No.8 December 15, 2009

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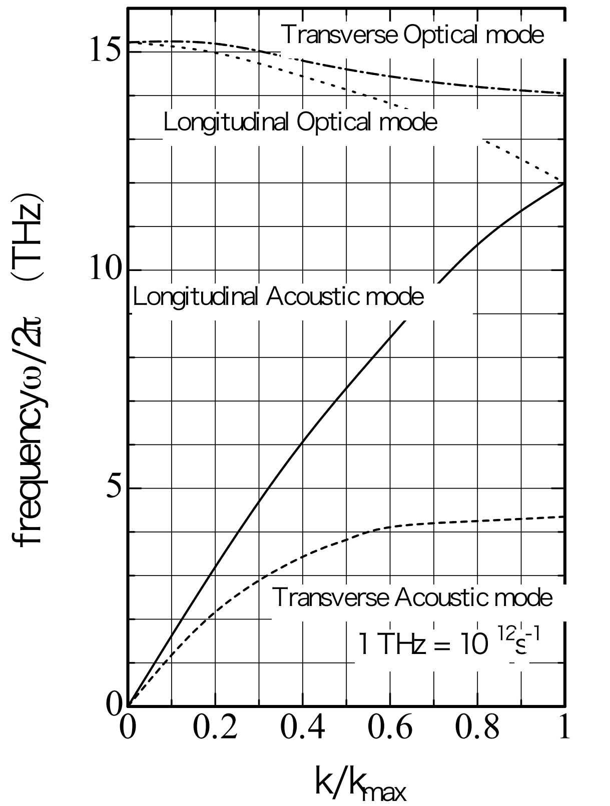
8. Consider the relationship between the lattice vibration and the elastic properties.
$\omega_k - k$ dispersion relation of [100] direction in Si is indicated in the right figure.

8-1 Explain the six modes of the lattice vibration in Si [100] direction.

8-2 Explain the relationship between three elastic waves and the lattice vibration in Si [100] direction.

Si[100] direction $\omega/2\pi$-k/k_{max} dispersion relation



8-3 Give the velocities of three elastic waves of Si [100] direction from the figure of $\omega_k - k$ dispersion.
Here $k_{\max} = 2\pi / a$, and a=5.4×10^{-10}m.

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9. Consider the elastic waves in cubic crystals in the [110] direction.

Here, ρ is the density [kg/m^3], u, v, w are the displacement in the x, y, z directions, respectively. C_{ij} is elastic stiffness constants [N/m^2]. Equations of motion in the x, y, z directions are given by

$$\rho \frac{\partial^2 u}{\partial t^2} = C_{11}\frac{\partial^2 u}{\partial x^2} + C_{44}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + (C_{12}+C_{44})\left(\frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial z}\right),$$

$$\rho \frac{\partial^2 v}{\partial t^2} = C_{11}\frac{\partial^2 v}{\partial y^2} + C_{44}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2}\right) + (C_{12}+C_{44})\left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 w}{\partial y \partial z}\right), \text{ and}$$

$$\rho \frac{\partial^2 w}{\partial t^2} = C_{11}\frac{\partial^2 w}{\partial z^2} + C_{44}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) + (C_{12}+C_{44})\left(\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 v}{\partial y \partial z}\right).$$

9-1 Consider elastic waves that propagate in the xy plane with the particle motion in the xy plane as $u = u_0 \exp[i(k_x x - k_y y - \omega t)]$, and $v = v_0 \exp[i(k_x x - k_y y - \omega t)]$.

Derive the propagating direction of this elastic wave, and calculate the values of k_x and k_y.

where, $k^2 = k_x^2 + k_y^2$, $k_x = k_y > 0$.

9-2 Calculate the velocities $\frac{\omega}{k}$ of longitudinal and transverse waves.

No. 10 January 12, 2010

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10-1-1 Illustrate the "new" coordinate vectors $\boldsymbol{e}_1'$, $\boldsymbol{e}_2'$ and $\boldsymbol{e}_3'$ which are transformed from $\boldsymbol{e}_1, \boldsymbol{e}_2$, and $\boldsymbol{e}_3$ by the following symmetries. Then, describe the unitary transformation matrices $\boldsymbol{a}$ for the symmetry elements. Note that 4 represents anticlockwise rotation of 90°:

a) $\bar{1}$, b) 2_z, c) 4_z, d) $\bar{4}_z$

10-1-2 Find the inverse unitary matrix $\boldsymbol{a}^{-1}(4_z)$, by transposing (with respect to the matrix diagonal axis) the elements of the matrix $\boldsymbol{a}(4_z)$ [note that $\boldsymbol{a}^{-1}(4_z)= \boldsymbol{a}^{T}(4_z)$].

No. 11 January 19, 2010

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11-1. i) Rewrite Eq. (4.1b) in conventional summation notation (instead of dummy suffix notation).

ii) Show that $\boldsymbol{B}$ = (B_1, B_2, B_3) and $\boldsymbol{D}$ = (D_1, D_2, D_3) are transformed by $\bar{2}_z$ as described in Eq. (4.4) as $\mathbf{B'} = |\mathbf{a}|\mathbf{aB}$ and $\mathbf{D'} = \mathbf{aD}$

iii) Using Eq. (4.1a), show that $\boldsymbol{B}$ is kept unchanged by $\bar{1}$.

Also show $\boldsymbol{D'}$ by using $\boldsymbol{D}$.

11-2. Gyration tensor $\boldsymbol{g}$ (g_{ij}) gives optical rotation (i.e. rotates azimuth angle of polarization of light).

We can change $\boldsymbol{g}$ by applying electric field $\boldsymbol{E}$, which is called "electric-field induced optical rotation." We can also change $\boldsymbol{g}$ by applying magnetic field $\boldsymbol{B}$, which is called "magneto-optical Faraday rotation."

The effects are represented as follows using tensors $\boldsymbol{A}$ and $\boldsymbol{B}$

$\Delta \boldsymbol{g} = \boldsymbol{A} \cdot \boldsymbol{E}$ and $\Delta \boldsymbol{g} = \boldsymbol{B} \cdot \boldsymbol{H}$.

What kind of tensors are $\boldsymbol{A}$ and $\boldsymbol{B}$? (Tell rank and whether axial or polar. Refer Table 4.1)

11-3. Tensor whose elements are symmetrical (namely such relation as $T_{ij} = T_{ji}$, $T_{ijk} = T_{jik}$ or $T_{ijkl} = T_{jikl}$ holds are called symmetrical tensor (for suffixes *i* and *j*). What change occurs in Eqs. (4.14) of [Example V] if the tensor $T = \begin{bmatrix} T_{11} & T_{12} & 0 \\ -T_{12} & T_{11} & 0 \\ 0 & 0 & T_{33} \end{bmatrix}$ becomes symmetrical ($T_{ij} = T_{ji}$) ?

11-4. On $BaTiO_3$ answer the following questions.

i) In the temperature region I the crystal has a cubic lattice ($a = b = c$, $\alpha = \beta = \gamma = 90^\circ$), which reduces in symmetry in other regions.
The lower-symmetry lattice is called "pseudo cubic lattice." What relations do *a*, *b*, *c*, α, β, and γ of the pseudo cubic lattice satisfy in regions II, III, and IV, respectively? [Hint: Fig. 4.8]

ii) Illustrate the spontaneous polarization ***p*** and the unit cell in respective regions II, III, and IV.

iii) In respective regions II ~ IV, show ***p*** and the mirror planes that the lattice has.

iv) In the region III, express *a'*, *b'*, and *c'* (lattice parameters of true lattice) in terms of *a*, *b*, *c*, and α (lattice parameters of pseudo cubic lattice). Also find an approximate relation which holds for *a'*, *b'*, and *c'*.

No. 12 January 26, 2010

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12-1. Rewrite the following matrix elements as tensor elements.

i) The piezoelectric matrix elements d_{12}, d_{25} and d_{34}.

ii) The stress matrix elements σ_2 and σ_6.

iii) The dielectric constant matrix elements $\varepsilon_3, \varepsilon_5$ and ε_6.

12-2. Rewrite the four equations given in Table 5.1 in conventional summation notation (rather than dummy suffix notation).

12-3. i) The piezoelectric coefficient of crystals belonging to 32 point group (given in Table 5.2) is expressed in the matrix notations as

$$[d_{ij}] = \begin{pmatrix} d_{11} & -d_{11} & 0 & d_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & -d_{14} & -2d_{11} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Following this example, express in the matrix notation the piezoelectric coefficient of crystals belonging to $\bar{4}2\,m$ and 3 point symmetry groups.

ii) Describe the non zero tensor elements d_{ijk} for crystals belonging to $\bar{4}2\,m$ point group. Also describe the relation holding for these elements.

12-4. A tensor stress σ_{11} is applied to a crystal belonging to point group 3 along the x_1 axis. Find the polarization induced by the piezoelectric effect.

[Hint: $\boldsymbol{P} = \boldsymbol{d\sigma}$ or $P_i = d_{ij}\sigma_j$]

12-5. An electric field E_1 is applied to a crystal belonging to $\bar{4}2\mathrm{m}$ point group along the x_1 axis. Find the strain induced by E_1.

[Hint: Put j=1 in $\varepsilon_j = d_{ij}E_i$]

No. 13 July 17, 2008

Electronic Materials B Y. Majima	Department	Laboratory	Matriculation number - -	Name

1. i) Along what direction(s) does a $BaTiO_3$ crystal exhibit pyroelectricity at -100℃, 0℃, 100℃, and 150℃? Answer on the “pseudo-cubic” lattice. [Hint: Fig. 4.4]

   ii) Along what direction(s) does a crystal belonging to 4*mm* point group exhibit pyroelectricity? [Hint: Table 3.3]

2. In Table 6.1, find the point group(s) to which the following crystals belong. (Note that some are not existing.)
   i) Polar crystal(s) with tetragonal symmetry (i.e. belonging to tetragonal system).

   ii) Cubic crystal(s) exhibiting piezoelectricity.

   iii) Cubic crystal(s) exhibiting pyroelectricity.

   iv) Crystal without $\bar{1}$ symmetry, but not exhibiting piezoelectricity.

   v) Hexagonal crystal(s) allowed by symmetry to exhibit ferroelectricity.

   vi) Cubic crystals allowed by symmetry to exhibit ferroelectricity.