The Epistemic Foundation of Rational Choice

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Techniques for Rational Analysis

Rational Choice and Epistemology

Rational choice is about choosing an alternative with the highest utility.

Q. Why should we bother epistemological foundation?
Ans. Decision makers are boundedly rational facing complex decision situations.

Two directions are possible dealing with complexity:

- simple heuristics and other non-rational approaches
- construct a reduced mental model of the world and choose rationally on the model

This lecture focuses on the latter.

The World and the Decision Frame

- Once the decision maker made the choice, the causal relationship between the choice (cause) and the consequence is objective (physical), independent of the mental frame.
- The interpretation of the consequences along with the setting of the decision frame (perceived set of alternatives and uncertain states of the world) is subjective.

Example (Photography)

- Choice of frame direction, range and resolution is subjective.
- Once the relevant setting is done, the relationship between the motif and its image on the photograph is objective.

The Basic Intuition underlying the Mental Frames

- Decision makers do not have a perfect mental model of the world.
- They make decisions with a tentative model that is not refuted at the moment.
- If the model gets refuted, the decision maker doubts the incompleteness of her mental model first, before she thinks that the characteristics of the external world has changed.

Strategic Use of Coarse Models

Coarse Evaluation

- Strategic decisions are often made without specifying the details.
  i.e.) rough strategic plan $\longrightarrow$ tactical details
  Q. Is it legitimate to make decisions such a way?
  Ans. Optimal substructure of set-valued solution provides a sufficient condition for the legitimacy.
- Set-valued solution characterizes hierarchical diversity in the society – finer coordination is possible within the expected range in the smaller communities

Local Dominance

Definition (Local Dominance)

- $a \in A$ locally dominates $a' \in A$ at the range of uncontrollable states $Y \subset X$ if
$$\forall x \in Y[u(a,x) \geq u(a',x)]$$
- $B \subset A$ is locally dominant at the range $Y \subset X$ if
$$\forall a \in B \forall a' \in A \setminus B[a \text{ locally dominates } a']$$

- Notice that the definition is applied to a subset of actions rather than a single best action.
- Local dominance describes the situation in which, if you know that the state of the world is within a certain range, then you can make choices from a certain range of alternatives.

Application to Interactive Decision Situations

Definition

$B \subset A$ is locally dominant if $\forall i \in N[B_i \text{ is locally dominant at } B_{-i}]$.

Example

$1 \backslash 2$	a	b	c	d
α	1, 1	2, 3	0, 0	0, 0
β	3, 2	1, 1	0, 0	0, 0
γ	0, 0	0, 0	1, 1	2, 3
δ	0, 0	0, 0	3, 2	1, 1

- $\{\alpha, \beta\} \times \{a, b\}$ is locally dominant.
- If player 1 knows that player 2 chooses within $\{a, b\}$, then she is better off choosing either α or β than choosing any other action.

Optimal Substructure

Theorem ([Kobayashi and Kijima(2009)])

Let $B \subset A$ be a locally dominant subspace. For $\forall C \subset B$, if C is locally dominant in the game restricted to B, then C is also locally dominant the original game.

Example

1 \2	a	b
α	1, 1	2, 3
β	3, 2	1, 1

$\Rightarrow$

1 \2	a	b	c	d
α	1, 1	2, 3	0, 0	0, 0
β	3, 2	1, 1	0, 0	0, 0
γ	0, 0	0, 0	1, 1	2, 3
δ	0, 0	0, 0	3, 2	1, 1

Partitioned Games

Definition (Partition game)

Let P^i be a frame on action space A_i for each player i, and $P := \times_{i \in N} P^i$. A partition game $\Gamma(P) = (N, P, \hat{u})$ of game Γ is a normal form game that satisfies

$$\forall i \in N, \forall B \in P : \min_{a \in B} u_i(a) \leq \hat{u}_i(B) \leq \max_{a \in B} u_i(a)$$

Theorem

Take a game Γ and a partition game $\Gamma(P)$. Assume that an action profile subspace $B^* \in P$. Then,
B^* is locally dominant in game $\Gamma \Rightarrow B^*$ is a Nash equilibrium of game $\Gamma(P)$

Example

Example

$1 \backslash 2$	$\{a, b\}$	$\{c, d\}$
$\{\alpha, \beta\}$	1, 3	0, 0
$\{\gamma, \delta\}$	0, 0	3, 2

- A partition game is obtained by labeling subsets by a coarse frame:
  - $A_1^1 = \{\{a, b\}, \{c, d\}\}$
  - $A_2^2 = \{\{\alpha, \beta\}, \{\gamma, \delta\}\}$
- $(\{a, b\}, \{c, d\}) \in A_1^1 \times A_2^2$ is a Nash equilibrium.
  (Recall that it is locally dominant in the original game.)

Implications

Local dominance along with partitioning of language are applicable to the following contexts:

- Strategic Decisions – If local dominance is applicable successively, then the succession characterizes the legitimacy of successively focusing on finer details in a narrower range.
- Hierarchical Diversity in the Society – finer coordination is possible within the expected range in the smaller communities

Nash Equilibrium as a Local Optimum
– Stability of Social Norms

Self-Reinforcing Nature of Stable Social Norms

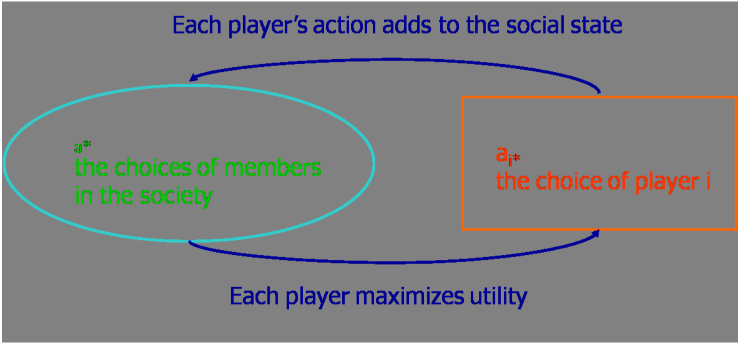

Figure: Micro-Macro Feedback Loop of Stable Social Norms [?]

Social Norm and Coordination

Example (Which Side to Drive?)

Table: Driving Game

1 \2	Left	Right
Left	1	0
Right	0	1

Social Norm as the Automatic Alternative

Example (A Native's Frame on the Driving Game in UK)

If the decision maker knows the correct answer, the set of alternatives along with the can be a singleton set.

1 ＼ 2	Left	Right
Left	1, 1	0, 0
Right	0, 0	1, 1

The British natives know the rule "you have to drive on the left side".

$$\forall i \in N[domP_i = \{(Left, Left)\}]$$

Social Norm as the Best Response

Example (A Foreigner's Frame on the Driving Game in UK)

＼ 2(natives) 1(foreigner)	Left	Right
Left	1, 1	0, 0
Right	0, 0	1, 1

A stranger (foreigner) may not be sure which side to drive but they can observe the natives driving on the left side. If he does not want to crush into the natives, it is optimal for him to drive on the left side.

- $\forall i \in Foreigners[domP_i = \{Left, Right\} \times \{Left\}]$
- $\forall i \in Natives[domP_i = \{(Left, Left)\}]$

Change in the Environment

Restricting the decision frame may cause inefficiency in the long-run.

Example (UK connected with France via EURO Tunnel)

1 \ 2	Left	Right
Left	1, 1	0, 0
Right	0, 0	1.2, 1.2

Restricting the perspective to the extant way, that is $domP_i = \{(Left, Left)\}$, deprives the decision makers from a better opportunity of driving on the right side.

M. Aoki.
Toward a Comparative Institutional Analysis.
MIT Press, Cambridge, 2001.

N. Kobayashi and K. Kijima.
Optimal substructure of set-valued solutions of normal-form games and coordination.
Journal of Systems Science and Complexity, 22(1):63–76, 03 2009.
URL http://dx.doi.org/10.1007/s11424-009-9147-9.