B-2). CIP (Constrained Interpolated Profile)法

Cubic-Spline: g_i^n の決定が煩雑 ⇒ どうにかならんか？

いっそのこと、g_i^n も f_i^n と同じように時間発展を計算したら？

1次元移流方程式（双曲型）

$$\frac{\partial f}{\partial t}+u\frac{\partial f}{\partial x}=0$$

微分を取る ⇒

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial t}+u\frac{\partial f}{\partial x}\right)=\frac{\partial}{\partial t}\left(\frac{\partial f}{\partial x}\right)+u\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)+\frac{\partial u}{\partial x}\frac{\partial f}{\partial x}=\frac{\partial g}{\partial t}+u\frac{\partial g}{\partial x}+\frac{\partial u}{\partial x}g=0$$

勾配 $g(t,x)=\left(\frac{\partial f(t,x)}{\partial x}\right)$ に対する基礎式

$$\therefore \frac{\partial g}{\partial t}+u\frac{\partial g}{\partial x}=-g\frac{\partial u}{\partial x} \qquad (1\text{-}52)$$

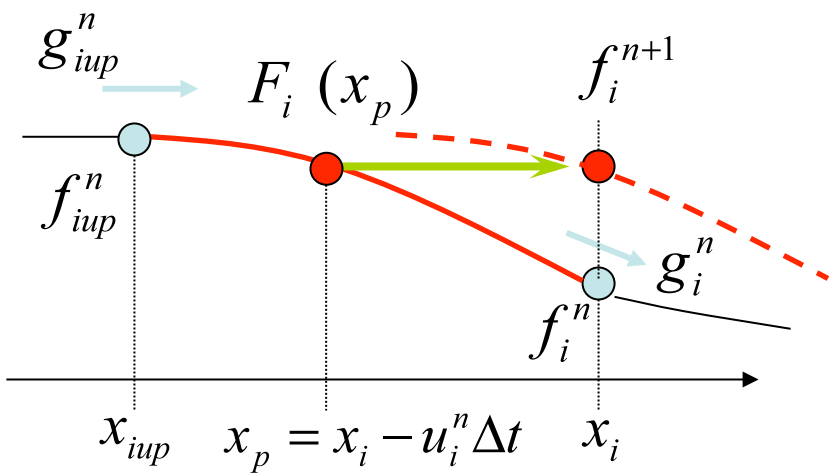
特に、uが空間的に一定 $\frac{\partial u}{\partial x}=0$ であれば、f と同じ移流方程式！！

$$\therefore \frac{\partial g}{\partial t}+u\frac{\partial g}{\partial x}=0$$

勾配も上流から流速uで流れる

B-2). CIP (Constrained Interpolated Profile)法

補間関数（3次多項式）:Cubic-Spline法(1-51)がそのまま使用可能。



区間$[x_i, x_{iup}]$での補間関数 $X_i = x - x_i$

$$F_i(x) = a_i X_i^3 + b_i X_i^2 + g_i^n X_i + f_i^n$$

$$a_i = -2(f_{iup}^n - f_i^n)/\Delta x_{i\to iup}^3 + (g_i^n + g_{iup}^n)/\Delta x_{i\to iup}^2$$

$$b_i = -3(f_i^n - f_{iup}^n)/\Delta x_{i\to iup}^2 - (2g_i^n + g_{iup}^n)/\Delta x_{i\to iup}$$

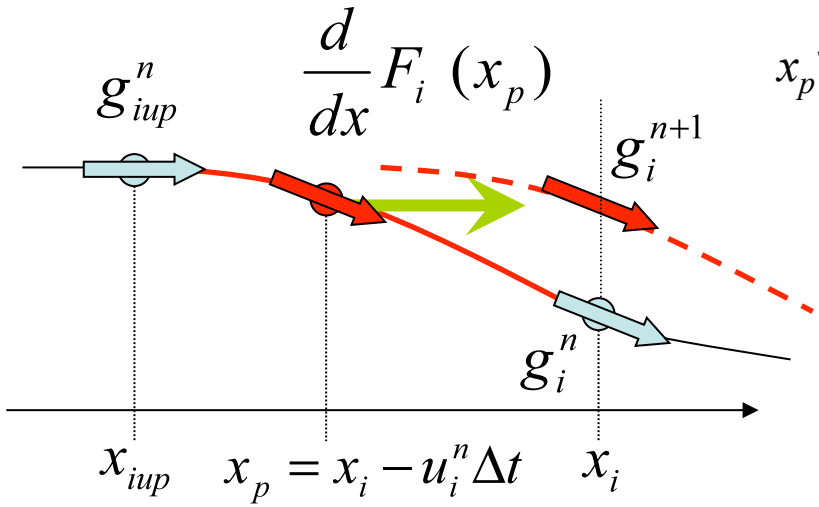
$$\therefore f_i^{n+1} = F_i\ (x_p) = a_i \xi^3 + b_i \xi^2 + g_i^n \xi + f_i^n$$

$$\xi = -u\Delta t$$

B-2). CIP (Constrained Interpolated Profile)法

$$\therefore \frac{\partial g}{\partial t} + u\frac{\partial g}{\partial x} = 0$$

勾配も上流から流速uで流れる



x_pでの勾配も補完関数で推定

$$\frac{d}{dx}F_i(x) = 3a_i X_i^2 + 2b_i X_i + g_i^n$$

$$\therefore g_i^{n+1} = \frac{d}{dx}F_i\ (x_p) = 3a_i\xi^2 + 2b_i\xi + g_i^n$$

B-2). CIP (Constrained Interpolated Profile)法

流速uは空間・時間の関数である場合：$\therefore \dfrac{\partial g}{\partial t} + u\dfrac{\partial g}{\partial x} = \underline{\underline{-g\dfrac{\partial u}{\partial x}}}$

１次元CIP法の計算式：

$$a_i = -2(f_{iup}^n - f_i^n)/\Delta x_{i\to iup}^3 + (g_i^n + g_{iup}^n)/\Delta x_{i\to iup}^2$$

$$b_i = -3(f_i^n - f_{iup}^n)/\Delta x_{i\to iup}^2 - (2g_i^n + g_{iup}^n)/\Delta x_{i\to iup}$$

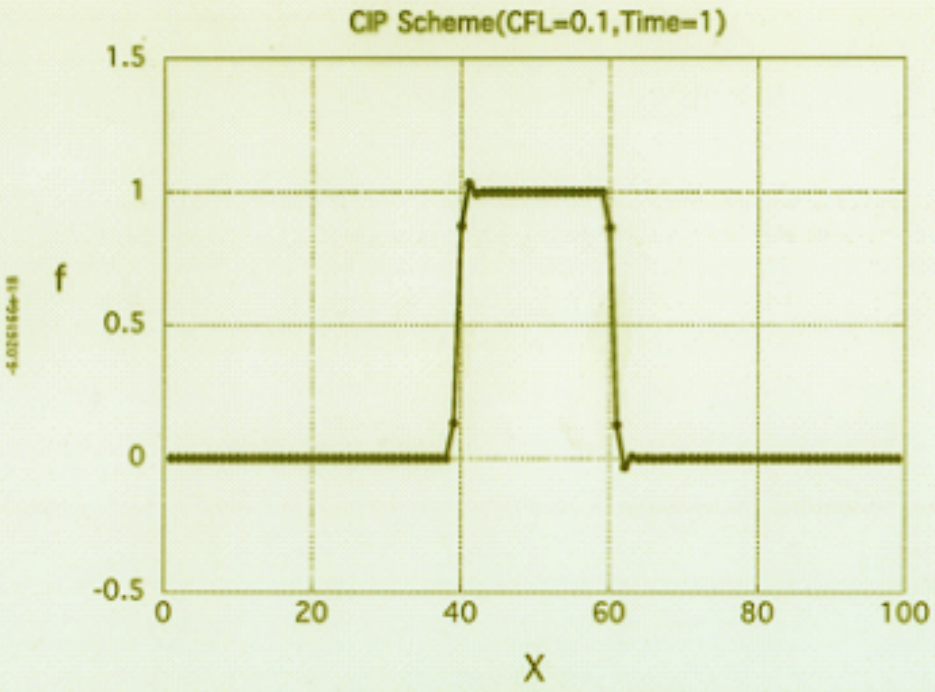
$$\begin{cases} f_i^{n+1} = a_i\xi_i^3 + b_i\xi_i^2 + g_i^n\xi_i + f_i^n \\ g_i^{n+1} = \underline{\underline{\left[1 - \left(\dfrac{\partial u}{\partial x}\right)_i^n \Delta t\right]}}\left(3a_i\xi_i^2 + 2b_i\xi_i + g_i^n\right) \end{cases}$$

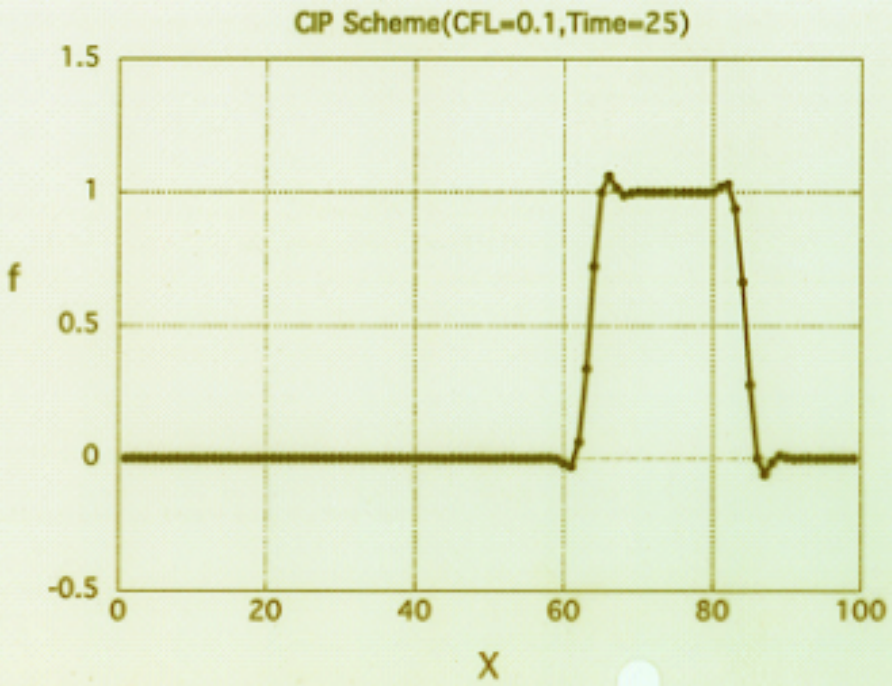
$$iup = i - \mathrm{sing}(u), \quad \Delta x_{i\to iup} = x_{iup} - x_i, \quad \xi_i = -u_i^n\Delta t$$

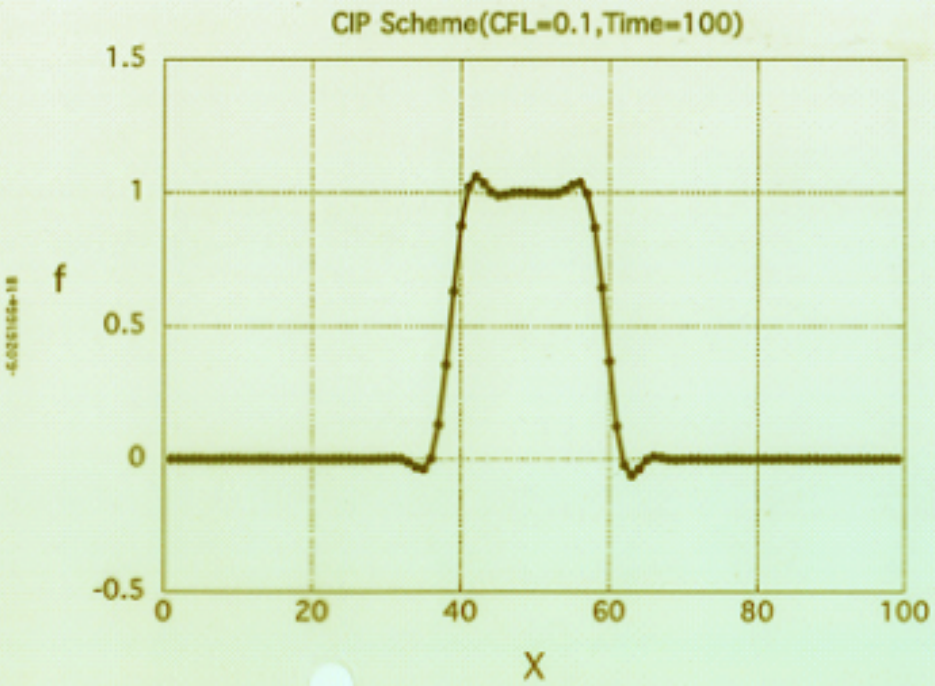
②補間による方法（CIP法）

- CIP法

低数値拡散
振動は成長しない







Solution of Advection Eq. By CIP

矩形波の移流計算（CFL=0.2)

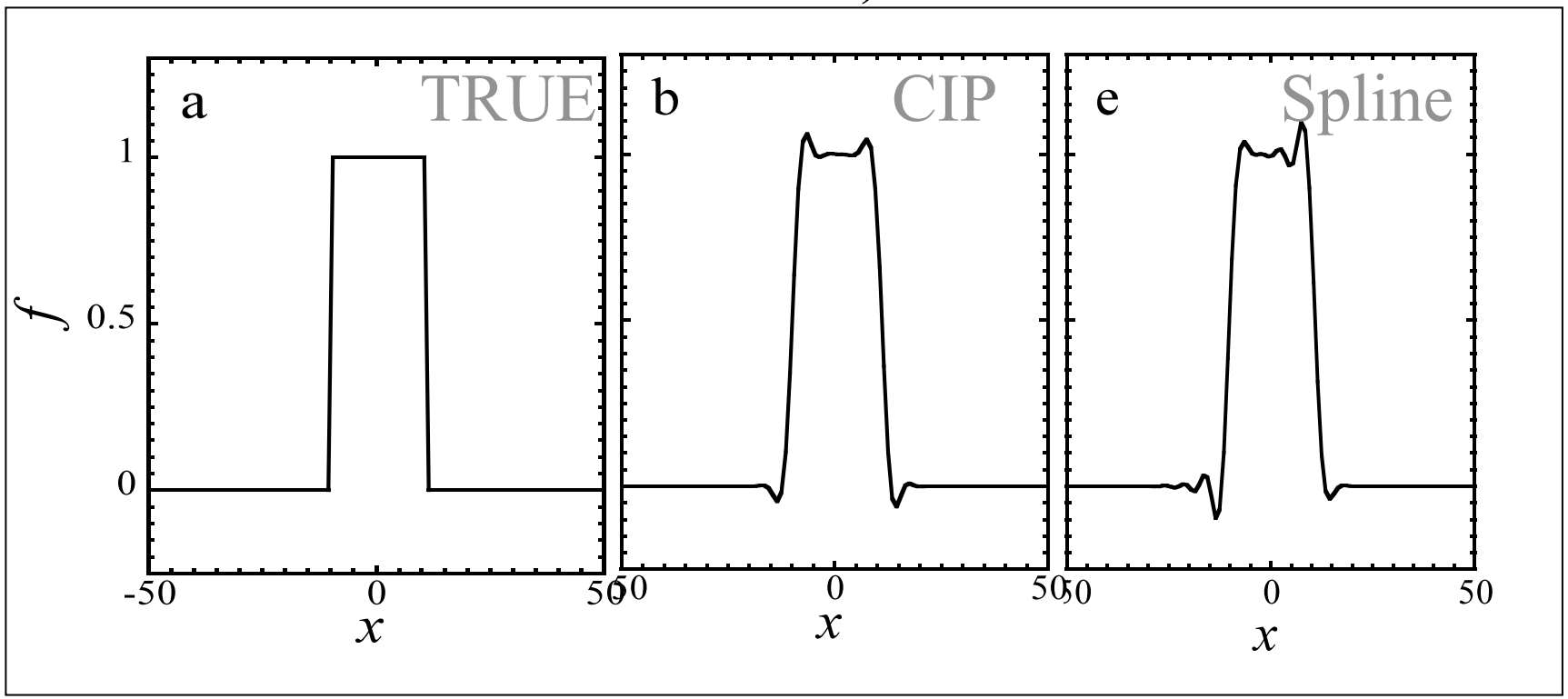
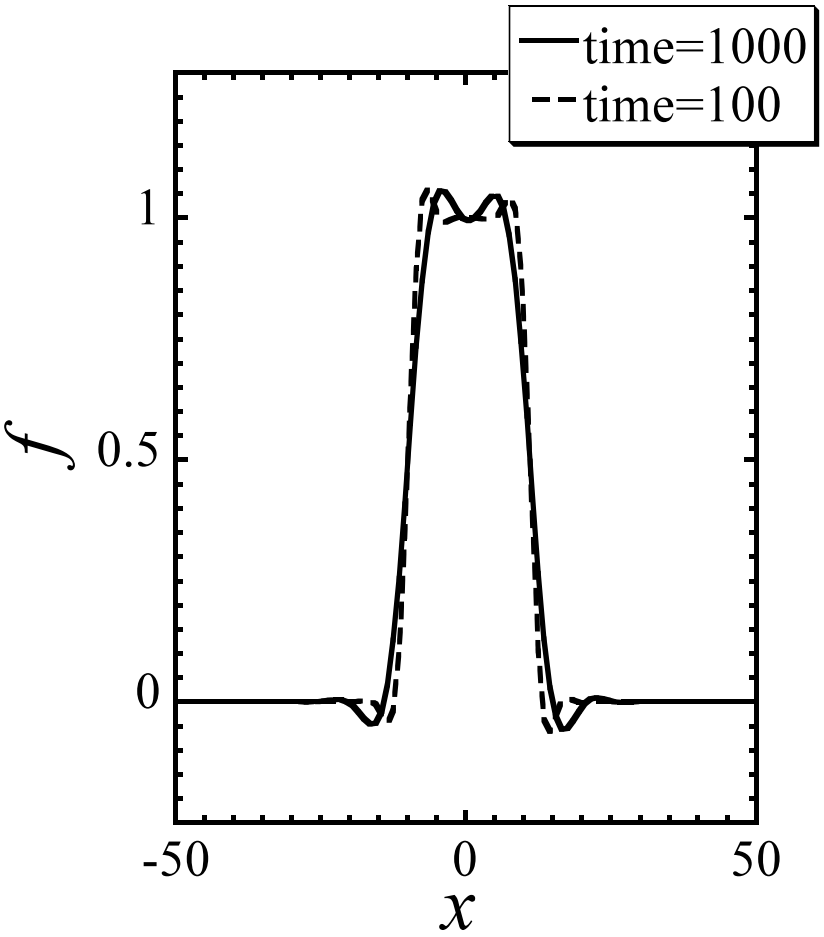


図2-2-2　矩形波の移流計算結果。格子間隔Δx=1、メッシュ数100、周期境界条件、速度場v=1、初期条件の矩形波は幅20、CFL数0.2ででの波形

Solution of Advection Eq. By CIP-2

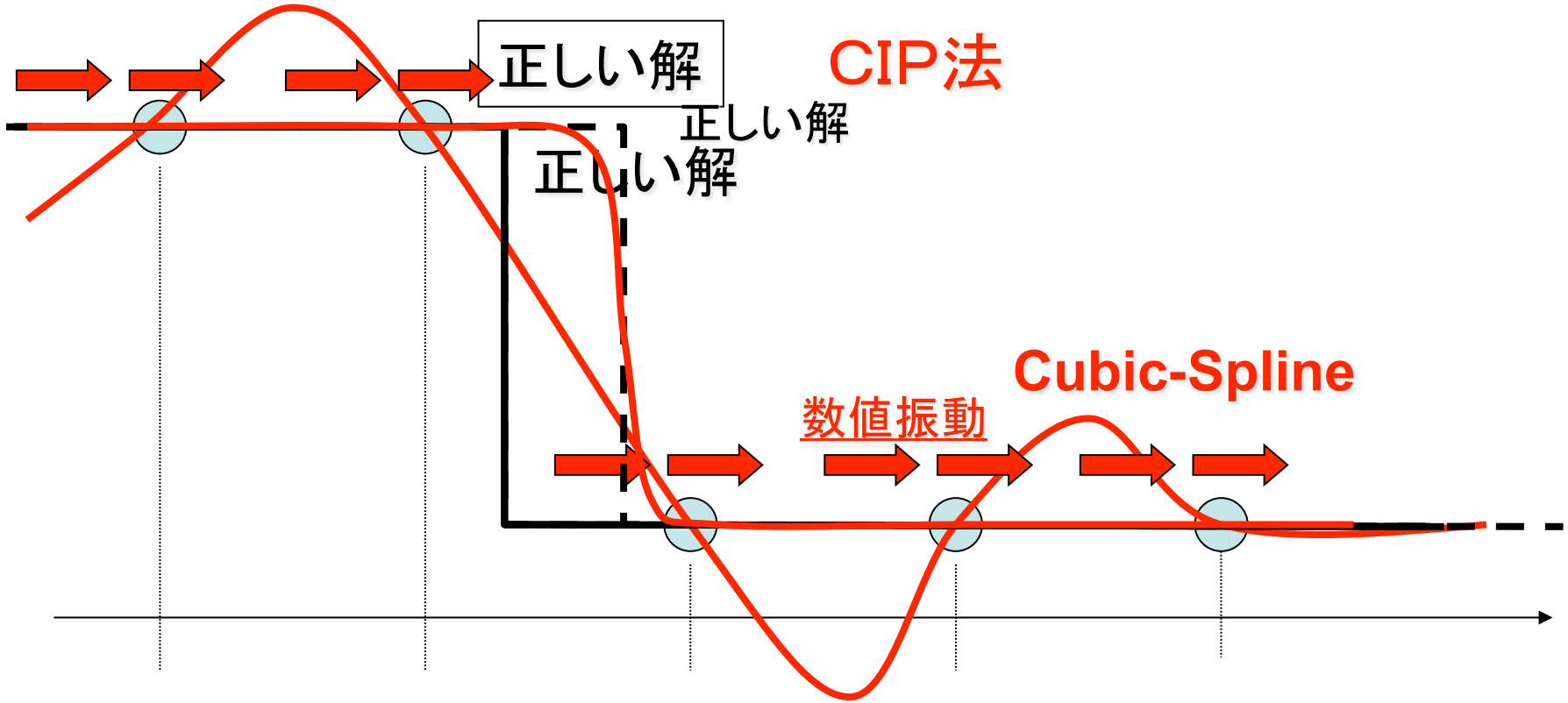


CIP法による矩形波の移流計算結果。異なる時間ステップにおける波形を表示

Over Shoot
Under Shoot
$\approx 10\%$

振動:成長は無い

Cubic-SplineとCIPの違い



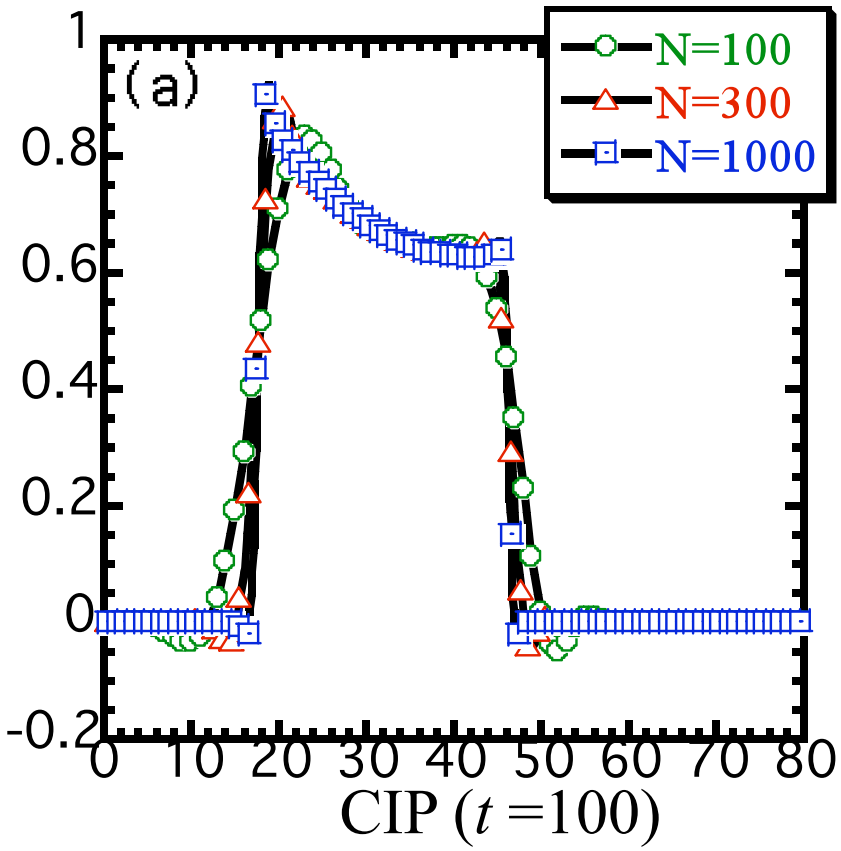
Cubic-Spline:格子点上で滑らかに接続するという“非物理的条件”から勾配を決定

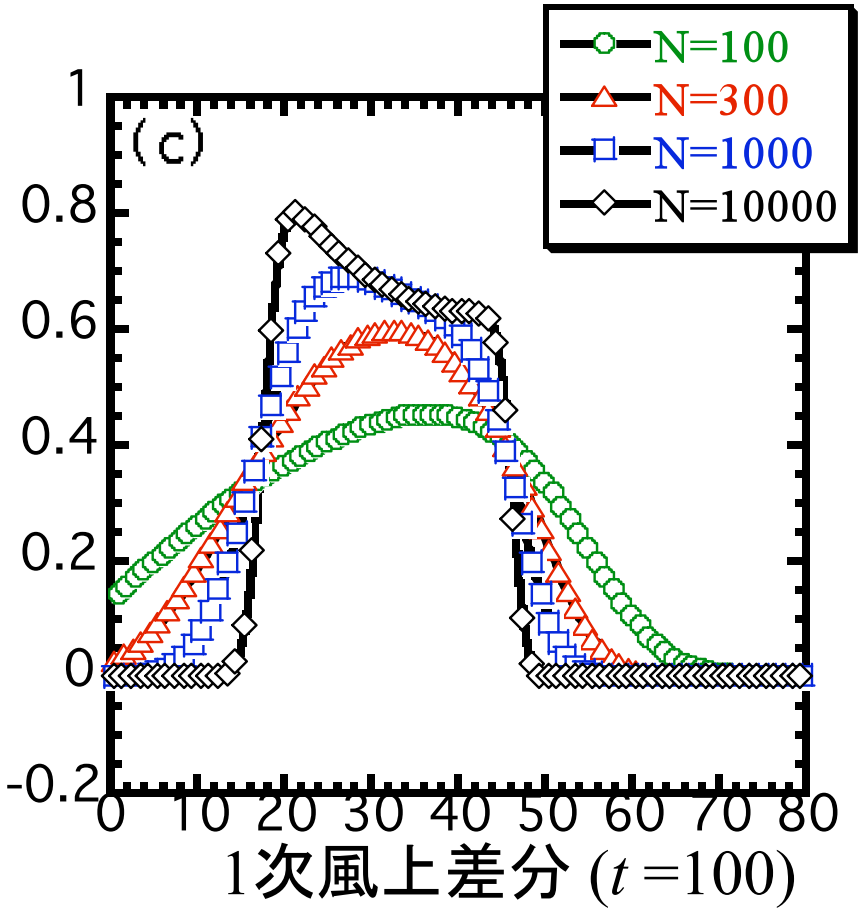
CIP法：勾配も物理モデル（移流方程式）に基づいて物理的に妥当に計算する。
数値振動の回避・低数値拡散の特徴

Conservative Eq. in Fluctuated Velocity

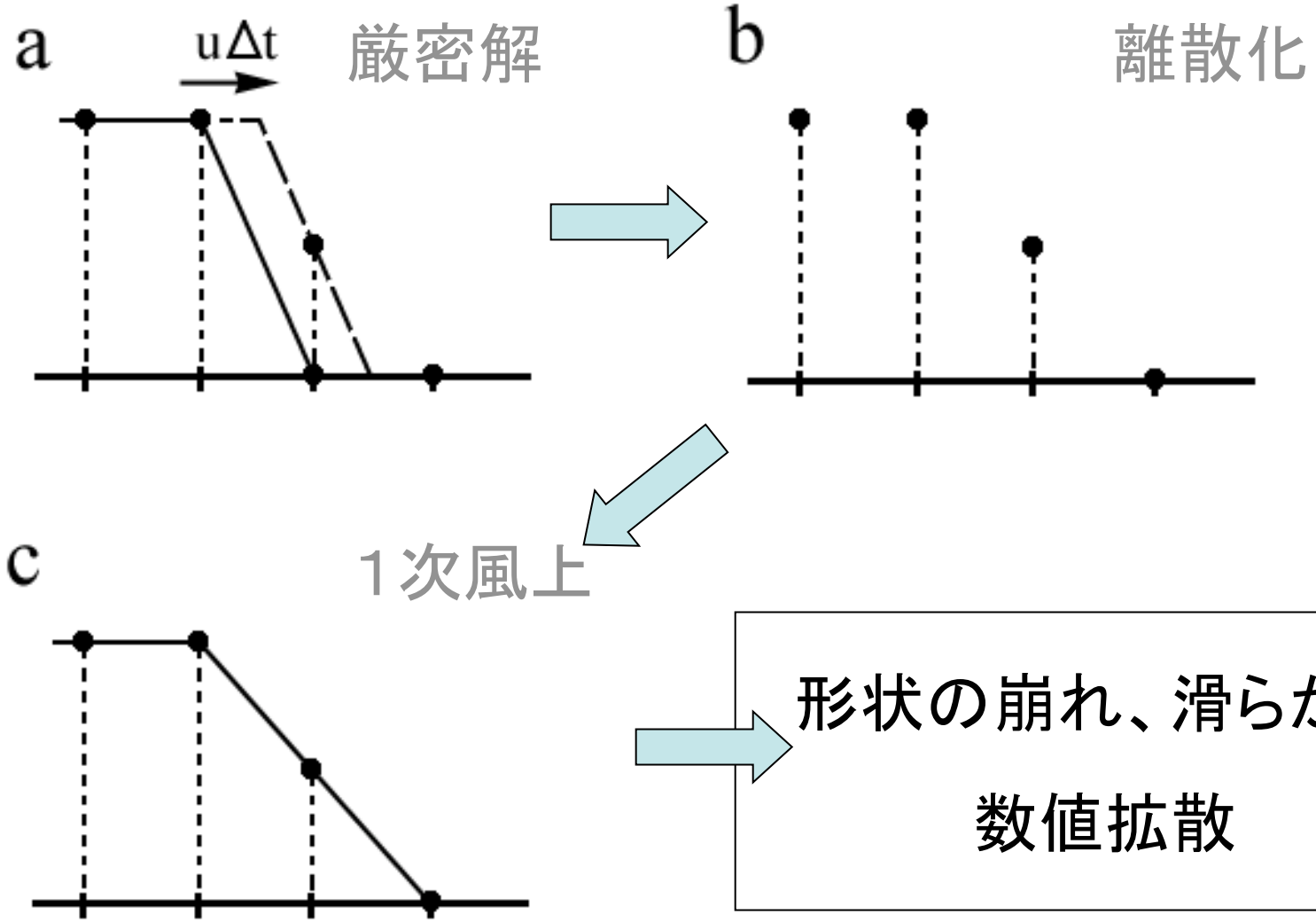
保存方程式 : $\dfrac{\partial f}{\partial t}+\dfrac{\partial (uf)}{\partial x}=0$ 計算条件 : $\begin{cases} u=1+0.5\sin(2\pi x/100) \\ \Delta x=100/(N-1) \quad N:\text{グリッド数} \\ CFL=0.1 \end{cases}$

幅20の矩形波を移流





Up-Wind 1st Order Scheme[風上差分]-4



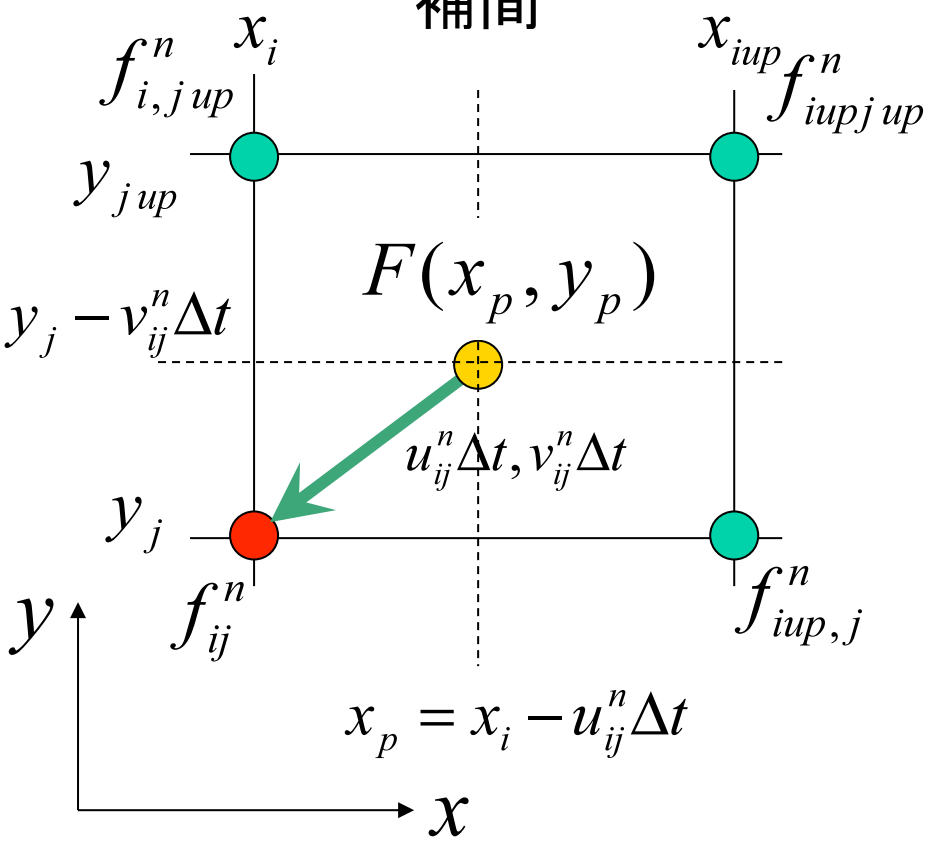
形状の崩れ、滑らかに

数値拡散

高次元移流方程式の解法

2次元移流方程式：$\frac{\partial f}{\partial t}+u\frac{\partial f}{\partial x}+v\frac{\partial f}{\partial x}=0$

補間



補間と特性線に基づく解法

格子点(x_i,y_j)における流速 $\left(u^n_{ij},v^n_{ij}\right)$

値 $f^n_{ij}=f(t_n,x_i,y_j)$ のΔt後の値は、上流点(x_p,y_p)での値を推定（補間）することで与えられる。

$$f^{n+1}_{ij}=F\left(x_p,y_p\right)$$

補間関数は移流原点(x_p,y_p)を含む4つの格子点から決定する。

$$(x_i,y_j),(x_{iup},y_{j\,up}),(x_i,y_{j\,up}),(x_{iup},y_{j\,up})$$

$$iup=i-sign(u^n_{ij}),\quad j\,up=j-sign(v^n_{ij})$$

高次元移流方程式の解法ー(斜め)風上１次補間

(斜め)風上１次補間

４格子点で囲まれる領域を１次関数で補間する。

$$F(x,y) = aX + bY + cXY + d, \quad X = x - x_i, Y = y - y_j$$

連続条件: $F(x_i, y_j) = f_{ij}^n, F(x_{iup}, y_j) = f_{iup,j}^n,$

$$F(x_i, y_{j\,up}) = f_{ij\,up}^n, F(x_{iup}, y_{j\,up}) = f_{iup,j\,up}^n$$

補間関数: $F(x,y) = (1-\alpha)(1-\beta) f_{ij}^n + \alpha(1-\beta) f_{iup,j}^n$

$$+ (1-\alpha)\beta f_{i,j\,up}^n + \alpha\beta f_{iup,j\,up}^n$$

$$\alpha = \frac{X}{\Delta x_{i \to iup}}, \quad \beta = \frac{Y}{\Delta y_{j \to j\,up}}$$

$$\Delta x_{i \to iup} = x_{iup} - x_i, \quad \Delta y_{j \to j\,up} = y_{j\,up} - y_j$$

高次元移流方程式の解法ー(斜め)風上1次補間

次時刻$t_{n+1}=t_n+\Delta t$ における値:

$$f_{ij}^{n+1} = F(x_p, y_p) = (1-C_x)(1-C_y) f_{ij}^n + C_x(1-C_y) f_{iup,j}^n + (1-C_x) C_y f_{i,j\,up}^n + C_x C_y f_{iup,j\,up}^n$$

$$C_x = \frac{-u_{ij}^n \Delta t}{\Delta x_{i\to iup}}, \quad C_y = \frac{-v_{ij}^n \Delta t}{\Delta y_{j\to j\,up}}$$

:x と y 方向へのCourant数

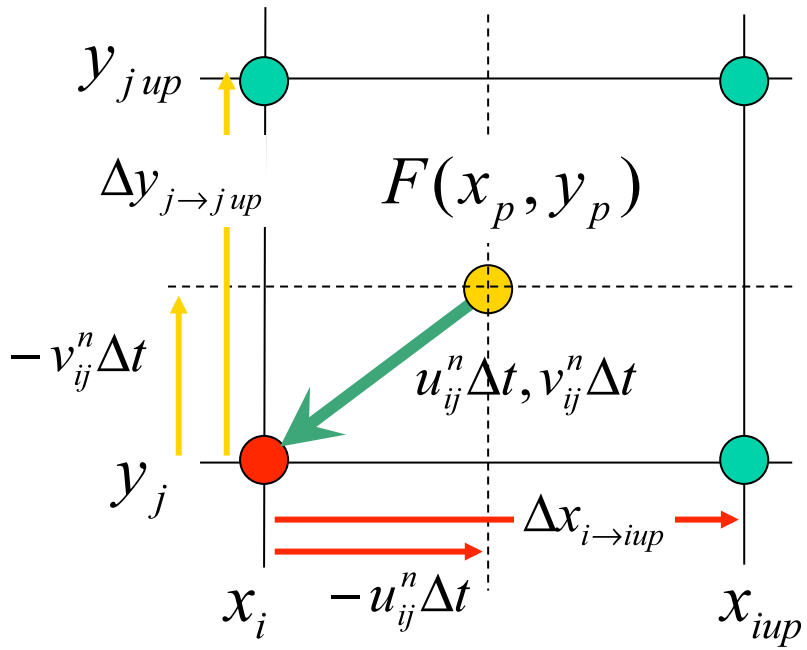
安定性のためには、格子の外側に飛び出てはいけない。

$$0 < C_x < 1$$

かつ

$$0 < C_y < 1$$

2次元のCFL条件

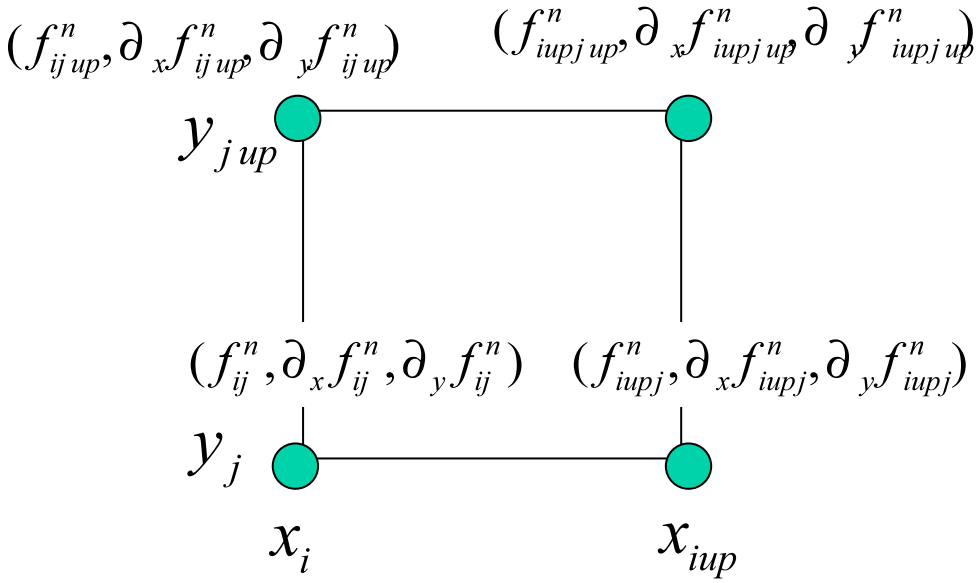


高次元移流方程式の解法ーCIP法

2次元:

x,y2方向の勾配も計算する。

$$\partial_x f_{ij}^n = \left(\frac{\partial f}{\partial x}\right)_{ij}^n, \quad \partial_y f_{ij}^n = \left(\frac{\partial f}{\partial y}\right)_{ij}^n$$



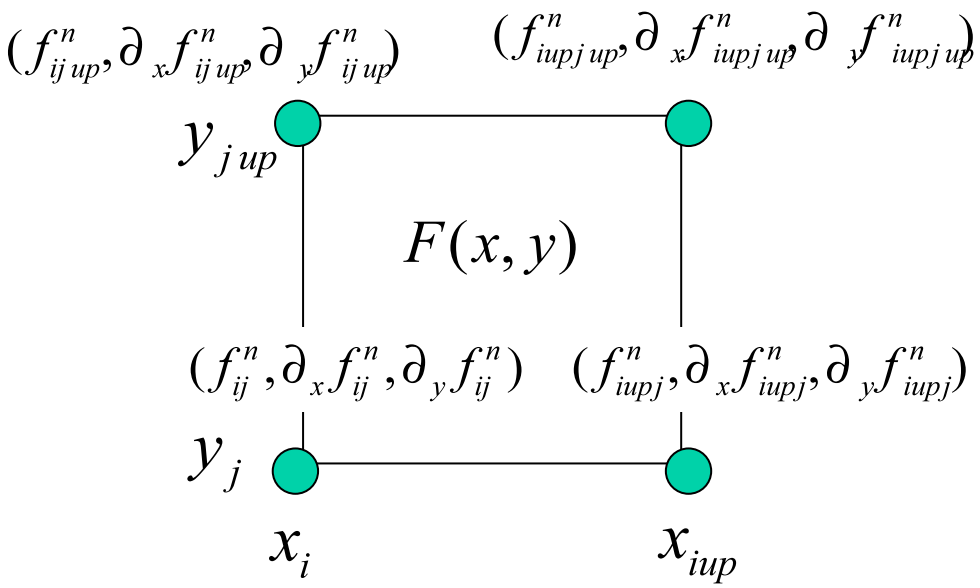
補間関数:***x,y***の3次多項式

$$F(x,y) = C_{30}X^3 + C_{20}X^2 + C_{10}X + C_{00} + C_{03}Y^3 + C_{02}Y^2 + C_{01}Y$$
$$+ C_{21}X^2Y + C_{11}XY + C_{12}XY^2 + C_{31}X^3Y + C_{13}XY^3$$
$$X = x - x_i, \quad Y = y - y_j$$

係数の決定には、4格子点上における連続条件を用いる。

高次元移流方程式の解法ーCIP法

連続条件（3変数×4格子点=12条件）:



値:

$$\begin{cases} F(x_i, y_j) = f^n_{ij} \\ F(x_{iup}, y_j) = f^n_{iupj} \\ F(x_i, y_{j\,up}) = f^n_{ij\,up} \\ F(x_{iup}, y_{j\,up}) = f^n_{iupj\,up} \end{cases}$$

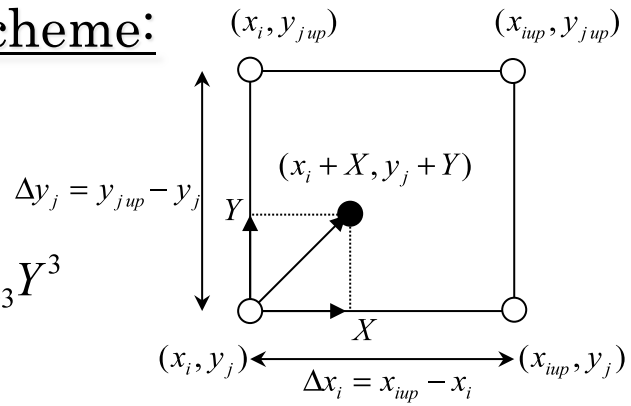
X勾配:

$$\begin{cases} \dfrac{\partial}{\partial x} F(x_i, y_j) = \partial_x f^n_{ij} \\ \dfrac{\partial}{\partial x} F(x_{iup}, y_j) = \partial_x f^n_{iupj} \\ \dfrac{\partial}{\partial x} F(x_i, y_{j\,up}) = \partial_x f^n_{ij\,up} \\ \dfrac{\partial}{\partial x} F(x_{iup}, y_{j\,up}) = \partial_x f^n_{iupj\,up} \end{cases}$$

Y勾配:

$$\begin{cases} \dfrac{\partial}{\partial y} F(x_i, y_j) = \partial_y f^n_{ij} \\ \dfrac{\partial}{\partial y} F(x_{iup}, y_j) = \partial_y f^n_{iupj} \\ \dfrac{\partial}{\partial y} F(x_i, y_{j\,up}) = \partial_y f^n_{ij\,up} \\ \dfrac{\partial}{\partial y} F(x_{iup}, y_{j\,up}) = \partial_y f^n_{iupj\,up} \end{cases}$$

Two Dimensional Interpolation Function in CIP scheme:



12元連立方程式を係数について解いて、

$$F_{ij}(X,Y) = C_{00} + C_{10}X + C_{01}Y + C_{20}X^2 + C_{02}Y^2 + C_{30}X^3 + C_{03}Y^3$$
$$+ C_{31}X^3Y + C_{21}X^2Y + C_{11}XY + C_{12}XY^2 + C_{13}XY^3$$

$$C_{00} = f_{ij} \qquad C_{10} = \partial_x f_{ij} \qquad C_{01} = \partial_y f_{ij}$$

$$C_{20} = 3(-f_{ij} + f_{iupj})/\Delta x_i^2 - (2\partial_x f_{ij} + \partial_x f_{iupj})/\Delta x_i$$

$$C_{02} = 3(-f_{ij} + f_{ij\,up})/\Delta y_j^2 - (2\partial_y f_{ij} + \partial_y f_{ij\,up})/\Delta y_j$$

$$C_{30} = 2(f_{ij} - f_{iupj})/\Delta x_i^3 + (\partial_x f_{ij} + \partial_x f_{iupj})/\Delta x_i^2$$

$$C_{03} = 2(f_{ij} - f_{ij\,up})/\Delta y_j^3 + (\partial_y f_{ij} + \partial_y f_{ij\,up})/\Delta y_j^2$$

$$C_{31} = -2A/\Delta x_i^3 \Delta y_j + (-\partial_x f_{ij} - \partial_x f_{iupj} + \partial_x f_{ij\,up} + \partial_x f_{iupj\,up})/\Delta x_i^2 \Delta y_j$$

$$C_{13} = -2A/\Delta x_i \Delta y_j^3 + (-\partial_y f_{ij} - \partial_y f_{ij\,up} + \partial_y f_{iupj} + \partial_y f_{iupj\,up})/\Delta x_i \Delta y_j^2$$

$$C_{21} = 3A/\Delta x_i^2 \Delta y_j + (2\partial_x f_{ij} + \partial_x f_{iupj} - 2\partial_x f_{ij\,up} - \partial_x f_{iupj\,up})/\Delta x_i \Delta y_j$$

$$C_{12} = 3A/\Delta x_i \Delta y_j^2 + (2\partial_y f_{ij} + \partial_y f_{ij\,up} - 2\partial_y f_{iupj} - \partial_y f_{iupj\,up})/\Delta x_i \Delta y_j$$

$$C_{11} = -A/\Delta x_i \Delta y_j + (-\partial_x f_{ij} + \partial_x f_{ij\,up})/\Delta y_j + (-\partial_y f_{ij} + \partial_y f_{iupj})/\Delta x_i$$

$$A = f_{ij} - f_{iupj} - f_{ij\,up} + f_{iupj\,up}$$

(CIP,Type-B)

Three Dimensional Interpolation Function in CIP scheme:

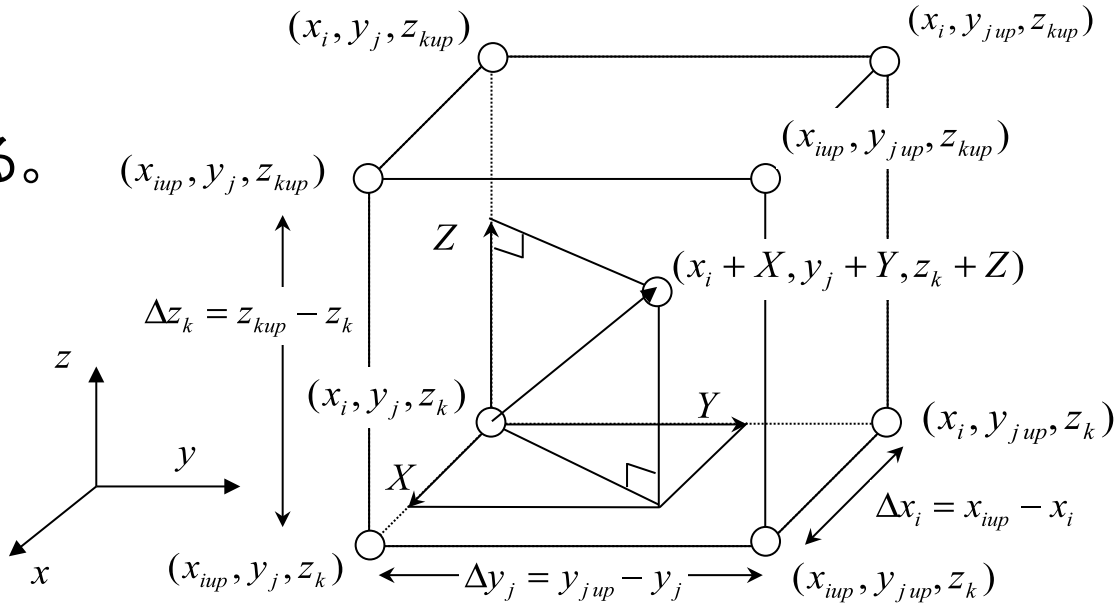
3次元：

x,y,z3方向の勾配も計算する。

$$\partial_x f_{ij}^n = \left(\frac{\partial f}{\partial x}\right)_{ij}^n,$$

$$\partial_y f_{ij}^n = \left(\frac{\partial f}{\partial y}\right)_{ij}^n,$$

$$\partial_z f_{ij}^n = \left(\frac{\partial f}{\partial z}\right)_{ij}^n$$



補間関数

$$F_{ijk}(X,Y,Z) = C_{000} + C_{100}X + C_{010}Y + C_{001}Z + C_{200}X^2 + C_{020}Y^2 + C_{002}Z^2 + C_{300}X^3 + C_{030}Y^3 + C_{003}Z^3$$
$$+ C_{310}X^3Y + C_{301}X^3Z + C_{130}XY^3 + C_{031}Y^3Z + C_{103}XZ^3 + C_{013}YZ^3$$
$$+ C_{210}X^2Y + C_{201}X^2Z + C_{120}XY^2 + C_{021}Y^2Z + C_{102}XZ^2 + C_{012}YZ^2$$
$$+ C_{110}XY + C_{011}YZ + C_{101}XZ + C_{113}XYZ^3$$
$$+ C_{311}X^3YZ + C_{131}XY^3Z + C_{112}XYZ^2 + C_{211}X^2YZ + C_{121}XY^2Z + C_{111}XYZ$$

$$A_{xy} = f_{iupj\,upk} - f_{iupj\,k} - f_{ij\,upk} + f_{ij\,k}$$
$$A_{yz} = f_{ij\,upkup} - f_{ij\,upk} - f_{ij\,kup} + f_{ij\,k}$$
$$A_{xz} = f_{iupj\,kup} - f_{iupj\,k} - f_{ij\,kup} + f_{ij\,k}$$
$$P = f_{iupj\,upkup} - f_{iupj\,upk} - f_{ij\,upkup} - f_{iupj\,kup} + f_{iupj\,k} + f_{ij\,upk} + f_{ij\,kup} - f_{ij\,k}$$

$$C_{000} = f_{ij\,k}$$
$$C_{100} = \partial_x f_{ij\,k}$$
$$C_{010} = \partial_y f_{ij\,k}$$
$$C_{001} = \partial_z f_{ij\,k}$$
$$C_{200} = 3(-f_{ij\,k} + f_{iupj\,k}) / \Delta x_i^2 - (2\partial_x f_{ij\,k} + \partial_x f_{iupj\,k}) / \Delta x_i$$
$$C_{020} = 3(-f_{ij\,k} + f_{ij\,upk}) / \Delta y_j^2 - (2\partial_y f_{ij\,k} + \partial_y f_{ij\,upk}) / \Delta y_j$$
$$C_{002} = 3(-f_{ij\,k} + f_{ij\,kup}) / \Delta z_k^2 - (2\partial_z f_{ij\,k} + \partial_z f_{ij\,kup}) / \Delta z_k$$
$$C_{300} = 2(f_{ij\,k} - f_{iupj\,k}) / \Delta x_i^3 + (\partial_x f_{ij\,k} + \partial_x f_{iupj\,k}) / \Delta x_i^2$$
$$C_{030} = 2(f_{ij\,k} - f_{ij\,upk}) / \Delta y_j^3 + (\partial_y f_{ij\,k} + \partial_y f_{ij\,upk}) / \Delta y_j^2$$
$$C_{003} = 2(f_{ij\,k} - f_{ij\,kup}) / \Delta z_k^3 + (\partial_z f_{ij\,k} + \partial_z f_{ij\,kup}) / \Delta z_k^2$$
$$C_{310} = (\partial_x f_{iupj\,upk} + \partial_x f_{ij\,upk} - \partial_x f_{iupj\,k} - \partial_x f_{ij\,k}) / \Delta x_i^2 \Delta y_j - 2A_{xy} / \Delta x_i^3 \Delta y_j$$
$$C_{130} = (\partial_y f_{iupj\,upk} + \partial_y f_{iupj\,k} - \partial_y f_{ij\,upk} - \partial_y f_{ij\,k}) / \Delta x_i \Delta y_j^2 - 2A_{xy} / \Delta x_i \Delta y_j^3$$
$$C_{013} = (\partial_z f_{ij\,upkup} + \partial_z f_{ij\,upk} - \partial_z f_{ij\,kup} - \partial_z f_{ij\,k}) / \Delta y_j \Delta z_k^2 - 2A_{yz} / \Delta y_j \Delta z_k^3$$
$$C_{031} = (\partial_y f_{ij\,upkup} + \partial_y f_{ij\,kup} - \partial_y f_{ij\,upk} - \partial_y f_{ij\,k}) / \Delta y_j^2 \Delta z_k - 2A_{yz} / \Delta y_j^3 \Delta z_k$$
$$C_{103} = (\partial_z f_{iupj\,kup} + \partial_z f_{iupj\,k} - \partial_z f_{ij\,kup} - \partial_z f_{ij\,k}) / \Delta x_i \Delta z_k^2 - 2A_{xz} / \Delta x_i \Delta z_k^3$$
$$C_{301} = (\partial_x f_{iupj\,kup} + \partial_x f_{ij\,kup} - \partial_x f_{iupj\,k} - \partial_x f_{ij\,k}) / \Delta x_i^2 \Delta z_k - 2A_{xz} / \Delta x_i^3 \Delta z_k$$

$$C_{210} = (-\partial_x f_{iupj\,upk} - 2\partial_x f_{ij\,upk} + \partial_x f_{iupj\,k} + 2\partial_x f_{ij\,k}) / \Delta x_i \Delta y_j + 3A_{xy} / \Delta x_i^2 \Delta y_j$$

$$C_{120} = (-\partial_y f_{iupj\,upk} - 2\partial_y f_{iupj\,k} + \partial_y f_{ij\,upk} + 2\partial_y f_{ij\,k}) / \Delta x_i \Delta y_j + 3A_{xy} / \Delta x_i \Delta y_j^2$$

$$C_{012} = (-\partial_z f_{ij\,upkup} - 2\partial_z f_{ij\,upk} + \partial_z f_{ij\,kup} + 2\partial_z f_{ij\,k}) / \Delta y_j \Delta z_k + 3A_{yz} / \Delta y_j \Delta z_k^2$$

$$C_{021} = (-\partial_y f_{ij\,upkup} - 2\partial_y f_{ij\,kup} + \partial_y f_{ij\,upk} + 2\partial_y f_{ij\,k}) / \Delta y_j \Delta z_k + 3A_{yz} / \Delta y_j^2 \Delta z_k$$

$$C_{102} = (-\partial_z f_{iupj\,kup} - 2\partial_z f_{iupj\,k} + \partial_z f_{ij\,kup} + 2\partial_z f_{ij\,k}) / \Delta x_i \Delta z_k + 3A_{xz} / \Delta x_i \Delta z_k^2$$

$$C_{201} = (-\partial_x f_{iupj\,kup} - 2\partial_x f_{ij\,kup} + \partial_x f_{iupj\,k} + 2\partial_x f_{ij\,k}) / \Delta x_i \Delta z_k + 3A_{xz} / \Delta x_i^2 \Delta z_k$$

$$C_{110} = -A_{xy} / \Delta x_i \Delta y_j + (-\partial_x f_{ij\,k} + \partial_x f_{ij\,upk}) / \Delta y_j + (-\partial_y f_{ij\,k} + \partial_y f_{iupj\,k}) / \Delta x_i$$

$$C_{011} = -A_{yz} / \Delta z_k \Delta y_j + (-\partial_z f_{ij\,k} + \partial_z f_{ij\,upk}) / \Delta y_j + (-\partial_y f_{ij\,k} + \partial_y f_{ij\,kup}) / \Delta z_k$$

$$C_{101} = -A_{xz} / \Delta x_i \Delta z_k + (-\partial_z f_{ij\,k} + \partial_z f_{iupj\,k}) / \Delta x_i + (-\partial_x f_{ij\,k} + \partial_x f_{ij\,kup}) / \Delta z_k$$

$$C_{311} = (\partial_x f_{iupj\,upkup} - \partial_x f_{iupj\,upk} + \partial_x f_{ij\,upkup} - \partial_x f_{iupj\,kup} + \partial_x f_{iupj\,k} - \partial_x f_{ij\,upk} - \partial_x f_{ij\,kup} + \partial_x f_{ij\,k}) / \Delta x_i^2 \Delta y_j \Delta z_k$$
$$- 2P / \Delta x_i^3 \Delta y_j \Delta z_k$$

$$C_{311} = (\partial_x f_{iupj\,upkup} - \partial_x f_{iupj\,upk} + \partial_x f_{ij\,upkup} - \partial_x f_{iupj\,kup} + \partial_x f_{iupj\,k} - \partial_x f_{ij\,upk} - \partial_x f_{ij\,kup} + \partial_x f_{ij\,k}) / \Delta x_i^2 \Delta y_j \Delta z_k$$
$$- 2P / \Delta x_i^3 \Delta y_j \Delta z_k$$

$$C_{131} = (\partial_y f_{iupj\,upkup} - \partial_y f_{iupj\,upk} + \partial_y f_{iupj\,kup} - \partial_y f_{ij\,upkup} + \partial_y f_{ij\,upk} - \partial_y f_{iupj\,k} - \partial_y f_{ij\,kup} + \partial_y f_{ij\,k}) / \Delta x_i \Delta y_j^2 \Delta z_k$$
$$- 2P / \Delta x_i \Delta y_j^3 \Delta z_k$$

$$C_{113} = (\partial_z f_{iupj\,upkup} - \partial_z f_{ij\,upkup} + \partial_z f_{iupj\,upk} - \partial_z f_{iupj\,kup} + \partial_z f_{ij\,kup} - \partial_z f_{ij\,upk} - \partial_z f_{iupj\,k} + \partial_z f_{ij\,k}) / \Delta x_i \Delta y_j \Delta z_k^2$$
$$- 2P / \Delta x_i \Delta y_j \Delta z_k^3$$

$$C_{211} = -(\partial_x f_{iupj\,upkup} - \partial_x f_{iupj\,upk} + 2\partial_x f_{ij\,upkup} - \partial_x f_{iupj\,kup} + \partial_x f_{iupj\,k} - 2\partial_x f_{ij\,upk} - 2\partial_x f_{ij\,kup} + 2\partial_x f_{ij\,k}) / \Delta x_i \Delta y_j \Delta z_k$$
$$+ 3P / \Delta x_i^2 \Delta y_j \Delta z_k$$

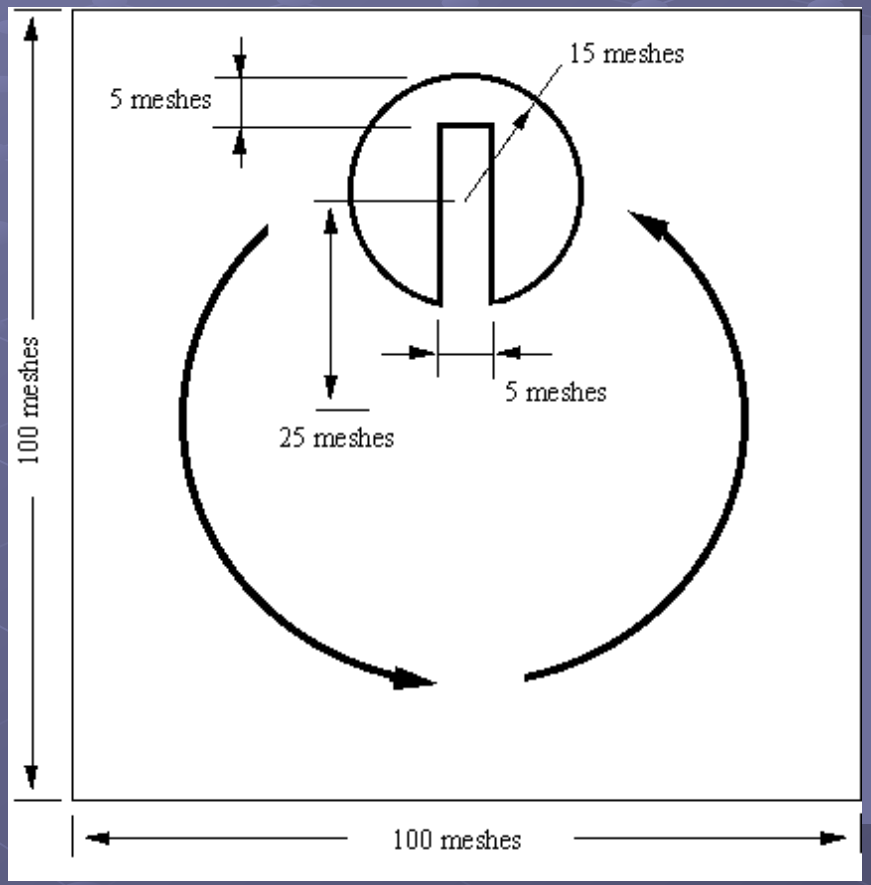
$$C_{121} = -(\partial_y f_{iupj\,upkup} - \partial_y f_{iupj\,upk} + 2\partial_y f_{iupj\,kup} - \partial_y f_{ij\,upkup} + \partial_y f_{ij\,upk} - 2\partial_y f_{iupj\,k} - 2\partial_y f_{ij\,kup} + 2\partial_y f_{ij\,k}) / \Delta x_i \Delta y_j \Delta z_k$$
$$+ 3P / \Delta x_i \Delta y_j^2 \Delta z_k$$

$$C_{112} = -(\partial_z f_{iupj\,upkup} - \partial_z f_{ij\,upkup} + 2\partial_z f_{iupj\,upk} - \partial_z f_{iupj\,kup} + \partial_z f_{ij\,kup} - 2\partial_z f_{ij\,upk} - 2\partial_z f_{iupj\,k} + 2\partial_z f_{ij\,k}) / \Delta x_i \Delta y_j \Delta z_k$$
$$+ 3P / \Delta x_i \Delta y_j \Delta z_k^2$$

$$C_{111} = (\partial_x f_{ij\,upkup} - \partial_x f_{ij\,upk} - \partial_x f_{ij\,kup} + \partial_x f_{ij\,k}) / \Delta y_j \Delta z_k + (\partial_y f_{iupj\,kup} - \partial_y f_{iupj\,k} - \partial_y f_{ij\,kup} + \partial_y f_{ij\,k}) / \Delta x_i \Delta z_k$$
$$+ (\partial_z f_{iupj\,upk} - \partial_z f_{iupj\,k} - \partial_z f_{ij\,upk} + \partial_z f_{ij\,k}) / \Delta x_i \Delta y_j - 2P / \Delta x_i \Delta y_j \Delta z_k$$

高次元CIP法の計算例

Zalesakの2次元剛体回転問題



$$f = \begin{cases} 1. & f\ or \quad inside\ of\ \ cylinda \\ 0 & otherwise \end{cases}$$

$$u = -\omega y, v = \omega x$$

$$\omega = 0.02, \Delta x = 1$$

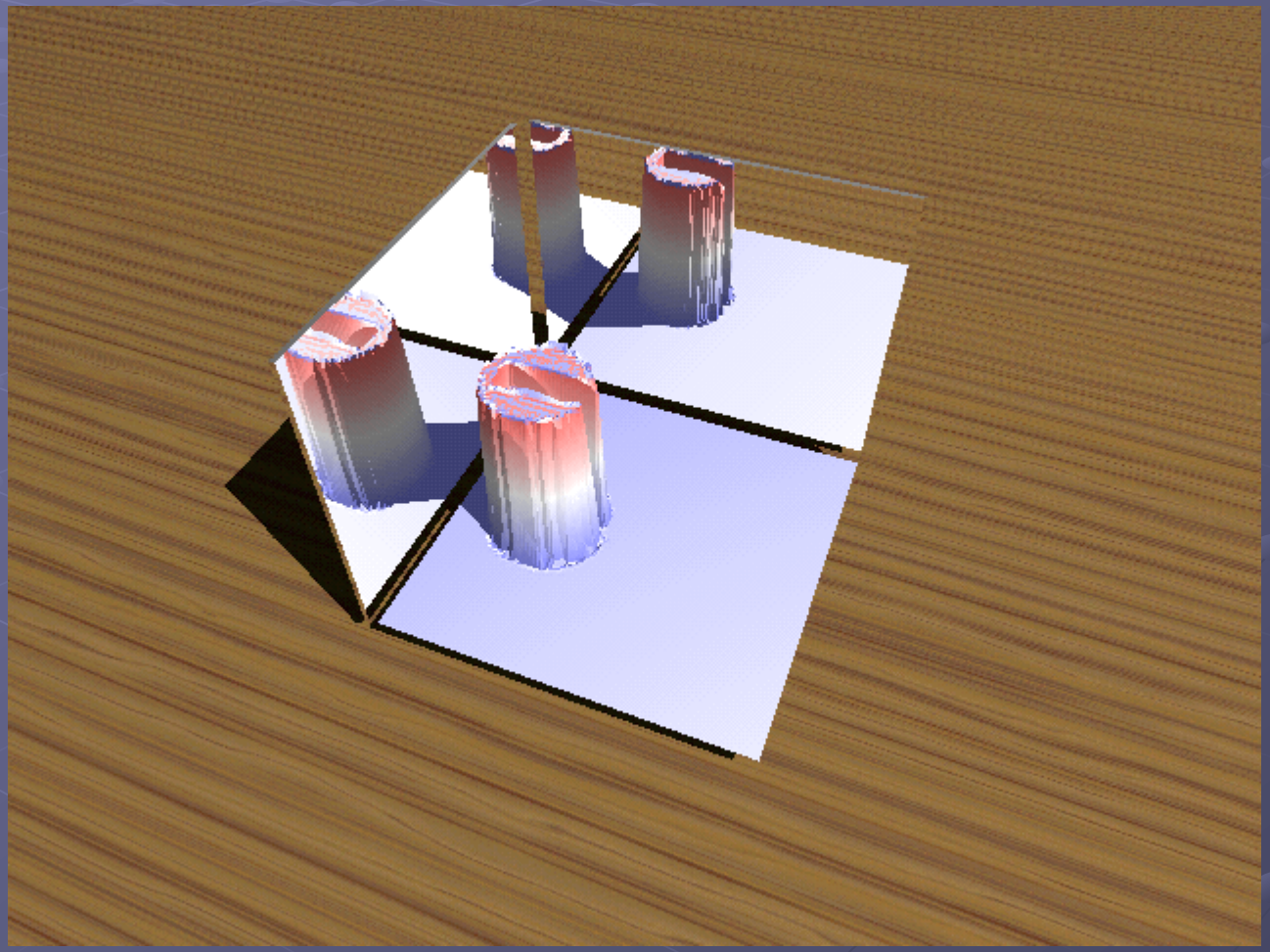
CFL=0.5

一定角速度で一周

Zalesakの2次元剛体回転問題(CIP-CSL2)

計算条件: $u = -2\pi y,\ v = 2\pi x$

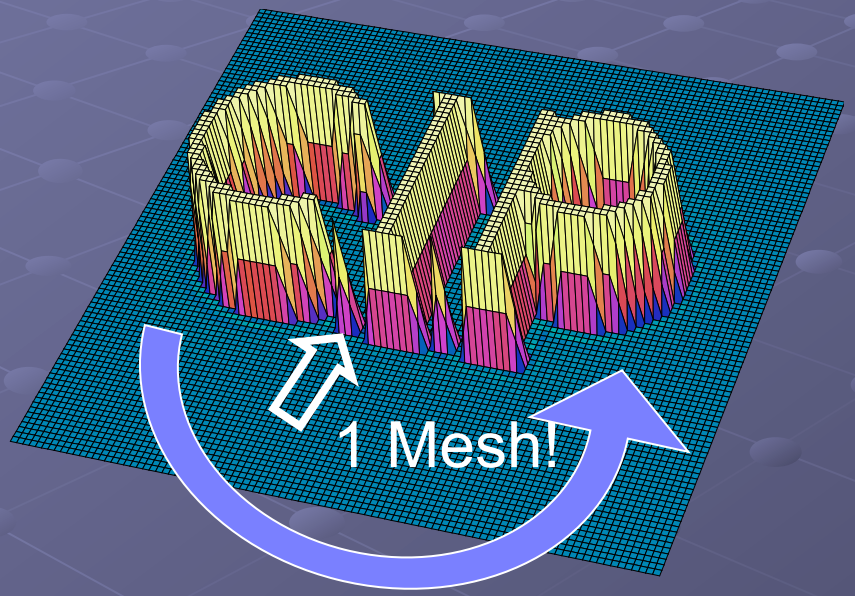
$\Delta x = 1,\ CFL = 0.2$



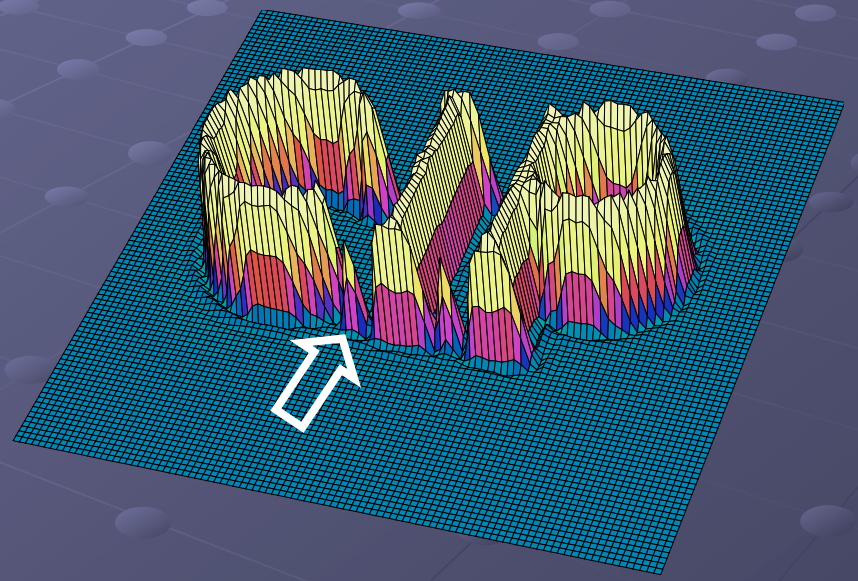
2次元剛体回転 (CIP-CSL4)

初期条件: •幅2メッシュ以下の文字
•1メッシュのドット

計算条件: $u = -2\pi y, v = 2\pi x$
$\Delta x = 1, CFL = 0.2$

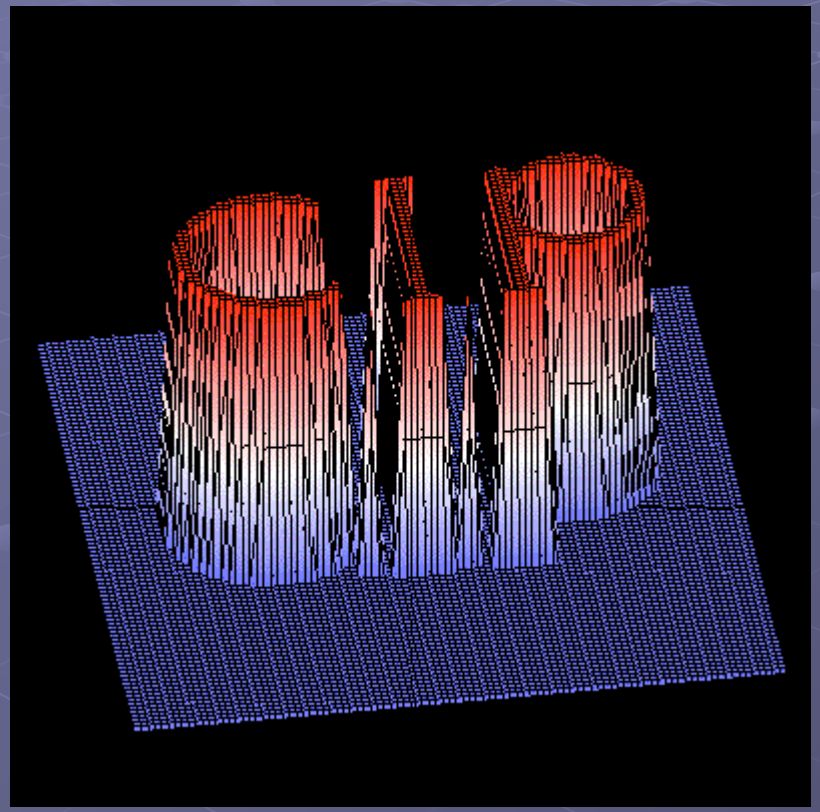


初期条件

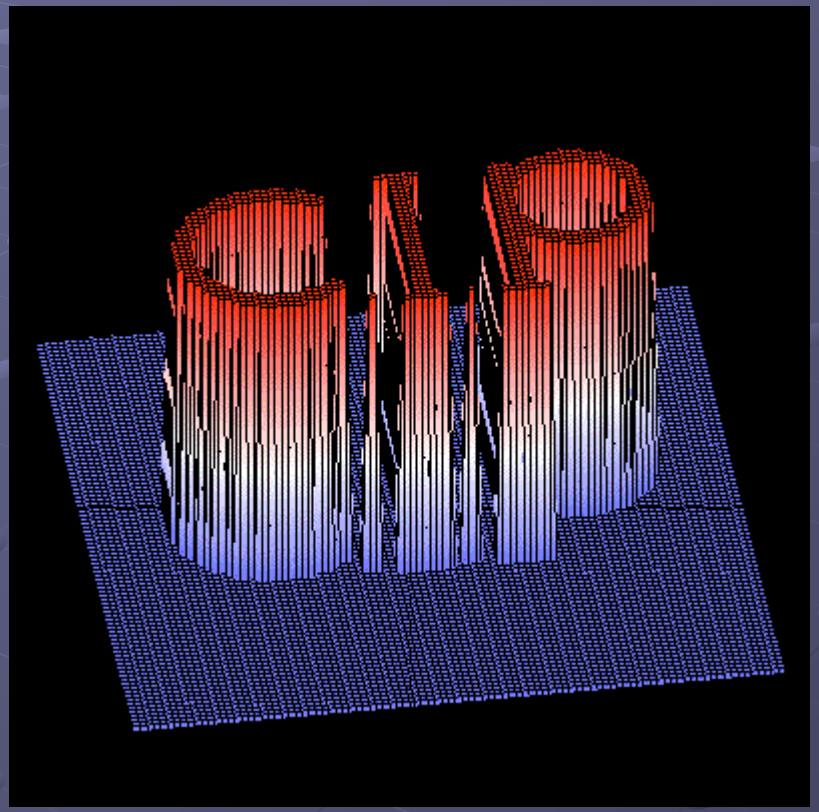


1回転後

2次元剛体回転 (CIP-CSL4)



CIP-CSL4



First Order Upwind