Solvent Effect Calculation

1. 連続体モデル 古典的反作用場モデル Polarizable continuum method (PCM) = apparent surface charge model 2. 分子動力学シミュレーション モンテカルロシミュレーション 3. QM/MM \mathfrak{V} \mathfrak{L} \mathfrak{L} \mathcal{V} \mathfrak{V} \mathfrak{V} 4. Car-Parrinelloシミュレーション 5. RISMモデル

Continuum solvation model

仮定:カノニカル集合(NVT一定)の系 溶媒QM Schrödinger方程式 (N≈10²³) 溶質 $\left[\sum \frac{\hbar^2}{2m_e} \nabla_{e,i}^2 + \sum \frac{\hbar^2}{2m_n} \nabla_{\alpha,i}^2 - \sum_{i,\alpha} \frac{Z_{\alpha}}{r_{i\alpha}} + \sum_{i,j} \frac{1}{r_{ij}}\right] \Psi = E \Psi$ QM 近似方程式の導出 溶質: quantum mechanics 溶媒: continuum 溶質 **OM** 溶質一溶媒間: effective hamiltonian continuum

Basic equations

Schrödinger equation

 $\mathbf{H}\Psi = E\Psi$



Simple example of reaction field

溶質分子:点双極子µ (Z軸に平行)

キャビティ:球形(半径a)

キャビティー内のポテンシャルを ϕ_1 、その外側のポテンシャルを ϕ_2 とすると、これらはLaplaceの方程式を次の境界条件のもとで満足しなければならない。

$$(\phi_{2})_{r \to \infty} = 0$$

$$(\phi_{1})_{r=a} = (\phi_{2})_{r=a}$$

$$\epsilon (\partial \phi_{2} / \partial r)_{r=a} = (\partial \phi_{1} / \partial r)_{r=a}$$

$$(\phi_{1})_{a \to \infty} = \mu / r^{2} \cdot \cos \theta$$

 ϕ_1 から誘電体が存在しないときの ポテンシャル $\phi_0 = \mu/r^2 \cdot \cos \theta$ を引いて、 $\mathbf{R} = \phi_1 - \phi_0$ あるいは、 $\mathbf{R} = \nabla \mathbf{R} = g \mu$ ここで、

 $g = 2(\epsilon - 1)/(2\epsilon + 1) \cdot 1/a^3$

con



Solvation process in the continuum model

1. Before solvation





2. Cavity formation

 $\Delta G_{cav} > 0$ Semiempirical evaluation





3. Solute-solvent interaction

 $\Delta G_{el} < 0$ quantum mechanical $\Delta G_{disp} < 0$ semiempirical



Derivation of ΔG_{el}

Guntelberg-Muller type charging process (仮想過程)

 ΔG_{el} はステップ3が可逆的に起こるものとする。いま、溶質分子が $e_1, e_2, ..., e_n$ の電 荷からできているものとする。この溶質分子をキャビティーに入れたときi番目の電 荷 e_i が溶媒から受ける反作用場を $R(e_i)$ とすれば、 ΔG_{solv} は電荷 i (=1, 2, ...,n) が反作用 場 $R(e_i)$ の中で0から e_i まで可逆的に増大するときの仕事と考えてよいから、

 $\Delta G_{solv} = \Sigma \int \lambda e_i \cdot R(e_i) d\lambda$

 $= 1/2 \Sigma \mathbf{e}_{i} \cdot \mathbf{R}(\mathbf{e}_{i})$

となる。ここで、λはcharging parameterである。

△G_{el}が平衡状態での溶質分子と溶媒との静電的相互作用

 $E_{elec} = \Sigma e_i \cdot R(e_i)$

のちょうど半分の値をもつということである。 ΔG_{el} の残りの部分は、主として溶媒の分極・配向に費やされる自由エネルギーと考えられるから、それを ΔG_{pol} とおけば

$$\Delta G_{el} = E_{elec} + \Delta G_{pol}$$

より、

$$\Delta G_{\text{pol}} = -1/2E_{\text{elec}}$$

となる。

Examples of solvation energy ΔG_{el} $\Delta G_{el} = \frac{1}{2} \int \rho_M(\mathbf{r}) R(\mathbf{r}) d\mathbf{r}$ $\rho_M(\mathbf{r})$: 溶質の電荷分布

Classical examples for a spherical cavity

 $\rho_{M}(\mathbf{r})$: monopole

$$\Delta G_{el} = -\frac{\varepsilon - 1}{2\varepsilon} \frac{q}{a}$$
 Bornの式



 $\rho_{\rm M}(\mathbf{r})$: dipole





Generalized Born (GB) Model

$$G_{elec} = -\frac{1}{2} \left(1 - \frac{1}{\varepsilon} \right) \sum_{k,k'}^{atoms} q_k q_{k'} \gamma_{kk'}$$

$$\gamma_{kk'} = (\gamma_{kk'}^2 + \alpha_k \alpha_{k'} e^{-r_{kk'}^2/d_{kk'} \alpha_k \alpha_{k'}})^{-1/2}$$

 q_k :原子kの電荷、 α_k :有効Born半径、d:パラメーター(通常は4)



History of PCM

任意形状のキャビティ、任意の電荷分布(|Ψ|²)に 適用可能な連続体モデル

- 1. S. Miertus, E. Scrocco and J. Tomasi, Chem. Phys., **55**,117-129 (1981). Iterative PCM
- 2. H. Hoshi, M. Sakurai, Y. Inoue and R. Chujo, J. Chem. Phys., 87, 1107-1115 (1987).
 PCM combined with the boundary element method (generalized reaction field theory)
- 3. A. Klamt and G. Schuurman,

J. Chem. Soc. Perkin Trans, 2, 799-805 (1993). Conductor-like Screening model (COSMO)

Feature of PCM



誘電体から生じる反作用場と 分子電荷分布との相互作用 真空中の表面電荷と電荷分布との相互作用

$$V = \sum_{i} q_{i} \int_{\Gamma} \frac{\sigma(\mathbf{s})}{|\mathbf{r}_{\mathbf{i}} - \mathbf{s}|} d\mathbf{s}$$

What is PCM ?

$$\varepsilon_i(s) \frac{\partial}{\partial n^+} \phi(s) = \varepsilon_o(s) \frac{\partial}{\partial n^-} \phi(s)$$

$$\sigma = -(\mathbf{P}_{o} - \mathbf{P}_{i}) \cdot \mathbf{n}$$

$$\mathbf{P}_{\mathbf{i}} = -(\varepsilon_i - 1)/4\pi \times \nabla \phi$$

$$\varepsilon_i = 1, P_i = 0$$
 $\varepsilon_o = \varepsilon, P_o = P$
 $\sigma(s) = -P \cdot n = (\varepsilon - 1)/4\pi \times \phi(s) \cdot n$

$$V = \sum_{i} q_{i} \int_{\Gamma} \frac{\sigma(\mathbf{s})}{|\mathbf{r}_{i} - \mathbf{s}|} d\mathbf{s}$$





Boundary element method

$$\phi(\mathbf{r}) = \int_{v} \frac{\rho(\mathbf{x})}{|\mathbf{r} - \mathbf{x}|} d\mathbf{x} + \int_{\Gamma} \frac{\sigma(\mathbf{s})}{|\mathbf{r} - \mathbf{s}|} d\mathbf{s},$$

$$\sigma(\mathbf{s}) = -\frac{1}{4\pi} \left(1 - \frac{\epsilon_o(\mathbf{s})}{\epsilon_i(\mathbf{s})} \right) \left(\frac{\partial \phi(\mathbf{s})}{\partial n^{-1}} \right)$$

Integral equation

$$\sigma(\mathbf{s}) + \frac{1}{4\pi} \left(1 - \frac{\epsilon_o(\mathbf{s})}{\epsilon_i(\mathbf{s})} \right) \left(\frac{\partial}{\partial n^-} \right)_{\mathbf{s}} \int_{\Gamma} \frac{\sigma(\mathbf{r})}{|\mathbf{s} - \mathbf{r}|} d\mathbf{r}$$
$$= -\frac{1}{4\pi} \left(1 - \frac{\epsilon_o(\mathbf{s})}{\epsilon_i(\mathbf{s})} \right) \left(\frac{\partial}{\partial n^-} \right)_{\mathbf{s}} \int_{\nu} \frac{\rho(\mathbf{r})}{|\mathbf{s} - \mathbf{r}|} d\mathbf{r}.$$



Solute-solvent interaction energy

 $E = \langle \Psi | \mathscr{H} | \Psi \rangle = \langle \Psi | \mathscr{H}_{0} | \Psi \rangle + \langle \Psi | V | \Psi \rangle$

Helmholtz free energy $H = \langle \Psi | \mathscr{H} | \Psi \rangle - 1/2 \int_{v} \rho(\mathbf{r}) \phi'(\mathbf{r}) d\mathbf{r}$ $H = \langle \Psi | \mathscr{H} | \Psi \rangle - 1/2 \langle \Psi | V | \Psi \rangle$

$$V = -\sum_{i,j} \sigma_j \int_{\Gamma_j} \frac{1}{|\mathbf{e}_i - \mathbf{s}|} d\mathbf{s}$$
$$+ \sum_{k,j} Z_k \sigma_j \int_{\Gamma_j} \frac{1}{|\mathbf{N}_k - \mathbf{s}|} d\mathbf{s}.$$

$$v = \langle \Psi | V | \Psi \rangle$$
$$v = \mathbf{P}' \mathbf{T} \mathbf{P} + \mathbf{P}' \mathbf{J}_1 + \mathbf{J}_2 \mathbf{P} + \mathbf{B}_3$$



H. Hoshi et al., J. Chem. Phys., **87**, 1107-1115 (1987). PCM combined with the boundary element method (generalized reaction field theory)

The relationship between the reaction field operators and the dielectric constant

The reaction field matrices T, J_1 , J_2 , and B in Eq. (5) are represented as the product of some matrices,¹⁰

$$T = C^{t} W^{-1} \partial^{-} C, \tag{A1a}$$

$$J_1 = C^t W^{-1} \partial^- M Z, \tag{A1b}$$

$$J_2 = Z^t M^t W^{-1} \partial^- C, \tag{A1c}$$

$$B = Z^{t} M^{t} W^{-1} \partial^{-} M Z. \tag{A1d}$$

In these equations, C and $\partial^- C$ are the matrices representing the electrostatic potential and the electric field generated from the electronic distribution of a solute molecule, respectively. Similarly, M and $\partial^- M$ are the matrices representing the electrostatic potential and the electric field generated from the nuclear charge distribution of the solute molecule, respectively. The diagonal element of the matrix W is inversely proportional to the reaction field factor f(u) [Eq. (33)],

$$W_{\vec{n}} \approx f(\varepsilon)^{-1}. \tag{A2}$$

As a result, T, J_1 , J_2 , and B are proportional to f(z).

Expression of Fock matrix

 $I = \langle \Psi | A | \Psi \rangle - \lambda \langle \Psi | \Psi \rangle,$ $A = \mathcal{H} - 1/2V = \mathcal{H}o + 1/2V,$

 λ :Lagrange's undeterminate multiplier. From the condition $\delta I = 0$ Fock matrix is derived as follows

$$F_{\mu\nu} = F^{o}_{\mu\nu} + F'_{\mu\nu} ,$$

$$F'_{\mu\nu} = 1/2\{(\mathbf{J}_1) + (\mathbf{J}_2)\}_{\mu\nu} + \{\mathbf{TP}\}_{\mu\nu}$$

What is COSMO ?

 $f(\varepsilon) = (\varepsilon - 1)/(\varepsilon + 1) \qquad f(80) = 0.975 \approx 1.000 = f(\infty)$

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Energy

 $E(\mathbf{q}) = 1/2\mathbf{Q}\mathbf{C}\mathbf{Q} + \mathbf{Q}\mathbf{B}\mathbf{q} + \frac{1}{2}\mathbf{q}\mathbf{A}\mathbf{q}$

A: inverse of **q-q** distance

B: inverse of **Q**-**q** distance

Minimization condition of $E(\mathbf{q})$ $\nabla_{\mathbf{q}} E(\mathbf{q}) |_{\mathbf{q}^*} = \mathbf{B}\mathbf{Q} + \mathbf{A}\mathbf{q}^* = 0$ $\mathbf{q}^* = -\mathbf{A}^{-1}\mathbf{B}\mathbf{Q}$

 $E(\mathbf{q}^*) = 1/2\mathbf{Q}(\mathbf{C} \cdot \mathbf{B}\mathbf{A}^{-1}\mathbf{B})\mathbf{Q} \qquad \Delta E = -1/2\mathbf{Q}\mathbf{B}\mathbf{A}^{-1}\mathbf{B}\mathbf{Q}$

Solvation energy evaluation in the continuum model



Solvent effect on the excitation energy of a molecule



Theoretical evaluation of medium effects on absorption maxima of molecular solutes. I. Formulation of a new method based on the self-consistent reaction field theory

H. Houjou, M. Sakurai and Y. Inoue J. Chem. Phys. 107, 5652-5660 (1997).

Expression of the free energy

$$A = \left\langle \Psi \mid \mathbf{H} \mid \Psi \right\rangle - \frac{1}{2} \left\langle \Psi \mid V \mid \Psi \right\rangle$$
$$= \left\langle \Psi \mid \mathbf{H}_{0} \mid \Psi \right\rangle + \frac{1}{2} \left\langle \Psi \mid V \mid \Psi \right\rangle$$

For the ground state (Hoshi et al., J. Chem. Phys. 87, 1107 (1987)) $A = \left\langle \Psi_0 \mid H_0 \mid \Psi_0 \right\rangle + \frac{1}{2} \left\langle \Psi_0 \mid V \mid \Psi_0 \right\rangle = E_0 + \frac{1}{2} V_0$ $V_0 = \mathbf{P}_0 \mathbf{T} \mathbf{P}_0 + \mathbf{P}_0 \mathbf{J}_1 + \mathbf{J}_2 \mathbf{P}_0 + \mathbf{B}$

For the excited state

$$A_{i} = \left\langle \Psi_{i} \left| \mathbf{H}_{0} \right| \Psi_{i} \right\rangle + \frac{1}{2} \left\langle \Psi_{i} \right\rangle \left| \Psi_{i} \right\rangle = \left\langle \Psi_{i} \left| \mathbf{H}_{0} \right| \Psi_{i} \right\rangle + \frac{1}{2} V_{i}$$

Expression of the excitation energy

Infinitely slow relaxation



$$\Delta A_t^s = E_t - E_0 + \frac{1}{2} [(\mathbf{P}_t - \mathbf{P}_0)\mathbf{T}\mathbf{P}_0 + (\mathbf{P}_t - \mathbf{P}_0)\mathbf{J}_1]$$
$$V_t^s = \mathbf{P}_t\mathbf{T}\mathbf{P}_0 + \mathbf{P}_t\mathbf{J}_1 + \mathbf{J}_2\mathbf{P}_0 + \mathbf{B}$$

Infinitely fast relaxation



$$\Delta A_{t} = E_{t} - E_{0} + \frac{1}{2} [(\mathbf{P}_{t} - \mathbf{P}_{0})\mathbf{T}_{stat}\mathbf{P}_{0} + (\mathbf{P}_{t} - \mathbf{P}_{0})\mathbf{J}_{1st} + \mathbf{P}_{t}\mathbf{T}_{opt}(\mathbf{P}_{t} - \mathbf{P}_{0}) + \mathbf{J}_{2opt}(\mathbf{P}_{t} - \mathbf{P}_{0})]$$

The simplified expression for solvent dependence of the excitation energy

$$(\mathbf{P}_{i} - \mathbf{P}_{0})\mathbf{T}_{stat}\mathbf{P}_{0}, (\mathbf{P}_{i} - \mathbf{P}_{0})\mathbf{J}_{1stat} \propto f(\varepsilon_{stat}), \varepsilon_{stat} = \varepsilon(0)$$

$$\mathbf{P}_{i}\mathbf{T}_{opt}(\mathbf{P}_{i} - \mathbf{P}_{0}), \mathbf{J}_{2opt}(\mathbf{P}_{i} - \mathbf{P}_{0}) \propto f(\varepsilon_{opt}), \varepsilon_{opt} = \varepsilon(\infty) = n^{2}$$

$$f\left(\varepsilon\right) = \frac{\varepsilon - 1}{\varepsilon + 1}$$

$$v_{\text{max}} = Af(\varepsilon_{\text{stat}}) + Bf(\varepsilon_{\text{opt}}) + C$$