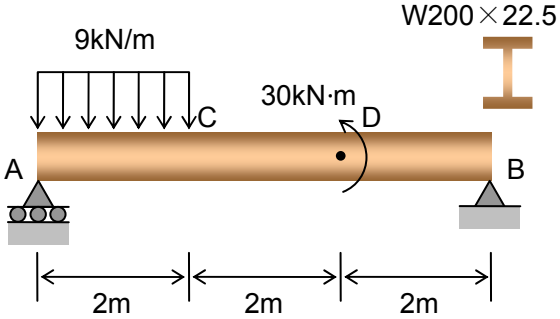
第 9 回課題解答(6/26 提出分)

問 1



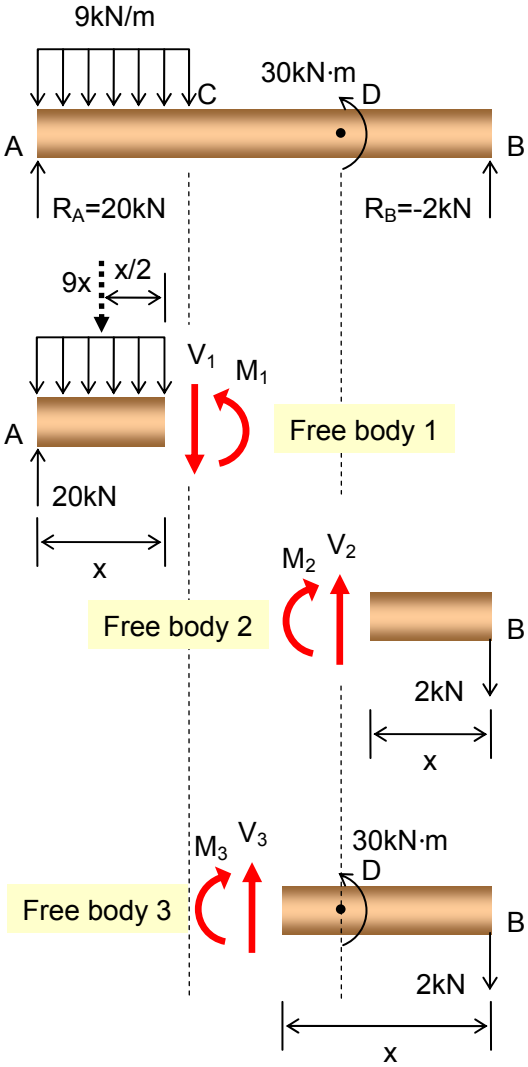
まず支点反力 R_A，R_B を求める

$$+\circlearrowleft \sum M_A = 0 \ :$$

$$-9\cdot 2\cdot 1 + 30 + R_B\cdot 6 = 0 \qquad R_B = -2\ (\mathrm{kN})$$

$$+\uparrow \sum F_y = 0 \ :$$

$$R_A - 9\cdot 2 + R_B = 0 \qquad R_A = 20\ (\mathrm{kN})$$



右図に示すように Free body1～3 を考え，それぞれにおける V および M を求める

ここでは計算を簡略化するため，Free body2 と 3 は支点 B からとっていることに注意してください

From A to C (Free body 1)

$$+\uparrow \sum F_y = 0 \ :$$

$$20 - 9x - V_1 = 0 \qquad \underline{V_1 = -9x + 20}$$

$$+\circlearrowleft \sum M_1 = 0 \ :$$

$$M_1 + 9x\cdot\frac{x}{2} - 20x = 0 \qquad \underline{M_1 = -\frac{9x^2}{2} + 20x}$$

From B to D (Free body 2)

$$+\uparrow \sum F_y = 0 \ :$$

$$-2 + V_2 = 0 \qquad \underline{V_2 = 2}$$

$$+\circlearrowright \sum M_2 = 0 \ :$$

$$M_2 + 2x = 0 \qquad \underline{M_2 = -2x}$$

From B to C (Free body 3)

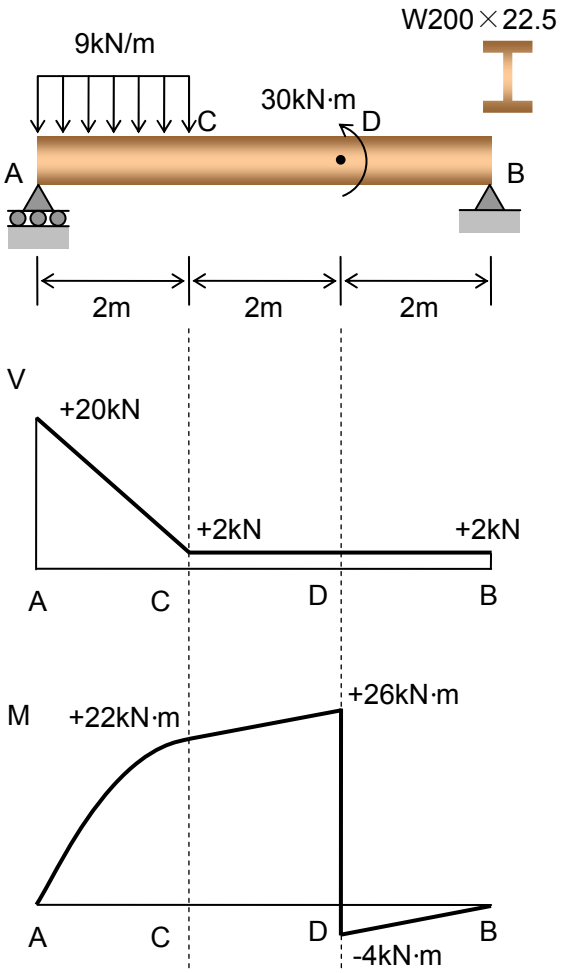
$$+\uparrow \sum F_y = 0 \ :$$

$$-2 + V_3 = 0 \qquad \underline{V_3 = 2}$$

$$+\circlearrowleft \sum M_3 = 0 \ :$$

$$M_3 - 30 + 2x = 0 \qquad \underline{M_3 = -2x + 30}$$

以上より，せん断力図，曲げモーメント図は
右図のようになる



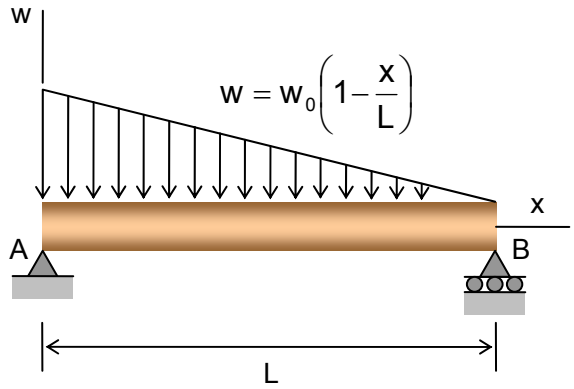
曲げモーメントの最大値 M_{max} は 26kN·m, W200×22.5 の S_x は Appendix C に示すように 194×10^3 (mm^3)である

$$\sigma_{max} = \frac{M_{max}}{S_x} \text{ より}$$

$$\sigma_{max} = \frac{26\times10^6 \ (N\cdot mm)}{194\times10^3 \ (mm^3)}$$

$$\underline{\sigma_{max} = 134 \ (MPa)}$$

問２

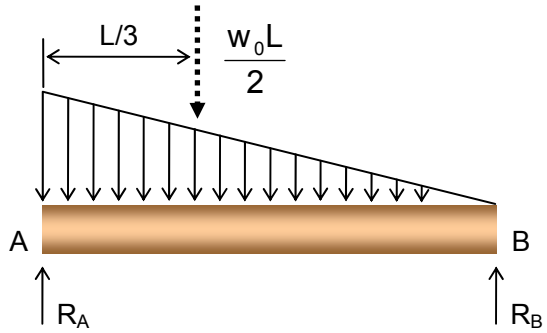


$\frac{dV}{dx} = -w$ より

$$V = -\int w dx = -\int w_0\left(1-\frac{x}{L}\right)dx$$

$$= -\left(w_0 x - \frac{w_0 x^2}{2L} + C_0\right)$$

$\frac{dM}{dx} = V$ より



$$M = \int V dx = \int\left(-w_0 x + \frac{w_0 x^2}{2L} + C_0\right)dx$$

$$= -\frac{w_0 x^2}{2} + \frac{w_0 x^3}{6L} + C_0 x + C_1$$

$x = 0, M_A = 0$ より　　$C_1 = 0$

$x = L, M_B = 0$ より　　$C_0 = -\frac{w_0 L}{3}$

よって

$$\underline{V = -w_0 x + \frac{w_0 x^2}{2L} + \frac{w_0 L}{3}} \qquad \underline{M = -\frac{w_0 x^2}{2} + \frac{w_0 x^3}{6L} + \frac{w_0 L}{3}x}$$

M の最大値が最大となるのは $\frac{dM}{dx} = V = 0$ のときであるから

$$V = -w_0 x + \frac{w_0 x^2}{2L} + \frac{w_0 L}{3} = 0$$

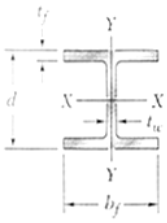
$$\frac{w_0}{6L}\left(3x^2 - 6Lx + 2L^2\right) = 0 \qquad x = \frac{3L \pm \sqrt{3}L}{3} \qquad x \le L \text{ より} \quad x = \frac{L\left(3+\sqrt{3}\right)}{3} \approx 1.58L$$

これを曲げモーメントの式に代入して

$$|M|_{max} = \left|-\frac{w_0(1.58L)^2}{2} + \frac{w_0(1.58L)^3}{6L} + \frac{w_0 L}{3}(1.58L)\right|$$

$$\underline{|M|_{max} = 0.0642 w_0 L^2}$$

Appendix C



Appendix C. Properties of Rolled-Steel Shapes
(SI Units)
Continued from page 751

W Shapes
(Wide-Flange Shapes)

Designation†	Area A, mm²	Depth d, mm	Flange Width b_f, mm	Flange Thickness t_f, mm	Web Thickness t_w, mm	Axis X-X I_x 10^6 mm⁴	Axis X-X S_x 10^3 mm³	Axis X-X r_x mm	Axis Y-Y I_y 10^6 mm⁴	Axis Y-Y S_y 10^3 mm³	Axis Y-Y r_y mm
W310 × 143	18200	323	309	22.9	14.0	348	2150	138	113	731	78.8
107	13600	311	306	17.0	10.9	248	1590	135	81.2	531	77.3
74	9480	310	205	16.3	9.4	165	1060	132	23.4	228	49.7
60	7590	303	203	13.1	7.5	129	851	130	18.3	180	49.1
52	6670	318	167	13.2	7.6	119	748	134	10.3	123	39.3
44.5	5690	313	166	11.2	6.6	99.2	634	132	8.55	103	38.8
38.7	4940	310	165	9.7	5.8	85.1	549	131	7.27	88.1	38.4
32.7	4180	313	102	10.8	6.6	65.0	415	125	1.92	37.6	21.4
23.8	3040	305	101	6.7	5.6	42.7	280	119	1.16	23.0	19.5
W250 × 167	21300	289	265	31.8	19.2	300	2080	119	98.8	746	68.1
101	12900	264	257	19.6	11.9	164	1240	113	55.5	432	65.6
80	10200	256	255	15.6	9.4	126	984	111	43.1	338	65.0
67	8580	257	204	15.7	8.9	104	809	110	22.2	218	51.0
58	7420	252	203	13.5	8.0	87.3	693	108	18.8	185	50.3
49.1	6250	247	202	11.0	7.4	70.6	572	106	15.1	150	49.2
44.8	5720	266	148	13.0	7.6	71.1	535	111	7.03	95.0	35.1
32.7	4180	258	146	9.1	6.1	48.9	379	108	4.73	64.8	33.7
28.4	3630	260	102	10.0	6.4	40.0	308	105	1.78	34.9	22.1
22.3	2850	254	102	6.9	5.8	28.9	228	101	1.23	24.1	20.8
W200 × 86	11000	222	209	20.6	13.0	94.7	853	92.4	31.4	300	53.2
71	9100	216	206	17.4	10.2	76.6	709	91.7	25.4	247	52.8
59	7560	210	205	14.2	9.1	61.1	582	89.9	20.4	199	51.9
52	6660	206	204	12.6	7.9	52.7	512	89.0	17.8	175	51.7
46.1	5860	203	203	11.0	7.2	45.5	448	87.9	15.3	151	51.1
41.7	5310	205	166	11.8	7.2	40.9	399	87.8	9.01	109	41.2
35.9	4580	201	165	10.2	6.2	34.4	342	86.7	7.64	92.6	40.8
31.3	4000	210	134	10.2	6.4	31.4	299	88.6	4.1	61.2	32.0
26.6	3390	207	133	8.4	5.8	25.8	249	87.2	3.3	49.6	31.2
22.5	2860	206	102	8.0	6.2	20.0	194	83.6	1.42	27.8	22.3
19.3	2480	203	102	6.5	5.8	16.6	164	81.8	1.15	22.5	21.5
W150 × 37.1	4730	162	154	11.6	8.1	22.2	274	68.5	7.07	91.8	38.7
29.8	3790	157	153	9.3	6.6	17.2	219	67.4	5.56	72.7	38.3
24.0	3060	160	102	10.3	6.6	13.4	168	66.2	1.83	35.9	24.5
18.0	2290	153	102	7.1	5.8	9.17	120	63.3	1.26	24.7	23.5
13.5	1730	150	100	5.5	4.3	6.87	91.6	63.0	0.918	18.4	23.0
W130 × 28.1	3580	131	128	10.9	6.9	10.9	166	55.2	3.81	59.5	32.6
23.8	3010	127	127	9.1	6.1	8.80	139	54.1	3.11	49.0	32.1
W100 × 19.3	2480	106	103	8.8	7.1	4.77	90.0	43.9	1.61	31.3	25.5

†A wide-flange shape is designated by the letter W followed by the nominal depth in millimeters and the mass in kilograms per meter.