What is Game Theory?

A theory of decision making with multiple agents in which
an agent's action affects the outcome to another agent
→ game situation

Two Branches of Game Theory

Non-cooperative Game Theory

No communication is allowed among agents.

Each agent chooses his/her action independently.

(e.g. price competition among firms)

Cooperative Game Theory

Communication is allowed.

Contract is signed once an agreement is reached.

→ Agreement is binding.

(e.g. mergers, negotiation among countries)

Representing A Game Situation

Strategic Form Game (Normal Form Game)

Decision maker	Player
A contingent plan of action	Strategy
Each agent’s valuation of an outcome	Payoff

Extensive Form Game

Each player’s actions in time are represented by a tree form.

Characteristic Function Form Game（Coalitional Game）

Main Representation of Cooperative Game Situations

History of Game Theory

J. von Neumann and O. Morgenstern

"Theory of Games and Economic Behavior" (1944)

2-person zero-sum games → **Non-cooperative**

Strategic Form Games, Extensive Form Games "Minimax Thm"

2-person nonzero-sum games → **Cooperative**

Games in Characteristic Function Form

Games with 3 or more players → **Cooperative** "Stable Set"

J.F. Nash (1994 Nobel Laureate)

2-person zero-sum game → **Non-cooperative** "Nash equilibrium"

Games with 3 or more players

2 person nonzero-sum games → **Cooperative**

Bargaining Game "Nash Bargaining Solution"

Developments in Game Theory

Completely Rational

Complete information of the structure of the game

Decision based on deduction of other players' actions

Refinement of Nash equilibria

→ R. Selten (1994 Nobel Laureate)

Cooperative solutions → R.J. Aumann (2005 Nobel Laureate)

L.S. Shapley ・・・

↓

Bounded Rational

Incomplete information → Games with incomplete information

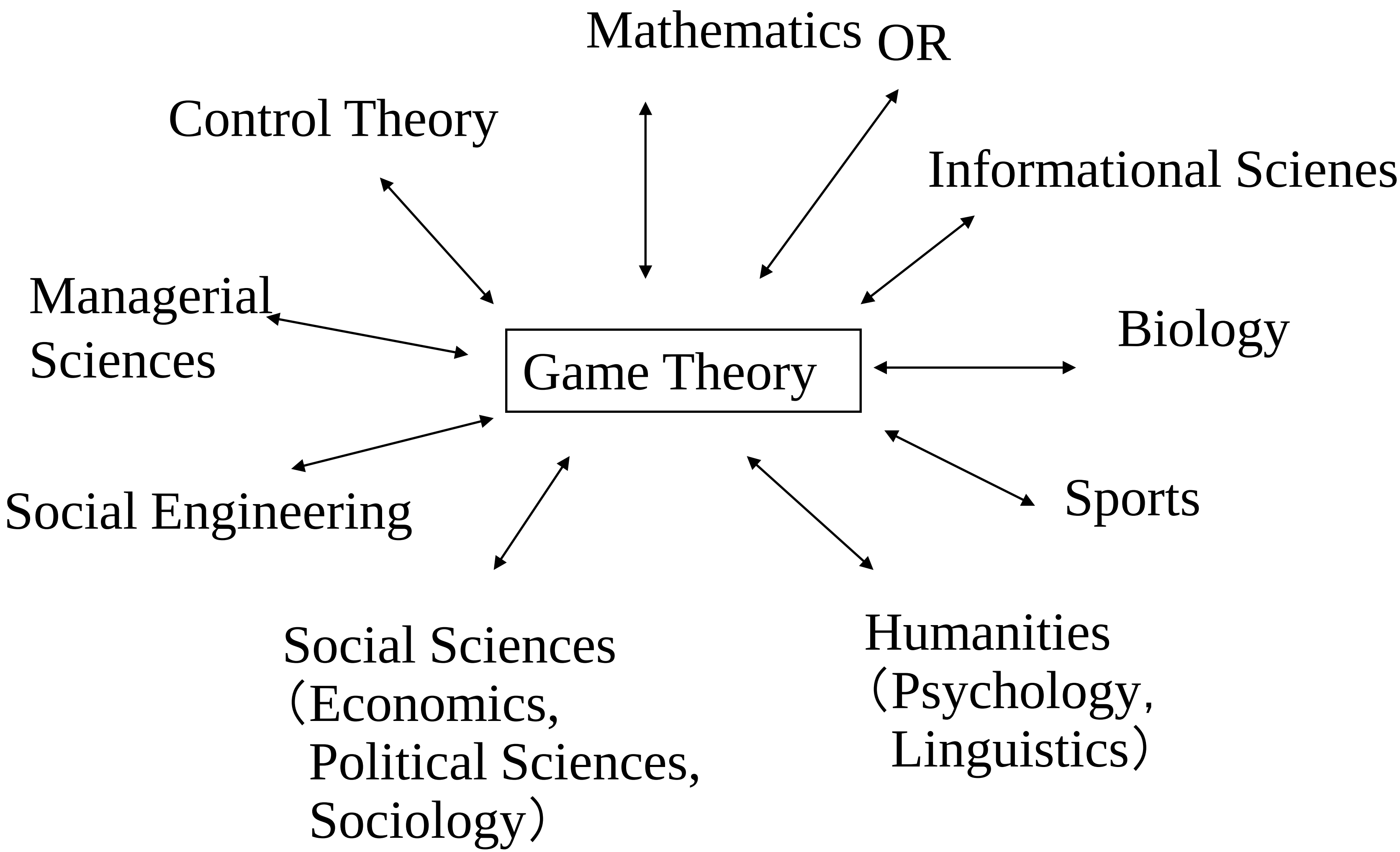
J.C. Harsanyi (1994 Nobel Laureate)

Simple decision making → Evolutionary game theory, Finite automata,

Neural network, Experimental game theory

Unification of Cooperative Game Theory and Noncooperative Game Theory

Game Theory and Related Fields



Overview of this Course

Cooperative Games

- Two-person cooperative games
  - Bargaining game
  - Nash bargaining solution
- Cooperative games with three or more players
  - Games in characteristic function form
  - Core, Nucleolus, Shapley value

Example 2－2

B	X		Y	
A				
X	6	4	0	0
Y	0	0	4	6

↓

Noncooperative game

→ （X, X), （Y, Y) Nash equilibria

Cooperative

→ What if A and B are able to communicate with each other?

Correlated Strategy

B / A	X		Y	
X	6	4	0	0
Y	0	0	4	6

A , B can coordinate their actions,

both play strategy X (payoff 6, 4),

or both play Y (payoff 4, 6)

or

using a fair coin (probability that heads comes up is ½)

if the coin lands heads, A plays X, B plays X,

and if the coin lands tails, A plays Y, B plays Y

expected payoff (5, 5)

→ correlated strategy

Feasible Payoffs under Correlated Strategies

B / A	X	Y
X	6 4	0 0
Y	0 0	4 6

Correlated strategy :

Let $r_{11}, r_{12}, r_{21}, r_{22}$ be the respective probabilities that (X, X), (X, Y), (Y, X), (Y, Y) will be played:

$$r_{11} + r_{12} + r_{21} + r_{22} = 1, \quad r_{11}, r_{12}, r_{21}, r_{22} \geq 0$$

Exp. payoff : A $u_A = 6r_{11} + 0r_{12} + 0r_{21} + 4r_{22} = 6r_{11} + 4r_{22}$

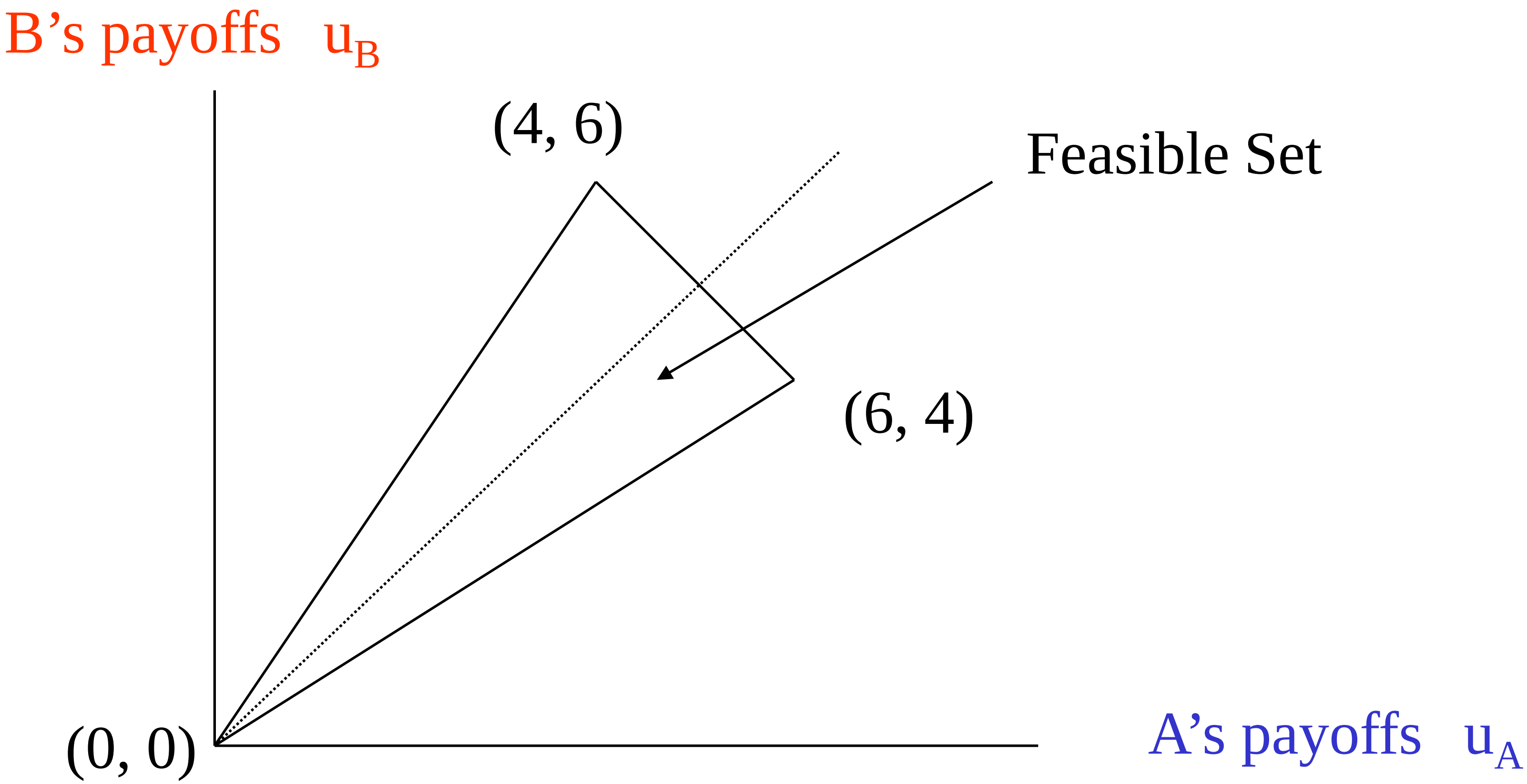
B $u_B = 4r_{11} + 0r_{12} + 0r_{21} + 6r_{22} = 4r_{11} + 6r_{22}$

Feasible Set

Exp. payoff : A $\quad u_A = 6r_{11} + 0r_{12} + 0r_{21} + 4r_{22} = 6r_{11} + 4r_{22}$

B $\quad u_B = 4r_{11} + 0r_{12} + 0r_{21} + 6r_{22} = 4r_{11} + 6r_{22}$

$$r_{11} + r_{12} + r_{21} + r_{22} = 1, \quad r_{11}, r_{12}, r_{21}, r_{22} \geq 0$$



Bargaining Game

Which payoff vector (or outcome) in the feasible set should players A and B agree on?

Disagreement Point（an outcome that results when negotiations between A and B break up）

(e.g. maximin value, Nash equilibrium)

Bargaining Game

Feasible Set R,　Disagreement point (u^0_A, u^0_B)

→ What will be the agreement point (u^*_A, u^*_B) ?

Applications： Price negotiations, Wage negotiations, Disarmament

Nash Bargaining Solution and Axioms

Bargaining Game $(R,\ u^0 = (u^0_A, u^0_B)) \rightarrow$

$u^* = (u^*_A, u^*_B)$ should satisfy the following four properties

1 Pareto optimality (or Pareto efficiency)

2 Symmetry

3 Independence of Strictly Positive Affine Transformation

4 Independence of Irrelevant Alternatives (IIA)

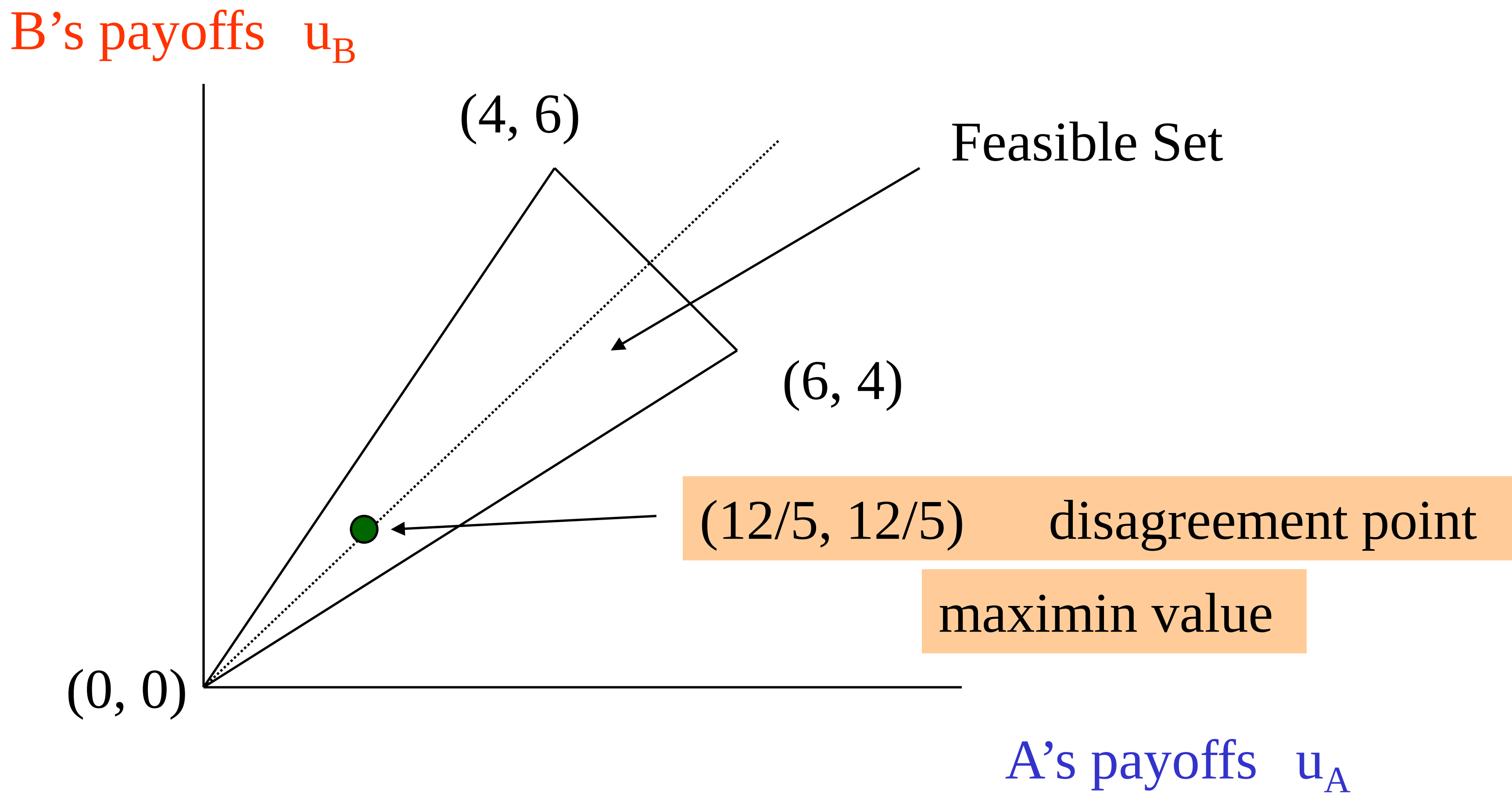
$\rightarrow$ Only one $u^* = (u^*_A, u^*_B)$ that satisfies 1-4 exists and

$$(u^*_A - u^0_A) \times (u^*_B - u^0_B)$$

$$= \text{Max}\ \{(u_A - u^0_A) \times (u_B - u^0_B) \mid u_A \geq u^0_A,\ u_B \geq u^0_B\}$$

$\longrightarrow$ Nash bargaining solution

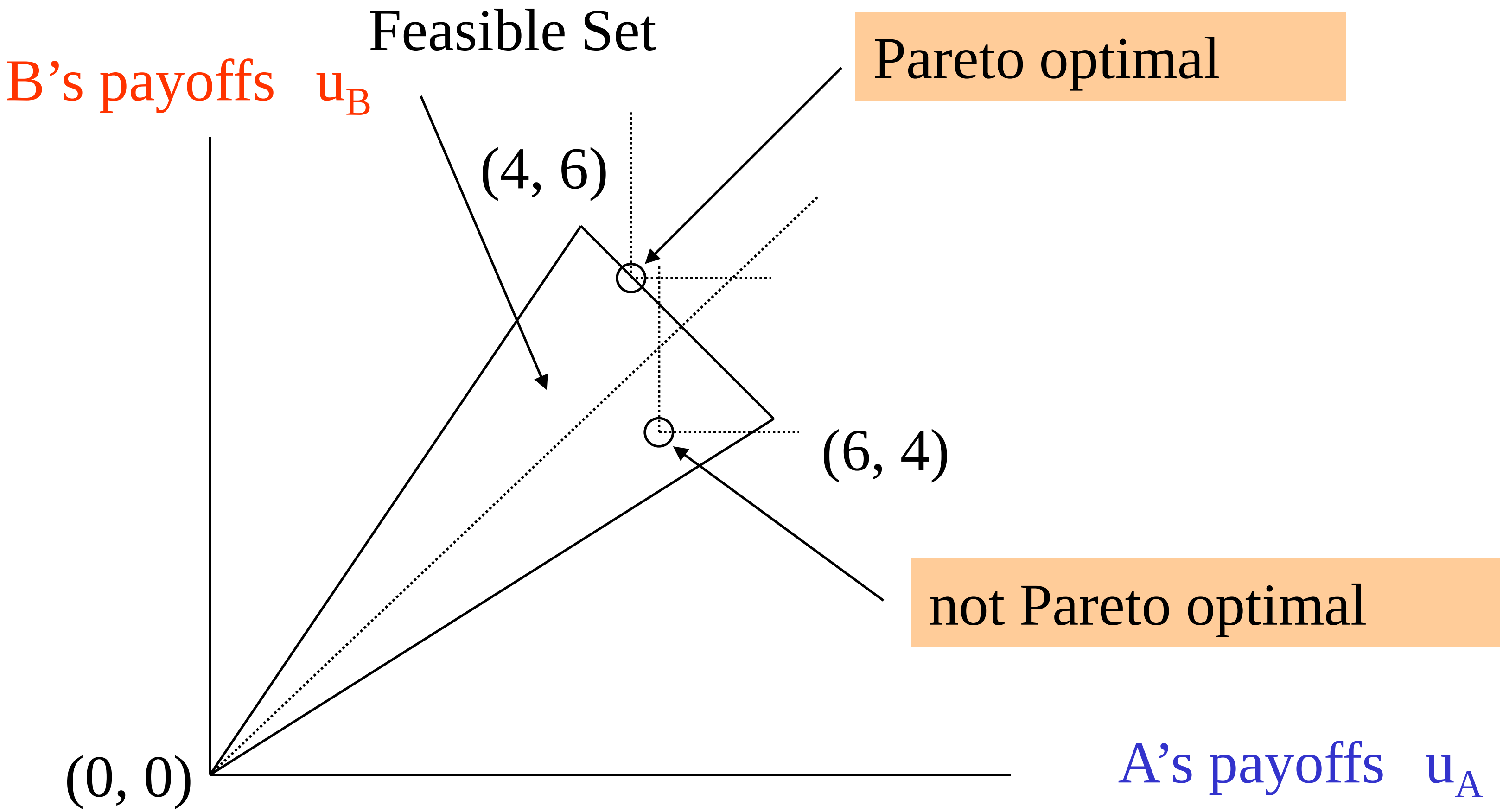
Example 5-1



B's payoffs u_B
(4, 6)
Feasible Set
(6, 4)
(12/5, 12/5) disagreement point
maximin value
(0, 0)
A's payoffs u_A

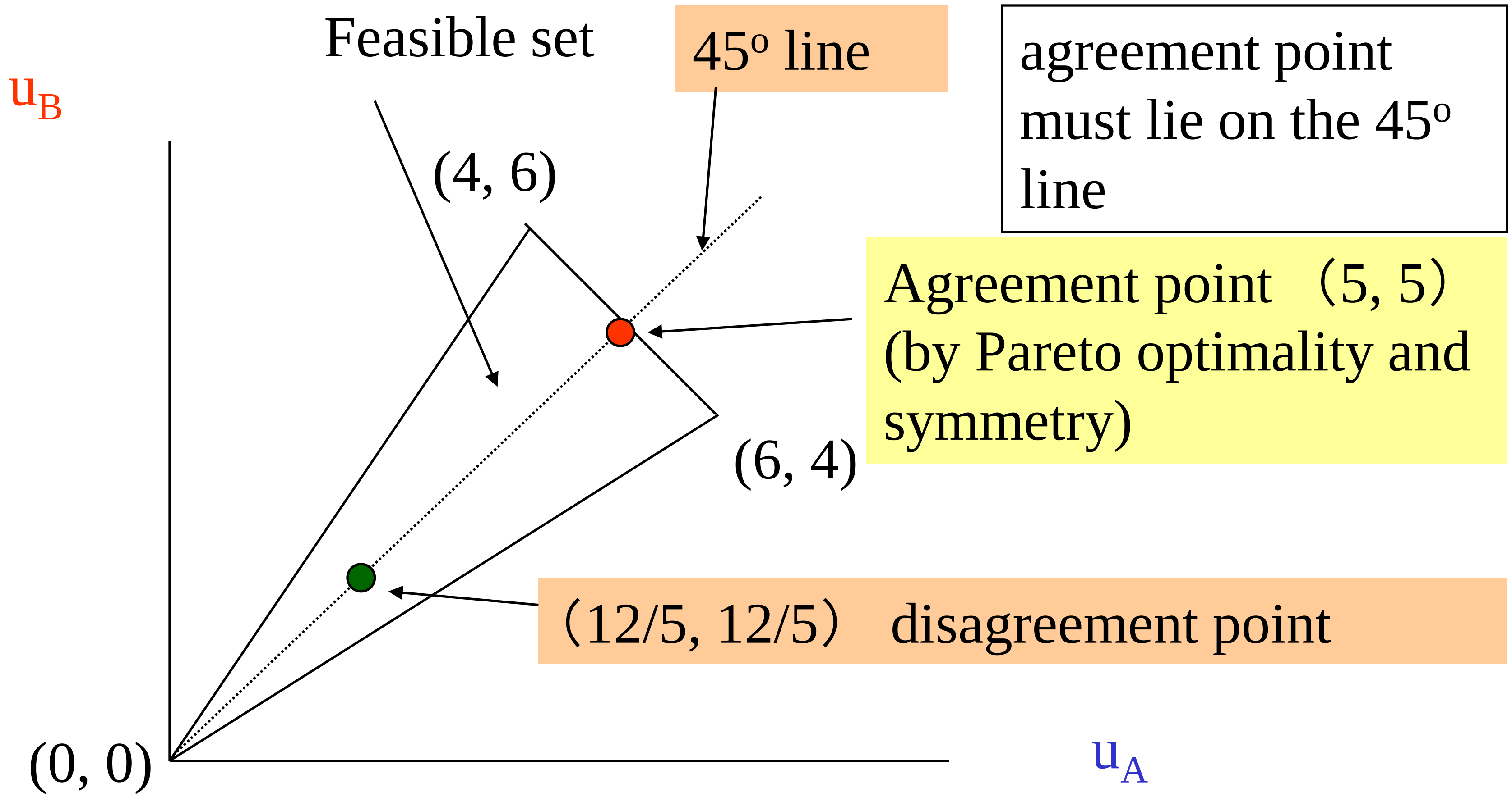
Pareto Optimality

At the agreement point, if one player's payoff is increased, the other player's payoff has to decrease as a result.



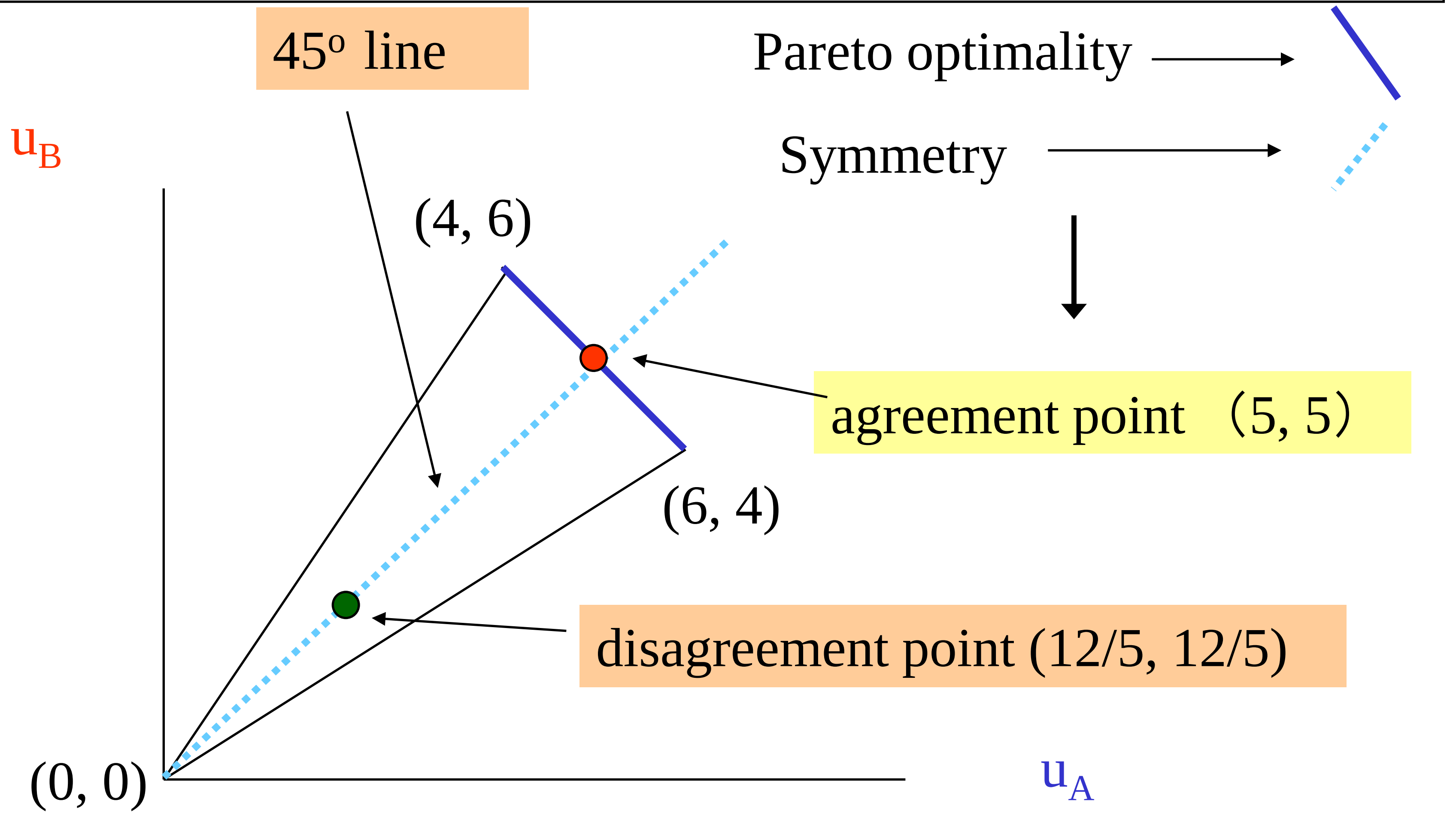
Symmetry

If both players receive the same payoffs at the disagreement point and if the feasible set is symmetric with respect to the 45° line, then both players' payoffs at the agreement point are equal.



Nash Bargaining Solution of Ex 5-1

Nash bargaining solution is（5, 5）by Pareto optimality and symmetry



Independence of Strictly Positive Affine Transformation（1）

The agreement point should not depend on the units and intervals in which payoffs are measured

B \ A	X		Y	
X	6	4	0	0
Y	0	0	4	6

Units : A: ¥ (100 million)
B: ¥ (100 million)

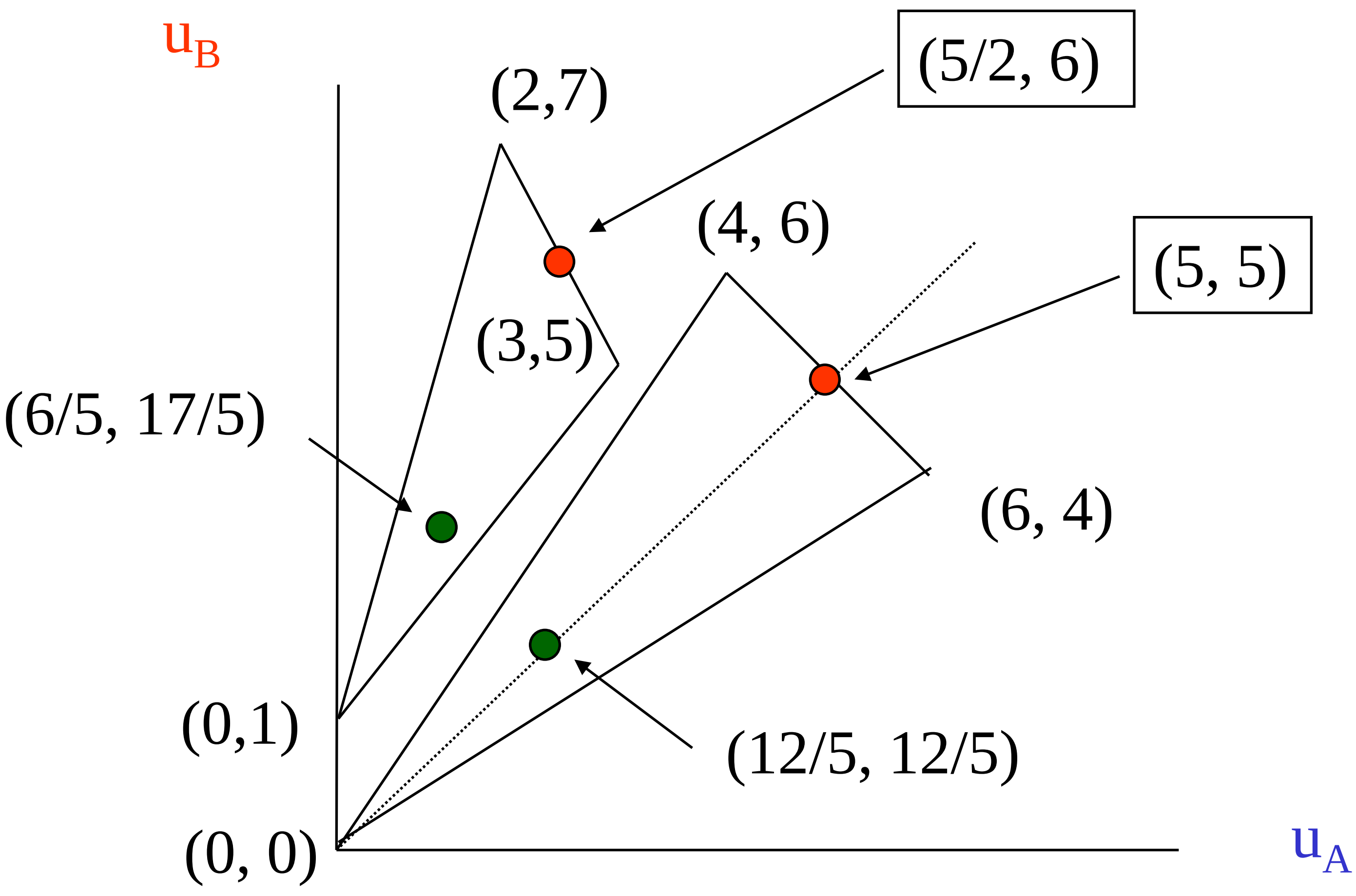
A's payoffs are now in dollars (1$=200¥)

B's payoffs are increased by 1

B \ A	X		Y	
X	3	5	0	1
Y	0	1	2	7

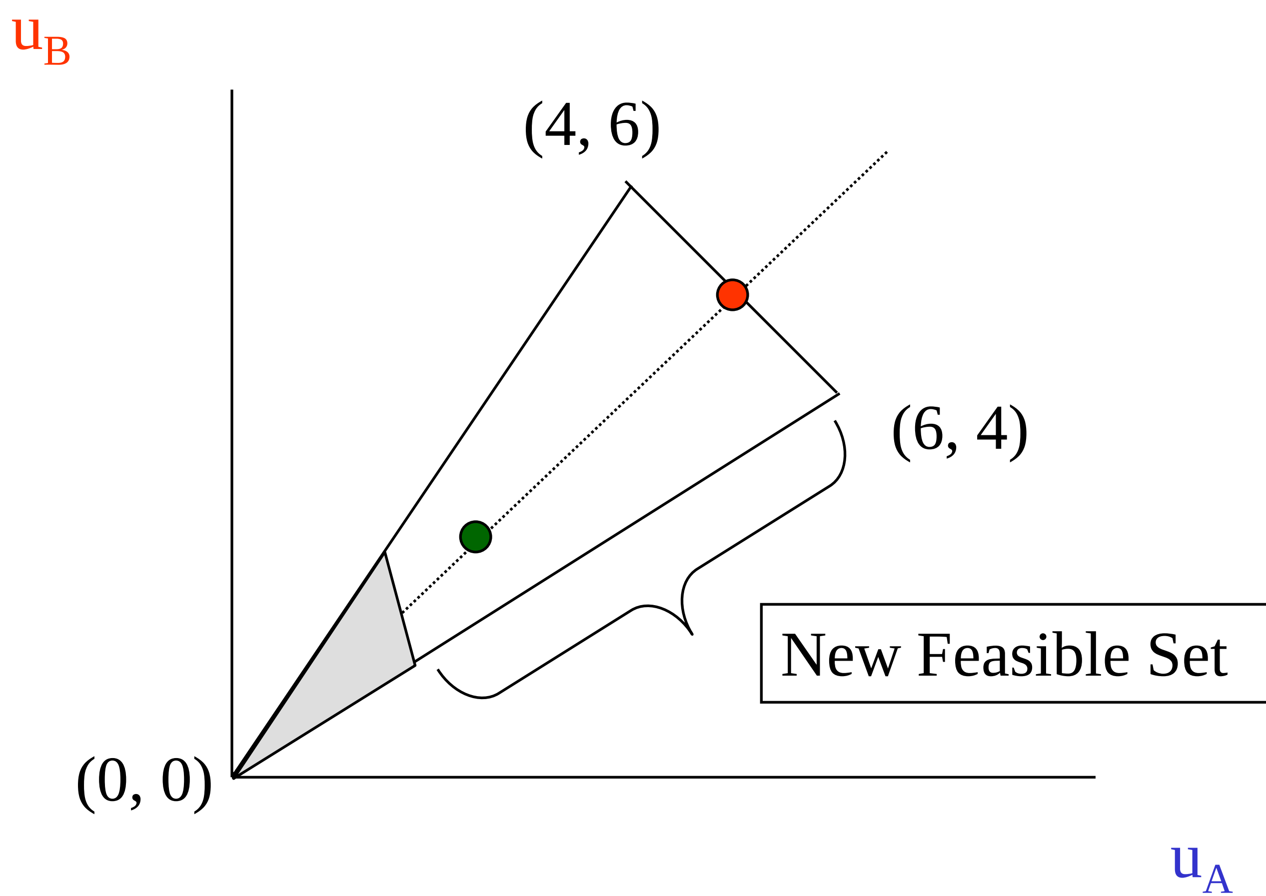
Units : A: $ (1 million)
B: ¥ (100 million)

Independence of Strictly Positive Affine Transformation（2）



Independence of Irrelevant Alternatives (IIA)

Even if a region that does not include the agreement point and the disagreement point are excluded from the feasible set, then the agreement point of the new set is the same.



Calculating Nash Bargaining Solution（Ex. 5-1）

Ex. 5-1　:　From Pareto optimality and symmetry　(5, 5)

Generally :　Set of Pareto optimal payoffs

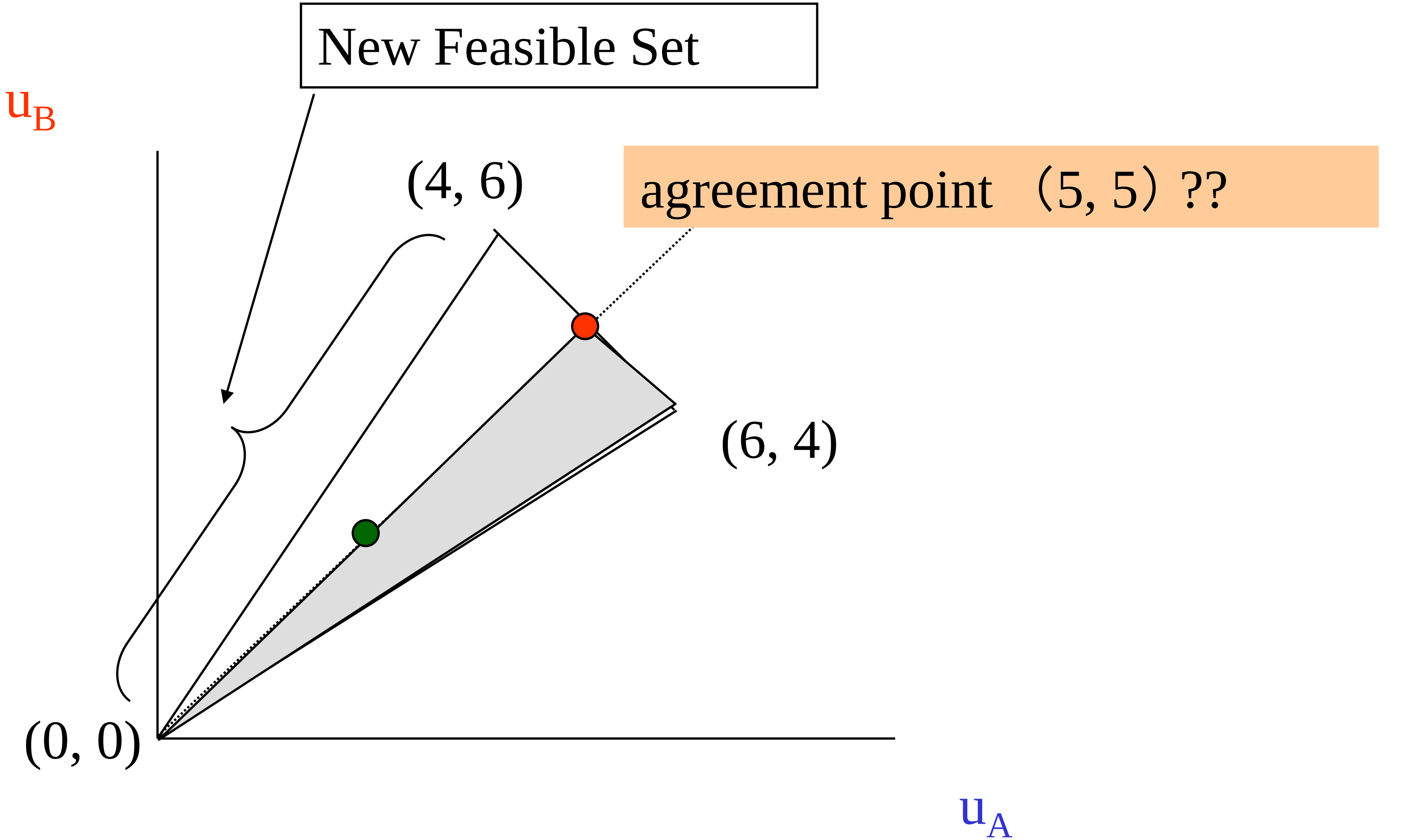
$$u_A + u_B = 10, \quad 4 \leq u_A, u_B \leq 6$$

$$(u_A - u^0{}_A)(u_B - u^0{}_B) = (u_A - 12/5)(10 - u_A - 12/5)$$

$$= -u_A{}^2 + 10u_A - 456/25 = -(u_A - 5)^2 + 169/25$$

maximum attained
when $u_A = u_B = 5$　→　Nash bargaining solution (5, 5)

Problems with IIA



Noncooperative Approach

Nash's approach

A, B choose their demands x_A, x_B simultaneously

If $(x_A, x_B) \in R$, then A receives x_A, B receives x_B If not, A receives $u^0{}_A$, B receives $u^0{}_B$

$\longrightarrow$ Multiple Nash equilibria

Rubinstein's approach

Player A first proposes a payoff vector (x_A, x_B) to player B

Player B either can accept this offer; A receives x_A, B receives x_B

If B rejects, B can now propose a different (x_A, x_B) to A

A chooses whether to accept or reject

Discount factor $\rightarrow$ 1

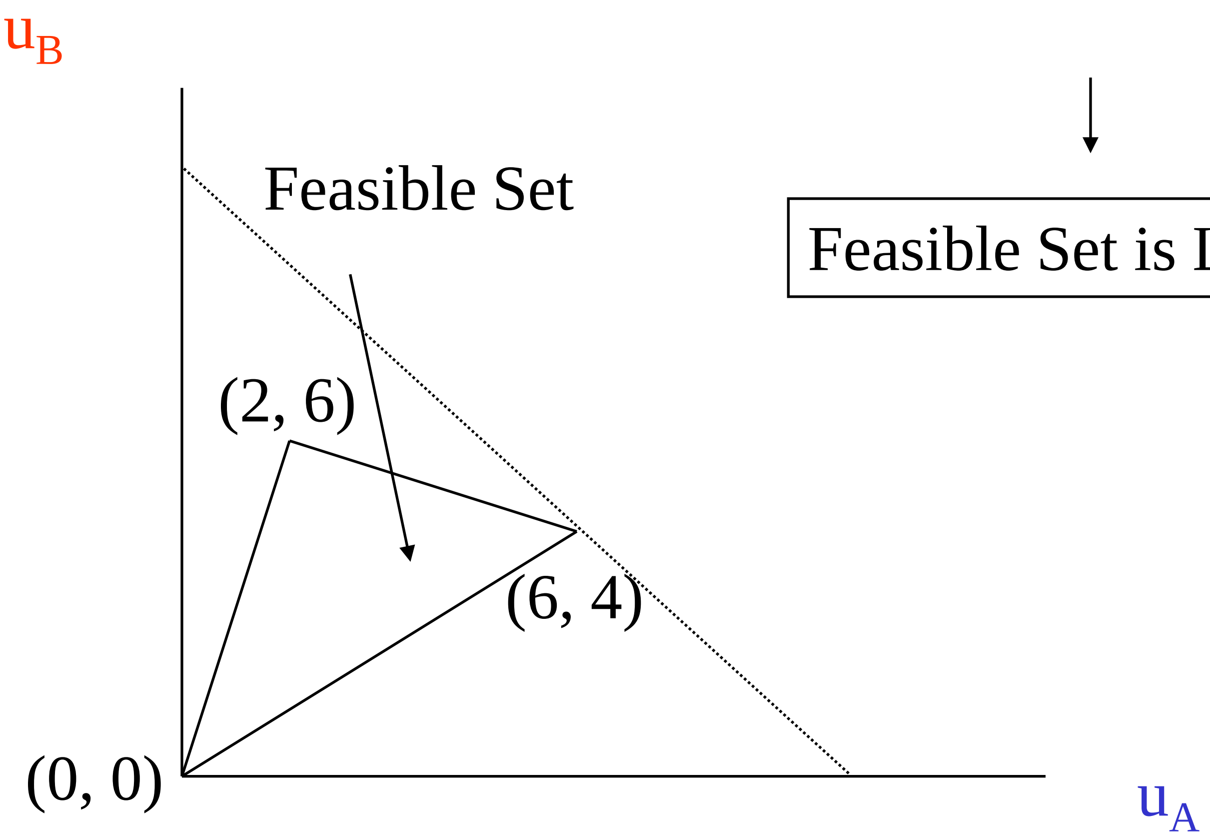
$\Rightarrow$ Subgame perfect equilibrium $\rightarrow$ Nash bargaining solution

Transferable Utility and Side Payment

A \ B	X		Y	
X	6	4	0	0
Y	0	0	2	6

A, B can receive (6, 4) and redistribute the total

Feasible Set is Larger



TU game and NTU game

Utility is increasing in proportion to the amount of money

→ Transferrable utility

（money as a medium of transfer）

Side Payement is possible

TU game : Transferrable utility,

Side payment is allowed

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NTU game

Assignment due Next Lecture

Reading assignment

"Introduction to Game Theory"： pp.139 -158

（"Game Theory"： pp.257 - 271）

Handout： Two-person Bargaining Game

Problem Set 1: #2

Homework

Problem Set 1: #1(a),(b),(c)

（Use A4-size paper,

and staple on the upper left-hand side）