Axiomatic Bargaining
with emphasis on renegotiation

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Pareto Order and Efficiency (Optimality)

Definition (Pareto Order)

For $x, y \in \Re^N$:

- $x \geq y \Leftrightarrow (\forall i \in N)(x_i \geq y_i)$
- $x > y \Leftrightarrow x \geq y \wedge x \neq y$
- $x \gg y \Leftrightarrow (\forall i \in N)(x_i > y_i)$

Definition (Pareto Efficiency (Optimality))

$x \in S \subset \Re^N$ is

- (weakly) Pareto efficient iff not $(\exists y \in S)(y \gg x)$
- strongly Pareto efficient iff not $(\exists y \in S)(y > x)$

Bargaining Games and Solutions

Definition (Bargaining problem)

A bargaining problem is a pair $(S, d) \in \Sigma$, where $S \subset \Re^N$ and $d \in S$.

- S is a utility possibility set (UPS).
- d is a disagreement point.

Definition (Solution)

A solution is a function $\varphi : \Sigma \to \Re^N$ such that $\forall (S, d) \in \Sigma : \varphi(S, d) \in S$.

Henceforth, without loss of generality, $d \equiv 0$ is assumed for simplicity of representation.

Decomposability and Comprehensiveness

Definition (Comprehensiveness)

$\forall x \in S, y \in \Re_{+}^{N}$:

$$y \leq x \Rightarrow y \in S$$

Comprehensiveness of S is necessary to obtain weak Pareto-efficiency of an egalitarian solution.

Decomposability and Strong Monotonicity [Kalai(1977)]

Definition (Decomposability (Step-by-Step Negotiation))

φ satisfies decomposability if it satisfies the following.
For $\forall S \subset \forall S'$, define $S'' \equiv \{x'' \in \Re^N_+ | \exists x' \in S' : x' = x'' + \varphi(S)\}$.
Then,

$$\varphi(S') = \varphi(S) + \varphi(S'')$$

Definition (Strong Monotonicity (Issue Monotonicity))

$$\forall S \subset \forall S' : \varphi(S) \leq \varphi(S')$$

Equivalence Theorem

Definition (Proportional)

Solution φ is proportional if $\exists p_1, ..., p_n > 0$, $\forall S \in \Sigma$

$$\varphi(S) = \max\{\lambda | \lambda p \in S\} p$$

Theorem (Kalai [1977])

Bargaining solution φ is decomposable $\Leftrightarrow$ φ is strongly monotonic

Thus we obtain the following collorary.

Theorem

Bargaining solution φ is decomposable $\Leftrightarrow$ φ is proportional

E. Kalai.
Proportional solutions to bargaining situations: interpersonal utility comparisons.
Econometrica, 45:1623–1630, 1977.