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Instructor : Hiroyuki Akama

## Differential Equation and others\*)

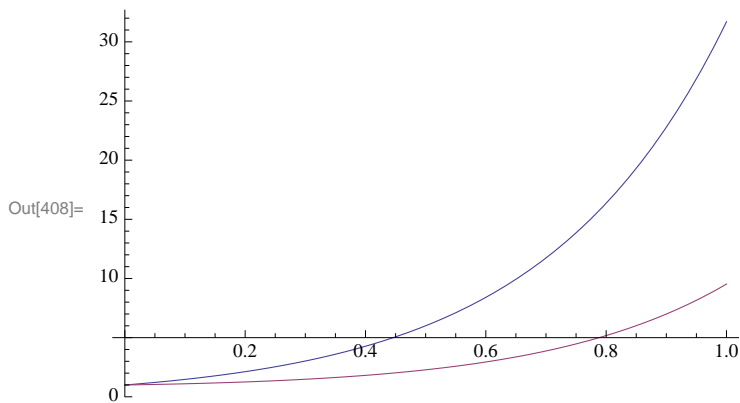
```
In[405]:= sol = NDSolve[{y'[x] == 3 * y[x] + z[x],
  z'[x] == y[x] - 0.2 * z[x], y[0] == z[0] == 1}, {y, z}, {x, 0, 1}]
? NDSolve
? InterpolatingFunction

Out[405]= {{y -> InterpolatingFunction[{{0., 1.}}, <>], z -> InterpolatingFunction[{{0., 1.}}, <>]}}
```

NDSolve[eqns, y, {x, x<sub>min</sub>, x<sub>max</sub>}] finds a numerical solution to the ordinary differential equations eqns for the function y with the independent variable x in the range x<sub>min</sub> to x<sub>max</sub>.  
 NDSolve[eqns, y, {x, x<sub>min</sub>, x<sub>max</sub>}, {t, t<sub>min</sub>, t<sub>max</sub>}] finds a numerical solution to the partial differential equations eqns.  
 NDSolve[eqns, {y<sub>1</sub>, y<sub>2</sub>, ...}, {x, x<sub>min</sub>, x<sub>max</sub>}] finds numerical solutions for the functions y<sub>i</sub>. >>

InterpolatingFunction[domain, table] represents  
 an approximate function whose values are found by interpolation. >>

```
In[408]:= fig = Plot[Evaluate[{y[x], z[x]} /. sol, {x, 0, 1}], PlotRange -> All]
```



```
In[409]:= solnew = {y, z} /. Flatten[sol]
```

```
Out[409]= {InterpolatingFunction[{{0., 1.}}, <>], InterpolatingFunction[{{0., 1.}}, <>]}
```

```
In[410]:= xarray = Table[xvals, {xvals, 0, 1, 0.1}]
```

```
Out[410]= {0., 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.}
```

```
In[411]:= {#, solnew[[1]][#], solnew[[2]][#]} & /@ xarray
```

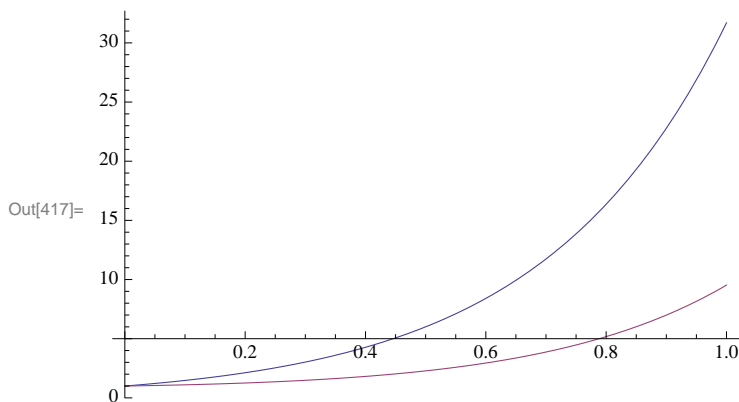
```
Out[411]= {{0., 1., 1.}, {0.1, 1.47166, 1.10138},
  {0.2, 2.12293, 1.25589}, {0.3, 3.02391, 1.48354}, {0.4, 4.27192, 1.81218},
  {0.5, 6.00219, 2.28053}, {0.6, 8.40252, 2.94237}, {0.7, 11.7338, 3.87248},
  {0.8, 16.3586, 5.17473}, {0.9, 22.7801, 6.99345}, {1., 31.6979, 9.52916}}
```

```
In[412]:= Clear[sol, fig, solnew, xarray];
OurNDSolve[p_, q_, step_] := Module[{sol, solnew, xvals, xarray, result}, sol =
  NDSolve[{y'[x] == p * y[x] + z[x], z'[x] == y[x] + q * z[x], y[0] == z[0] == 1}, {y, z}, {x, 0, 1}];
  solnew = {y, z} /. Flatten[sol]; xarray = Table[xvals, {xvals, 0, 1, step}];
  result = {sol, {#, solnew[[1]][#], solnew[[2]][#]} & /@ xarray};
```

```
In[414]:= answer = OurNDSolve[3, -0.2, 0.01];
answer[[1]]
answer[[2]]
fig = Plot[Evaluate[{y[x], z[x]} /. answer[[1]]], {x, 0, 1}, PlotRange -> All]

Out[415]= {{y -> InterpolatingFunction[{{0., 1.}}, <>], z -> InterpolatingFunction[{{0., 1.}}, <>]}}
```

```
Out[416]= {{0., 1., 1.}, {0.01, 1.04065, 1.00819}, {0.02, 1.08262, 1.01678},
{0.03, 1.12595, 1.02578}, {0.04, 1.17071, 1.0352}, {0.05, 1.21692, 1.04506},
{0.06, 1.26464, 1.05537}, {0.07, 1.31392, 1.06614},
{0.08, 1.36481, 1.07739}, {0.09, 1.41738, 1.08913}, {0.1, 1.47166, 1.10138},
{0.11, 1.52772, 1.11416}, {0.12, 1.58563, 1.12749}, {0.13, 1.64543, 1.14137},
{0.14, 1.7072, 1.15584}, {0.15, 1.771, 1.1709}, {0.16, 1.8369, 1.18658},
{0.17, 1.90497, 1.2029}, {0.18, 1.97528, 1.21987}, {0.19, 2.04791, 1.23753},
{0.2, 2.12293, 1.25589}, {0.21, 2.20043, 1.27497}, {0.22, 2.28049, 1.29481},
{0.23, 2.36318, 1.31541}, {0.24, 2.44862, 1.33682}, {0.25, 2.53687, 1.35905},
{0.26, 2.62804, 1.38213}, {0.27, 2.72223, 1.40609}, {0.28, 2.81953, 1.43096},
{0.29, 2.92005, 1.45676}, {0.3, 3.02391, 1.48354}, {0.31, 3.1312, 1.51132},
{0.32, 3.24204, 1.54013}, {0.33, 3.35656, 1.57001}, {0.34, 3.47488, 1.60099},
{0.35, 3.59712, 1.63312}, {0.36, 3.72341, 1.66642}, {0.37, 3.8539, 1.70093},
{0.38, 3.98871, 1.7367}, {0.39, 4.128, 1.77377}, {0.4, 4.27192, 1.81218},
{0.41, 4.42061, 1.85198}, {0.42, 4.57425, 1.8932}, {0.43, 4.73299, 1.93591},
{0.44, 4.897, 1.98014}, {0.45, 5.06647, 2.02594}, {0.46, 5.24157, 2.07338},
{0.47, 5.4225, 2.1225}, {0.48, 5.60944, 2.17336}, {0.49, 5.8026, 2.22601},
{0.5, 6.00219, 2.28053}, {0.51, 6.20841, 2.33696}, {0.52, 6.4215, 2.39537},
{0.53, 6.64169, 2.45583}, {0.54, 6.8692, 2.5184}, {0.55, 7.10429, 2.58316},
{0.56, 7.34721, 2.65018}, {0.57, 7.59821, 2.71953}, {0.58, 7.85758, 2.79129},
{0.59, 8.12559, 2.86554}, {0.6, 8.40252, 2.94237}, {0.61, 8.68869, 3.02185},
{0.62, 8.98439, 3.10409}, {0.63, 9.28995, 3.18916}, {0.64, 9.60568, 3.27716},
{0.65, 9.93194, 3.36819}, {0.66, 10.2691, 3.46236}, {0.67, 10.6175, 3.55976},
{0.68, 10.9774, 3.66051}, {0.69, 11.3494, 3.76471}, {0.7, 11.7338, 3.87248},
{0.71, 12.1311, 3.98394}, {0.72, 12.5415, 4.0992}, {0.73, 12.9657, 4.21841},
{0.74, 13.404, 4.34169}, {0.75, 13.8569, 4.46917}, {0.76, 14.325, 4.601},
{0.77, 14.8086, 4.73731}, {0.78, 15.3084, 4.87827}, {0.79, 15.8249, 5.02402},
{0.8, 16.3586, 5.17473}, {0.81, 16.9101, 5.33055}, {0.82, 17.48, 5.49166},
{0.83, 18.0689, 5.65824}, {0.84, 18.6775, 5.83047}, {0.85, 19.3064, 6.00854},
{0.86, 19.9563, 6.19263}, {0.87, 20.6278, 6.38295}, {0.88, 21.3219, 6.57973},
{0.89, 22.039, 6.78315}, {0.9, 22.7801, 6.99345}, {0.91, 23.5459, 7.21085},
{0.92, 24.3374, 7.4356}, {0.93, 25.1552, 7.66795}, {0.94, 26.0004, 7.90813},
{0.95, 26.8737, 8.15641}, {0.96, 27.7762, 8.41307}, {0.97, 28.7089, 8.67838},
{0.98, 29.6727, 8.95263}, {0.99, 30.6686, 9.23612}, {1., 31.6979, 9.52916}}
```



```
In[418]:= ymodelfunction = y /. Flatten[answer[[1]]][[1]];
zmodelfunction = z /. Flatten[answer[[1]]][[2]];
error = Transpose[{Table[0, {i, 1, Length[answer[[2]]]}], Table[0.01 * Random[Real],
    {i, 1, Length[answer[[2]]]}], Table[0.01 * Random[Real], {i, 1, Length[answer[[2]]]}]}];
realdata = answer[[2]] + error;
```

General::spell1 :

New symbol name "zmodelfunction" is similar to existing symbol "ymodelfunction" and may be misspelled. >>

```
In[422]:= realy = {#[[1]], #[[2]]} & /@ realdata;
realz = {#[[1]], #[[3]]} & /@ realdata;
```

General::spell1 : New symbol name "realz" is similar to existing symbol "realy" and may be misspelled. >>

```
In[424]:= yrealdatafunction = Interpolation[realy];
zrealdatafunction = Interpolation[realz];
? Interpolation
```

General::spell1 :

New symbol name "zrealdatafunction" is similar to existing symbol "yrealdatafunction" and may be misspelled. >>

`Interpolation[{ $f_1$ ,  $f_2$ , ...}]` constructs an interpolation of the function values  $f_i$ , assumed to correspond to  $x$  values 1, 2, ... .

`Interpolation[{ $\{x_1, f_1\}$ ,  $\{x_2, f_2\}$ , ...}]` constructs an interpolation of the function values  $f_i$  corresponding to  $x$  values  $x_i$ .

`Interpolation[{ $\{x_1, y_1, \dots\}$ ,  $f_1$ },  $\{x_2, y_2, \dots\}$ ,  $f_2$ }, ...}]` constructs an interpolation of multidimensional data.

`Interpolation[{ $\{x_1, \dots\}$ ,  $f_1$ ,  $df_1$ , ...}, ...}]` constructs an interpolation that reproduces derivatives as well as function values.

`Interpolation[data,  $x$ ]` find an interpolation of *data* at the point  $x$ . >>

```
In[427]:= yrealdatafunction[0.1]
ymodelfunction[0.1]
yrealdatafunction[0.1] - ymodelfunction[0.1]
```

Out[427]= 1.4728

Out[428]= 1.47166

Out[429]= 0.00113721

```
In[430]:= yerrorfunction[x_] := yrealdatafunction[x] - ymodelfunction[x]
yerrorfunction[0.3]
```

Out[431]= 0.000449473

```
In[432]:= Plus@@(yerrorfunction[#] & /@ Table[xvals, {xvals, 0, 1, 0.1}])
```

Out[432]= 0.0516319