In[3]:= (*Instructor：Hiroyuki Akama

Basic Built–in Functions of Mathematica
Numerical Calculation and Algebraic Calculation*)

$$\int_{-1}^{1} \frac{\sqrt{x}}{x} \, dx$$

Out[3]= 2 - 2 i

In[4]:= $\sum_{i=1}^{3} i\,(i+1)$

Out[4]= 20

In[5]:= x^y

Out[5]= x^y

In[6]:= a_1

Out[6]= a_1

```
In[7]:= N[Pi, 100]
```

Out[7]= 3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482
5342117068

```
In[8]:= N[Pi]
```

Out[8]= 3.14159

```
In[9]:= N[E]
```

Out[9]= 2.71828

```
In[10]:= N[E, 10]
```

Out[10]= 2.718281828

```
In[11]:= N[i]
```

Out[11]= i

```
In[12]:= Sqrt[2]
```

Out[12]= $\sqrt{2}$

```
In[13]:= % // N
```

Out[13]= 1.41421

```
In[14]:= Det[{{4, -3}, {2, 1}}]
         (*Compute a determinant*)
         ? Det
```

Out[14]= 10

Det[*m*] gives the determinant of the square matrix *m*. ≫

In[16]:=
```
Limit[(x^2 + x - 6) / (x^2 + 2 x - 8), x → 2]
? Limit
```

Out[16]= $\frac{5}{6}$

Limit[*expr*, $x \to x_0$] finds the limiting value of *expr* when x approaches x_0. ≫

In[18]:=
```
Series[Sin[x], {x, 0, 5}]
? Series
```

Out[18]= $x - \frac{x^3}{6} + \frac{x^5}{120} + O[x]^6$

Series[f, {x, x_0, n}] generates a power series expansion for f about the point $x = x_0$ to order $(x - x_0)^n$.
Series[f, {x, x_0, n_x}, {y, y_0, n_y}, …] successively finds series expansions with respect to x, then y, etc. ≫

In[20]:=
```
Series[Sin[x], {x, 0, 2}]
```

Out[20]= $x + O[x]^3$

In[21]:=
```
Series[Sin[x], {x, 0, 7}]
```

Out[21]= $x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + O[x]^8$

In[22]:=
```
Apart[x / (x^2 - 3 x - 4)]
? Apart
```

Out[22]= $\frac{4}{5(-4 + x)} + \frac{1}{5(1 + x)}$

Apart[*expr*] rewrites a rational expression as a sum of terms with minimal denominators.
Apart[*expr*, *var*] treats all variables other than *var* as constants. ≫

In[24]:=
```
D[x Sin[x], x]
```

Out[24]= $x \operatorname{Cos}[x] + \operatorname{Sin}[x]$

In[25]:=
```
Together[1 + 1 / x]
? Together
```

Out[25]= $\frac{1 + x}{x}$

Together[*expr*] puts terms in a sum over a common denominator, and cancels factors in the result. ≫

In[27]:=
```
Factor[5 x^2 - 8 x - 4]
? Factor
```

Out[27]= $(-2 + x)(2 + 5x)$

Factor[*poly*] factors a polynomial over the integers.
Factor[*poly*, Modulus –> p] factors a polynomial modulo a prime p.
Factor[*poly*, Extension –> {a_1, a_2, …}] factors a polynomial
allowing coefficients that are rational combinations of the algebraic numbers a_i. ≫

In[29]:=
```
Eigensystem[{{-17, 15}, {20, 18}}]
% // N

? Eigensystem
?? Eigensystem
```

Out[29]= $\left\{\left\{\frac{1}{2}\left(1+5\sqrt{97}\right), \frac{1}{2}\left(1-5\sqrt{97}\right)\right\}, \left\{\left\{-\frac{9}{10}+\frac{1}{40}\left(1+5\sqrt{97}\right), 1\right\}, \left\{-\frac{9}{10}+\frac{1}{40}\left(1-5\sqrt{97}\right), 1\right\}\right\}\right\}$

Out[30]= {{25.1221, -24.1221}, {{0.356107, 1.}, {-2.10611, 1.}}}

Eigensystem[*m*] gives a list {*values*, *vectors*} of the eigenvalues and eigenvectors of the square matrix *m*.
Eigensystem[{*m*, *a*}] gives the generalized eigenvalues and eigenvectors of *m* with respect to *a*.
Eigensystem[*m*, *k*] gives the eigenvalues and eigenvectors for the first *k* eigenvalues of *m*.
Eigensystem[{*m*, *a*}, *k*] gives the first *k* generalized eigenvalues and eigenvectors. ≫

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Eigensystem[{*m*, *a*}, *k*] gives the first *k* generalized eigenvalues and eigenvectors. ≫

Attributes[Eigensystem] = {Protected}

Options[Eigensystem] = {Cubics → False, Method → Automatic, Quartics → False, ZeroTest → Automatic}

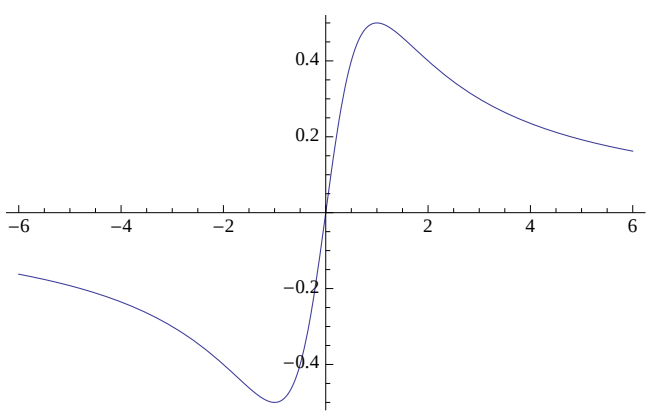
In[33]:=
```
Inverse[{{-4, 3}, {4, -4}}]
? Inverse
```

Out[33]= $\left\{\left\{-1, -\frac{3}{4}\right\}, \{-1, -1\}\right\}$

Inverse[*m*] gives the inverse of a square matrix *m*. ≫

In[36]:=
```
Plot[x / (x^2 + 1), {x, -6, 6}]
? Plot
```

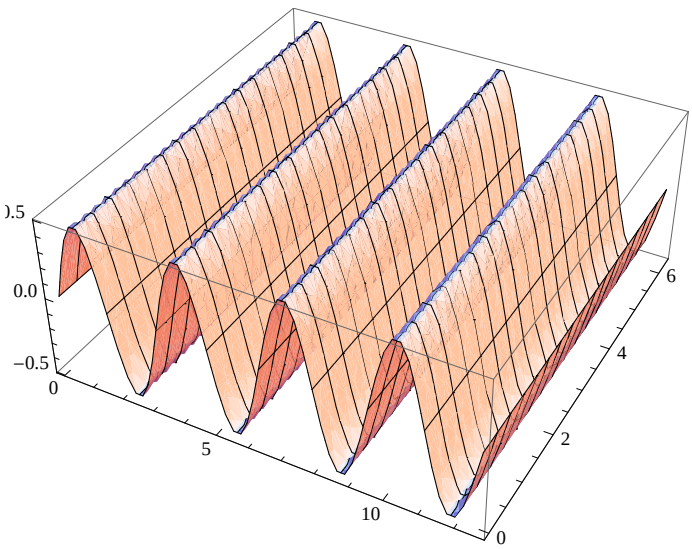
Out[36]=



Plot[*f*, {*x*, x_{min}, x_{max}}] generates a plot of *f* as a function of *x* from x_{min} to x_{max}.
Plot[{f_1, f_2, …}, {*x*, x_{min}, x_{max}}] plots several functions f_i. ≫

In[38]:=
```
Plot3D[Sin[x] Cos[x], {x, 0, 4 Pi}, {y, 0, 2 Pi}]
? Plot3D
```

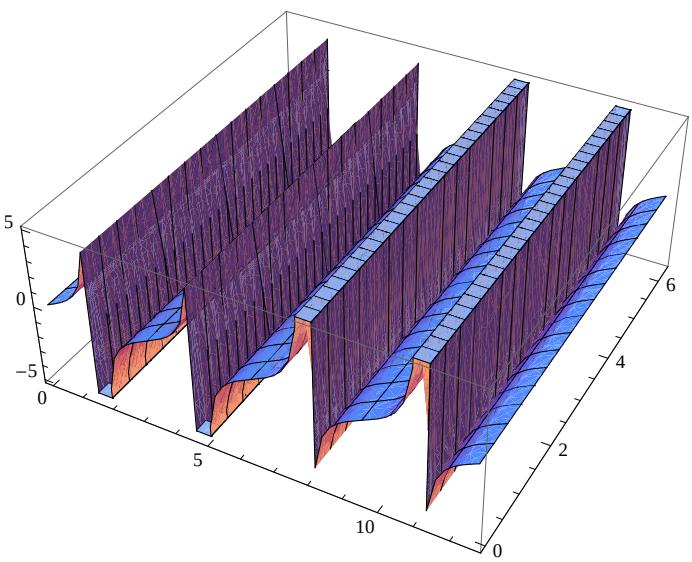
Out[38]=



Plot3D[f, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}] generates a three−dimensional plot of f as a function of x and y.
Plot3D[{f_1, f_2, …}, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}] plots several functions. ≫

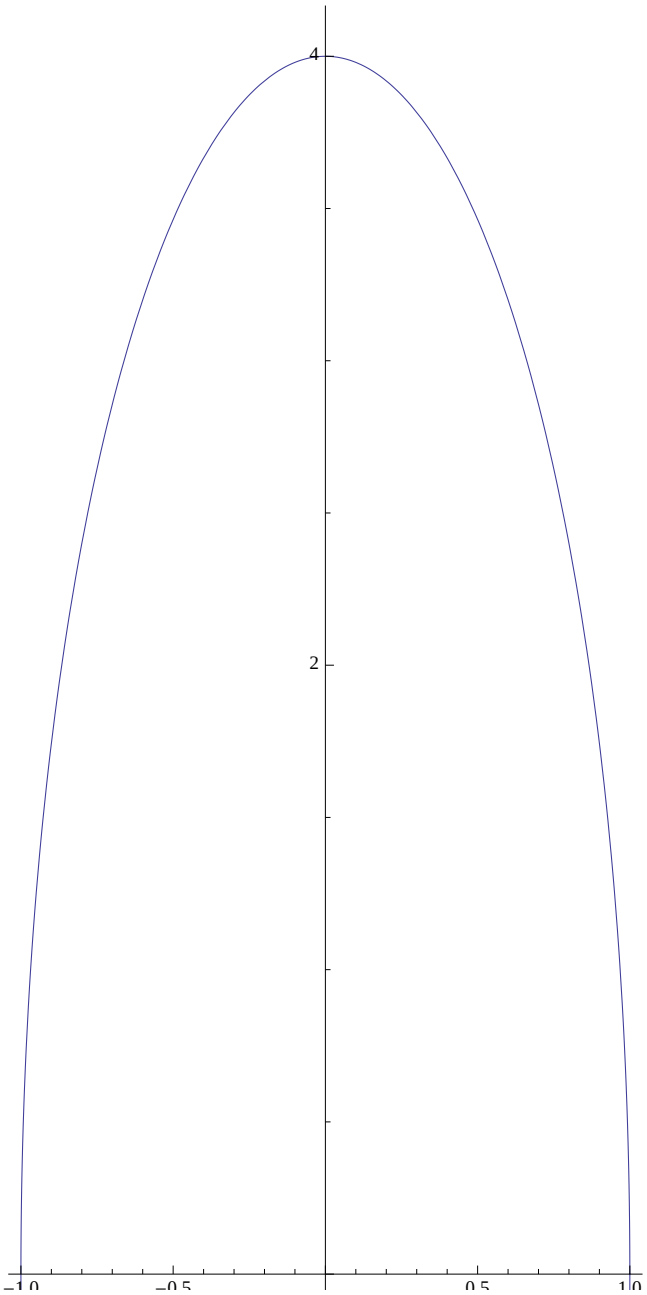
In[40]:=
```
Plot3D[Sin[x] / Cos[x], {x, 0, 4 Pi}, {y, 0, 2 Pi}]
```

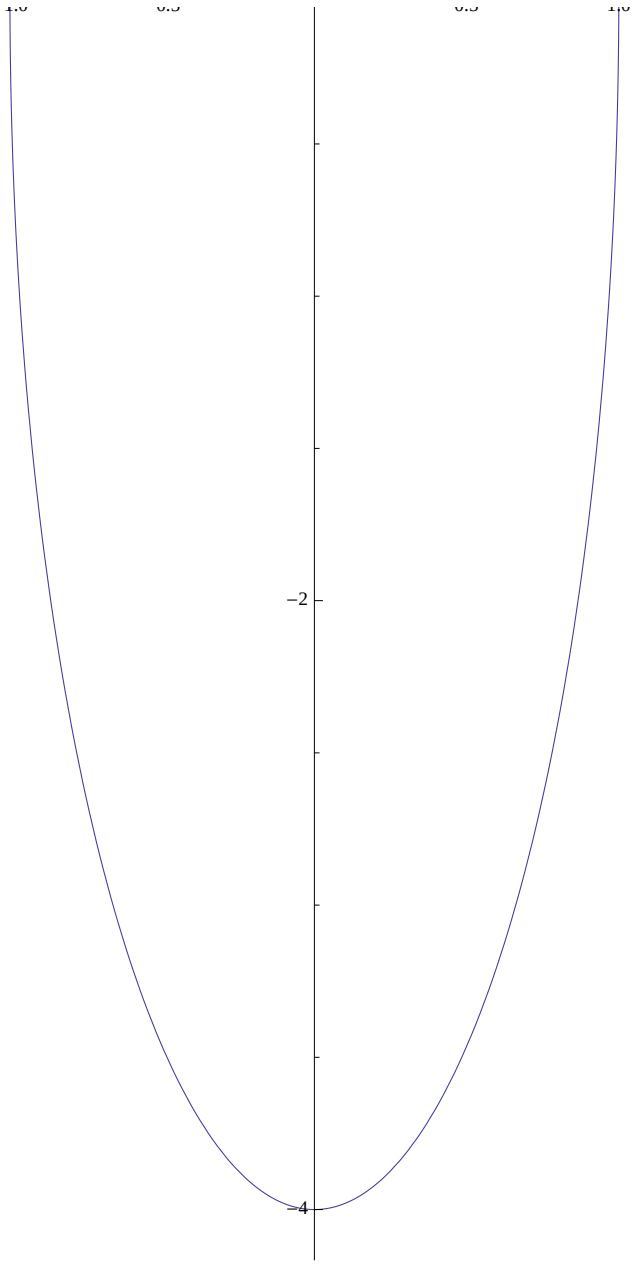
Out[40]=



In[41]:=
```
ParametricPlot[{Cos[t], 4 Sin[t]}, {t, 0, 2 Pi}]
? ParametricPlot
```

Out[41]=





ParametricPlot[{f_x, f_y}, {u, u_{min}, u_{max}}] generates a parametric plot of a curve with x and y coordinates f_x and f_y as a function of u.
ParametricPlot[{{f_x, f_y}, {g_x, g_y}, …}, {u, u_{min}, u_{max}}] plots several parametric curves.
ParametricPlot[{{f_x, f_y}, {u, u_{min}, u_{max}}, {v, v_{min}, v_{max}}] plots a parametric region.
ParametricPlot[{{f_x, f_y}, {g_x, g_y}, …}, {u, u_{min}, u_{max}}, {v, v_{min}, v_{max}}] plots several parametric regions. ≫

In[44]:=
```
Integrate[x Sin[x], x]
? Integrate
```

Out[44]= $-x\,\mathrm{Cos}[x] + \mathrm{Sin}[x]$

Integrate[*f*, *x*] gives the indefinite integral $\int f\,dx$.

Integrate[*f*, {*x*, x_{min}, x_{max}}] gives the definite integral $\int_{x_{min}}^{x_{max}} f\,dx$.

Integrate[*f*, {*x*, x_{min}, x_{max}}, {*y*, y_{min}, y_{max}}, …] gives the multiple integral $\int_{x_{min}}^{x_{max}} dx \int_{y_{min}}^{y_{max}} dy \ldots f$. ≫

In[46]:=
```
Expand[(5 x + 2) (x - 2)]
? Expand
```

Out[46]= $-4 - 8x + 5x^2$

Expand[*expr*] expands out products and positive integer powers in *expr*.
Expand[*expr*, *patt*] leaves unexpanded any parts of *expr* that are free of the pattern *patt*. ≫

In[48]:=
```
MatrixPower[{{-1, -3}, {-1, 3}}, 5]
? MatrixPower
```

Out[48]= {{68, -372}, {-124, 564}}

MatrixPower[*m*, *n*] gives the nth matrix power of the matrix *m*.
MatrixPower[*m*, *n*, *v*] gives the nth matrix power of the matrix *m* applied to the vector *v*. ≫

In[50]:=
```
Cancel[(x - 1) / (x^2 - 1)]
? Cancel
```

Out[50]= $\frac{1}{1+x}$

Cancel[*expr*] cancels out common factors in the numerator and denominator of *expr*. ≫

In[52]:=
```
DSolve[y'[x] == 1 + y[x], y[x], x]
? DSolve
```

Out[52]= $\{\{y[x] \to -1 + e^x\, C[1]\}\}$

DSolve[*eqn*, *y*, *x*] solves a differential equation for the function *y*, with independent variable *x*.
DSolve[{eqn_1, eqn_2, …}, {y_1, y_2, …}, *x*] solves a list of differential equations.
DSolve[*eqn*, *y*, {x_1, x_2, …}] solves a partial differential equation. ≫

In[54]:=
```
Solve[{2 x - 7 == 7, 4 x + 2 y == 2}]
? Solve
```

Out[54]= {{x → 7, y → -13}}

Solve[*eqns*, *vars*] attempts to solve an equation or set of equations for the variables *vars*.
Solve[*eqns*, *vars*, *elims*] attempts to solve the equations for *vars*, eliminating the variables *elims*. ≫

In[56]:=
```
Solve[x^2 - 4 x - 5 == 0]
```

Out[56]=
```
{{x → -1}, {x → 5}}
```