  versus  $Q_{depl}$  characteristics.
- In the MOS structure, Poisson's equation is solved. The equations representing accumulation, depletion, weak-inversion, and strong-inversion are obtained.
- Quasi Fermi level is introduced to discuss carriers in non-equilibrium semiconductors.
- Carriers in the gated PN junction is discussed.

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