In[473]:=
```
(*Instructor：Hiroyuki Akama
  Annex–3*)
<< Combinatorica`
```

In[474]:=
```
binrandommatrix = Table[Table[RandomInteger[], {i, 1, 100}], {i, 1, 100}];
```

In[475]:=
```
randomadjacencymatrix = (binrandommatrix + Transpose[binrandommatrix]) /. x_ /; x ≠ 0 → 1;
% // SymmetricQ
edgesnotselfloop =
  Select[#[[1]] & /@ Most[ArrayRules[SparseArray[randomadjacencymatrix]]], #[[1]] ≠ #[[2]] &];
arrule = (# → 1) & /@ edgesnotselfloop;
sparaserandomadjacencymatrix = SparseArray[arrule, {100, 100}]
```

Out[476]= True

Out[479]= SparseArray[<7354>, {100, 100}]

In[480]:=
```
nodeselected = 34;
edgesselected = Select[arrule, #[[1]][[1]] == nodeselected &]
neighboringnodes = #[[1, 2]] & /@ edgesselected
```

Out[481]=
```
{{34, 1} → 1, {34, 3} → 1, {34, 7} → 1, {34, 8} → 1, {34, 10} → 1, {34, 12} → 1, {34, 13} → 1,
 {34, 14} → 1, {34, 16} → 1, {34, 17} → 1, {34, 18} → 1, {34, 19} → 1, {34, 21} → 1,
 {34, 22} → 1, {34, 23} → 1, {34, 24} → 1, {34, 25} → 1, {34, 26} → 1, {34, 30} → 1,
 {34, 31} → 1, {34, 32} → 1, {34, 33} → 1, {34, 35} → 1, {34, 36} → 1, {34, 37} → 1,
 {34, 38} → 1, {34, 40} → 1, {34, 41} → 1, {34, 42} → 1, {34, 43} → 1, {34, 44} → 1,
 {34, 45} → 1, {34, 46} → 1, {34, 47} → 1, {34, 48} → 1, {34, 49} → 1, {34, 51} → 1,
 {34, 53} → 1, {34, 54} → 1, {34, 55} → 1, {34, 57} → 1, {34, 58} → 1, {34, 59} → 1,
 {34, 60} → 1, {34, 61} → 1, {34, 62} → 1, {34, 63} → 1, {34, 64} → 1, {34, 65} → 1,
 {34, 67} → 1, {34, 71} → 1, {34, 73} → 1, {34, 74} → 1, {34, 75} → 1, {34, 76} → 1,
 {34, 77} → 1, {34, 78} → 1, {34, 80} → 1, {34, 81} → 1, {34, 82} → 1, {34, 84} → 1,
 {34, 85} → 1, {34, 86} → 1, {34, 88} → 1, {34, 90} → 1, {34, 91} → 1, {34, 93} → 1,
 {34, 94} → 1, {34, 95} → 1, {34, 96} → 1, {34, 97} → 1, {34, 99} → 1, {34, 100} → 1}
```

Out[482]=
```
{1, 3, 7, 8, 10, 12, 13, 14, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 30, 31, 32, 33, 35, 36, 37, 38,
 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67,
 71, 73, 74, 75, 76, 77, 78, 80, 81, 82, 84, 85, 86, 88, 90, 91, 93, 94, 95, 96, 97, 99, 100}
```

In[483]:=
```
allpossiblecombinations = KSubsets[neighboringnodes, 2];
existentcombinations = Intersection[edgesnotselfloop, allpossiblecombinations];
curvatureofnodeselected = Length[existentcombinations] / Length[allpossiblecombinations] // N
```

Out[485]= 0.745814

In[486]:=
```
samplepairs = {{1, 2}, {1, 3}, {1, 11}, {2, 3}, {3, 4}, {4, 5}, {4, 6}, {4, 7}, {4, 8},
   {5, 6}, {5, 7}, {5, 8}, {6, 7}, {6, 8}, {7, 8}, {8, 9}, {9, 10}, {9, 11}, {10, 11}};
```

In[487]:=
```
edgenumvsedge = Table[{k, samplepairs[[k]]}, {k, 1, Length[samplepairs]}]
```

Out[487]=
```
{{1, {1, 2}}, {2, {1, 3}}, {3, {1, 11}}, {4, {2, 3}}, {5, {3, 4}}, {6, {4, 5}}, {7, {4, 6}},
 {8, {4, 7}}, {9, {4, 8}}, {10, {5, 6}}, {11, {5, 7}}, {12, {5, 8}}, {13, {6, 7}},
 {14, {6, 8}}, {15, {7, 8}}, {16, {8, 9}}, {17, {9, 10}}, {18, {9, 11}}, {19, {10, 11}}}
```

In[488]:=
```
? Thread
? Distribute
Flatten[Thread[#] & /@ edgenumvsedge, 1]
```

Thread[*f*[*args*]] "threads" *f* over any lists that appear in *args*.
Thread[*f*[*args*], *h*] threads *f* over any objects with head *h* that appear in *args*.
Thread[*f*[*args*], *h*, *n*] threads *f* over objects with head *h* that appear in the first *n* *args*. ≫

Distribute[$f[x_1, x_2, \ldots]$] distributes *f* over Plus appearing in any of the x_i.
Distribute[*expr*, *g*] distributes over *g*.
Distribute[*expr*, *g*, *f*] performs the distribution only if the head of *expr* is *f*. ≫

Out[490]= {{1, 1}, {1, 2}, {2, 1}, {2, 3}, {3, 1}, {3, 11}, {4, 2}, {4, 3}, {5, 3}, {5, 4},
{6, 4}, {6, 5}, {7, 4}, {7, 6}, {8, 4}, {8, 7}, {9, 4}, {9, 8}, {10, 5}, {10, 6},
{11, 5}, {11, 7}, {12, 5}, {12, 8}, {13, 6}, {13, 7}, {14, 6}, {14, 8}, {15, 7},
{15, 8}, {16, 8}, {16, 9}, {17, 9}, {17, 10}, {18, 9}, {18, 11}, {19, 10}, {19, 11}}

In[491]:=
```
Sort[Reverse[#] & /@ Flatten[Thread[#] & /@ edgenumvsedge, 1]]
```

Out[491]= {{1, 1}, {1, 2}, {1, 3}, {2, 1}, {2, 4}, {3, 2}, {3, 4}, {3, 5}, {4, 5}, {4, 6},
{4, 7}, {4, 8}, {4, 9}, {5, 6}, {5, 10}, {5, 11}, {5, 12}, {6, 7}, {6, 10}, {6, 13},
{6, 14}, {7, 8}, {7, 11}, {7, 13}, {7, 15}, {8, 9}, {8, 12}, {8, 14}, {8, 15},
{8, 16}, {9, 16}, {9, 17}, {9, 18}, {10, 17}, {10, 19}, {11, 3}, {11, 18}, {11, 19}}

In[492]:=
```
Split[Sort[Reverse[#] & /@ Flatten[Thread[#] & /@ edgenumvsedge, 1]], #1[[1]] == #2[[1]] &]
```

Out[492]= {{{1, 1}, {1, 2}, {1, 3}}, {{2, 1}, {2, 4}},
{{3, 2}, {3, 4}, {3, 5}}, {{4, 5}, {4, 6}, {4, 7}, {4, 8}, {4, 9}},
{{5, 6}, {5, 10}, {5, 11}, {5, 12}}, {{6, 7}, {6, 10}, {6, 13}, {6, 14}},
{{7, 8}, {7, 11}, {7, 13}, {7, 15}}, {{8, 9}, {8, 12}, {8, 14}, {8, 15}, {8, 16}},
{{9, 16}, {9, 17}, {9, 18}}, {{10, 17}, {10, 19}}, {{11, 3}, {11, 18}, {11, 19}}}

In[493]:=
```
Rassemble[list_] := {list[[1, 1]], Last[#] & /@ list};
```

In[494]:=
```
incidencerelations = Rassemble[#] & /@
  Split[Sort[Reverse[#] & /@ Flatten[Thread[#] & /@ edgenumvsedge, 1]], #1[[1]] == #2[[1]] &]
```

Out[494]= {{1, {1, 2, 3}}, {2, {1, 4}}, {3, {2, 4, 5}}, {4, {5, 6, 7, 8, 9}},
{5, {6, 10, 11, 12}}, {6, {7, 10, 13, 14}}, {7, {8, 11, 13, 15}},
{8, {9, 12, 14, 15, 16}}, {9, {16, 17, 18}}, {10, {17, 19}}, {11, {3, 18, 19}}}

In[495]:=
```
numberofnodes = 11;
numberofedges = Max[Flatten[#[[2]] & /@ incidencerelations]]
```

Out[496]= 19

In[497]:=
```
incidencedata = Flatten[Distribute[Thread[#]] & /@ incidencerelations, 1]
```

Out[497]= {{1, 1}, {1, 2}, {1, 3}, {2, 1}, {2, 4}, {3, 2}, {3, 4}, {3, 5}, {4, 5}, {4, 6},
{4, 7}, {4, 8}, {4, 9}, {5, 6}, {5, 10}, {5, 11}, {5, 12}, {6, 7}, {6, 10}, {6, 13},
{6, 14}, {7, 8}, {7, 11}, {7, 13}, {7, 15}, {8, 9}, {8, 12}, {8, 14}, {8, 15},
{8, 16}, {9, 16}, {9, 17}, {9, 18}, {10, 17}, {10, 19}, {11, 3}, {11, 18}, {11, 19}}

In[498]:=
```
Table[incidencedata[[k]] → 1, {k, 1, Length[incidencedata]}]
# → 1 & /@ incidencedata
```

Out[498]= {{1, 1} → 1, {1, 2} → 1, {1, 3} → 1, {2, 1} → 1, {2, 4} → 1, {3, 2} → 1, {3, 4} → 1, {3, 5} → 1,
{4, 5} → 1, {4, 6} → 1, {4, 7} → 1, {4, 8} → 1, {4, 9} → 1, {5, 6} → 1, {5, 10} → 1, {5, 11} → 1,
{5, 12} → 1, {6, 7} → 1, {6, 10} → 1, {6, 13} → 1, {6, 14} → 1, {7, 8} → 1, {7, 11} → 1, {7, 13} → 1,
{7, 15} → 1, {8, 9} → 1, {8, 12} → 1, {8, 14} → 1, {8, 15} → 1, {8, 16} → 1, {9, 16} → 1,
{9, 17} → 1, {9, 18} → 1, {10, 17} → 1, {10, 19} → 1, {11, 3} → 1, {11, 18} → 1, {11, 19} → 1}

Out[499]= {{1, 1} → 1, {1, 2} → 1, {1, 3} → 1, {2, 1} → 1, {2, 4} → 1, {3, 2} → 1, {3, 4} → 1, {3, 5} → 1,
{4, 5} → 1, {4, 6} → 1, {4, 7} → 1, {4, 8} → 1, {4, 9} → 1, {5, 6} → 1, {5, 10} → 1, {5, 11} → 1,
{5, 12} → 1, {6, 7} → 1, {6, 10} → 1, {6, 13} → 1, {6, 14} → 1, {7, 8} → 1, {7, 11} → 1, {7, 13} → 1,
{7, 15} → 1, {8, 9} → 1, {8, 12} → 1, {8, 14} → 1, {8, 15} → 1, {8, 16} → 1, {9, 16} → 1,
{9, 17} → 1, {9, 18} → 1, {10, 17} → 1, {10, 19} → 1, {11, 3} → 1, {11, 18} → 1, {11, 19} → 1}

In[500]:=
```
im = SparseArray[%, {numberofnodes, numberofedges}]
```

Out[500]=
```
SparseArray[<38>, {11, 19}]
```

In[501]:=
```
Transpose[im].im
```

Out[501]=
```
SparseArray[<123>, {19, 19}]
```

In[502]:=
```
% // Normal
```

Out[502]=
```
{{2, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {1, 2, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {1, 1, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1},
 {1, 1, 0, 2, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 1, 0, 1, 2, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 1, 2, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 1, 1, 2, 1, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 1, 1, 1, 2, 1, 0, 1, 0, 1, 0, 1, 0, 0, 0, 0},
 {0, 0, 0, 0, 1, 1, 1, 1, 2, 0, 0, 1, 0, 1, 1, 1, 0, 0, 0},
 {0, 0, 0, 0, 0, 1, 1, 0, 0, 2, 1, 1, 1, 1, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 2, 1, 1, 0, 1, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 1, 0, 0, 1, 1, 1, 2, 0, 1, 1, 1, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 2, 1, 1, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 0, 1, 1, 2, 1, 1, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 1, 1, 2, 1, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 1, 1, 2, 1, 1, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 1, 1},
 {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 2, 1},
 {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 2}}
```

In[503]:=
```
% - 2 * IdentityMatrix[Length[%]]
```

Out[503]=
```
{{0, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {1, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1},
 {1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 1, 0, 1, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 1, 0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 1, 1, 0, 1, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 1, 1, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 0, 0, 0},
 {0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 1, 0, 1, 1, 1, 0, 0, 0},
 {0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0, 1, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 1, 0, 0, 1, 1, 1, 0, 0, 1, 1, 1, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 1, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 1, 1, 0, 1, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 1, 1, 0, 1, 1, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1},
 {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1},
 {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0}}
```