## 5. SEISMIC RESPONSE OF BRIDGES

### 5.1 Seismic Response Characteristics of Standard Bridges

Fig. 5.1 shows a response acceleration on the top of a column of Itajima Bridge during the M7.5 Hyuga-nada earthquake in April, 1968. The epicentral distance was 100 km . A ground acceleration at a free-field 400 m apart from the bridge is also presented. Both longitudinal and transverse responses were measured. As shown in Fig. 5.2, the Itajima Bridge is a 125 m long 5 span simply supported plate girder bridge supported by 4 caissons at columns and 2 pile foundations at abutments. The column and the caisson where the column responses were recorded are 5 m tall and 15.5 m long, respectively. The peak accelerations on the ground are 170 gal and 186 gal in the longitudinal and transverse directions, respectively, and the peak response accelerations on the column are 219 gal and 310 gal in the longitudinal and transverse directions, respectively. The column response accelerations are larger than the ground accelerations in both directions in this earthquake.

(3) Vertical Direction
(a) Top of the Column

(1) Longitudinal Direction

(2) Transverse Direction
(b) Ground Surface

Fig. 5.1 Response accelerations at Itajima Bridge during Hyuga-nada Earthquake in April 1968


Fig. 5.2 Itajima Bridge
On the other hand, the Itajima Bridge was subjected to an M6.6 event at 11 km from the bridge in August 1968, and response accelerations as shown in Fig. 5.3 were obtained. The records were obtained at the same locations on the top of the columns and the free-field ground surface. The peak accelerations on the ground are 441 gal and 353 gal in the longitudinal and transverse directions, respectively, and the peak response accelerations on the column are 199 gal and 231 gal in the longitudinal and transverse directions, respectively. On the contrary to the previous earthquake, the column response accelerations are smaller than the ground accelerations in both directions in this earthquake. Consequently, amplifications of column responses depend on ground motion characteristics.


Fig. 5.3 Response Accelerations at Itajima Bridge during an M6.6 Event in August 1968
Another example of recorded responses is presented in Fig. 5.4 for Kaihoku Bridge during an M6.7 event in February 1978. The Kaihoku Bridge is a 285 m long 5 span continuous steel box girder bridge as shown in Fig. 5.5. The column responses were measured on the top of column 2 (P2), and the ground accelerations were measured on the surface of free-field ground 60 m from P2. The peak accelerations on the ground are 76 gal and 141 gal in the longitudinal
and transverse directions, respectively, and the peak response accelerations on the column are 326 gal and 197 gal in the longitudinal and transverse directions, respectively. Subsequent to this earthquake, the Kaihoku Bridge was subjected to the Miyagi-ken oki earthquake with a magnitude 7.4 at 90 km in June 1978, and response accelerations as shown in Fig. 5.6 were obtained. The peak accelerations on the ground are 192 gal and 288 gal in the longitudinal and transverse directions, respectively, and the peak response accelerations on the column are over 500 gal and 332 gal in the longitudinal and transverse directions, respectively. Since the capacity of accelerograph was 500 gal , the response acceleration on the column in the longitudinal direction was not recorded due to an overscale. Again, the amplification of the column response depends on the ground motion characteristics.

(a) Top of the Column

(b) Ground Surface

Fig. 5.4 Response Accelerations at Kaihoku Bridge during an M6.7 Event in February 1978


Fig. 5.5 Kaihoku Bridge


Fig. 5.6 Response Accelerations at Kaihoku Bridge during Miyagi-ken-oki Earthquake in June 1978

At Kaihoku Bridge, response accelerations have been recorded by nine earthquakes including the above two earthquakes as shown in Table 5.1. Fig. 5.7 shows the peak ground accelerations $a_{g}$ vs. peak column response accelerations $a_{p}$ relations. Scattering of the data is not considerable, and they may be fitted as

$$
a_{p}=\left\{\begin{array}{cc}
2.41 \times a_{g}^{1.10} & \text { longitudinal direction }  \tag{5.1}\\
2.03 \times a_{g}^{0.91} & \text { transverse direction }
\end{array}\right.
$$

Table 5.1 Peak Response Accelerations (cm/s2) at Kaihoku Bridge

| No. | Date | Magnitude | Epicentral <br> Distance $(\mathrm{km})$ | Ground |  | LG | TR |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |



MAX ACCELERATION ON GBOUND SURFACE [GAL]
Fig. 5.7 Peak Column Accelerations vs. Peak Ground Accelerations at Kaihoku Bridge
It is known that the amplifications of column response accelerations depend on the type of foundations. Fig. 5.8 shows $a_{p}$ vs. $a_{g}$ relations for 135 records measured at 12 bridges. The type of foundations are classified into caissons, piles, and spread foundations. The amplifications may be fitted as

$$
\begin{align*}
& a_{p}=\left\{\begin{array}{rr}
1.097 \times a_{g}{ }_{g}^{0.960} & \text { caisssons } \\
1.50 \times a_{g}{ }^{0.933} & p i l e s \\
7.95 \times a_{g}{ }^{0.748} & \text { spread foundations }
\end{array}\right.  \tag{5.2}\\
& a_{p}=\left\{\begin{array}{rr}
2.53 \times a_{g}{ }_{g}^{0.801} & \text { (Longitudinal direction) } \\
11.50 \times a_{g}{ }^{0.860} & \text { caisssons } \\
75.56 \times a_{g}{ }^{0.773} & \text { spread foundations }
\end{array}\right. \tag{5.3}
\end{align*} \text { (Transverse direction) }
$$

The amplification is remarkably large at spread foundations than other types foundations.


Fig. 5.8 Dependence of Amplifications of Column Response Accelerations on Types of Foundations

### 5.2 Seismic Response Analysis of Kaihoku Bridge

An analytical simulation for the records at the Kaihoku Bridge (refer to Figs. 5.4, 5.5 and 5.6) was conducted (Iwasaki, Kawashima, Takagi and Aizawa 1982). As described above, the bridge consists of a 5 span continuous steel box girder. The deck is supported by fixed bearing at P1 with other supports being supported by movable bearings. Viscous damper stoppers (shock transmission units) are provided at P 2 so that P 2 , in addition to P 1 , resists seismic lateral force of the deck in the longitudinal direction during an earthquake. Seismic lateral force of the deck in the transverse direction is supported by all substructures.

Since ground accelerations and response accelerations on the top of the column are recorded, the frequency response functions between the ground and the top of the column $H_{G P}(f)$ and the coherency function $C_{P G}(f)$ are evaluated as

$$
\begin{gather*}
H_{G P}(f)=\frac{\bar{S}_{G P}}{\bar{S}_{G}}  \tag{5.4}\\
C_{G P}(f)=\sqrt{\frac{\left|\bar{S}_{G P}\right|}{\bar{S}_{G} \cdot \bar{S}_{P}}} \tag{5.5}
\end{gather*}
$$

where, $\bar{S}_{G}, \bar{S}_{P}$ and $\bar{S}_{G P}$ an assemble average of the power spectra of ground, accelerations $S_{G i}$, power spectra of response accelerations on the top of the column $S_{P i}$ and cross spectra between the ground accelerations and the response accelerations $S_{G P i}$ of each record as

$$
\begin{equation*}
\bar{S}_{G}=\sum_{i} S_{G i} ; \quad \bar{S}_{P}=\sum_{i} S_{P i} ; \quad \bar{S}_{G P}=\sum_{i} S_{G P i} \tag{5.6}
\end{equation*}
$$

Fig. 5.9 shows the frequency response function $H_{G P}(f)$ and coherency function
$C_{G P}(f)$ averaged over the 7 records (refer to Table 5.1). The averaged frequency response function in the longitudinal and transverse directions may be approximated as

$$
\begin{align*}
& H_{G P}(f)= \begin{cases}1 & 0<\mathrm{f}<6 \mathrm{~Hz} \\
3 & 6<\mathrm{f}<12 \mathrm{~Hz} \\
2 & 12<\mathrm{f}<15 \mathrm{~Hz}\end{cases}  \tag{5.7}\\
& H_{G P}(f)= \begin{cases}1 & 0<\mathrm{f}<11 \mathrm{~Hz} \\
0.5 & 11<\mathrm{f}<15 \mathrm{~Hz}\end{cases} \tag{5.8}
\end{align*} \text { (tongitudinal direction) } \text { (transverse direction) } \$ \text { ( }
$$



Fig. 5.9 Averaged Frequency Response Functions and Coherency Functions between the Ground and the Top of the Column

Eqs. (5.7) and (5.8) shows that the amplification of the column P2 is limited under 6 Hz and 11 Hz in the longitudinal and transverse directions, respectively, and the column responded in a similar way with the ground under those frequencies.

The bridge is idealized by a two dimensional discrete linear analytical model as shown in Fig. 5.10. The ground on the base-rock was idealized by 6 soil columns with the Ramberg-Osgood hystereses. After computing the soil responses by prescribing the measured ground acceleration at the base-rocks of 6 soil columns, the computed soil responses are applied to bridge through soil springs (Penzien, Scheffey and Parmelee 1964). The stiffness of soil springs are evaluated according to the Design Specifications of Highway Bridges. Shear moduli of soil which are compatible to shear stains induced during the earthquakes (Schnabel, Lysmer and Seed 1972) are used to evaluate the stiffness of soil springs. Damping ratios are evaluated by Eq. (2.6) assuming 0.02 for the deck and the columns, and 0.1 for the foundations. Fig. 5.11 shows major mode shapes in the transverse direction. Deformation at P3 and P4 is predominant in the first and the second modes, respectively. Deformation at P2 is predominant in the 12 nd mode with the natural frequency of 7.8 Hz .


Fig. 5.10 Analytical Model of Kaihoku Bridge


Fig. 5.11 Major Mode Shapes in the Transverse Direction
Fig. 5.12 and 5.13 show correlations of the measured accelerations on the top of the column (P2) for the February, 1978 earthquake and the June, 1978 earthquake, respectively. The measured response accelerations agree well with the measured responses.


Fig. 5.12 Correlation of the Response acceleration on the Top of the Column (February, 1978)


Fig. 5.13 Correlation of the Response acceleration on the Top of the Column (June, 1978)

### 5.3 Effect of Multiple Excitation

Because bridges are lengthy in the longitudinal direction, ground motions at each support are not identical. Because of its unique characteristics, various studies have been conducted for the effect of multiple excitation, or non-synchronous ground motions (Werner et al 1979, Somaini 1987, Monti et al 1994). As an example of such analyses, two analyses are introduced here. The first is a linear dynamic response analysis of a 6 -span continuous bridge subjected to rigid and multiple excitations in the longitudinal direction (Monti, Nuti, Pinto and Vanzi 1994).

A deck consisting of six 50 m long spans is supported by 5 piers and 2 abutments as shown in Fig. 5.14. The deck is transversely hinged to the piers and the abutments. Pier heights were varied as $7.5 \mathrm{~m}, 10 \mathrm{~m}$ and 15 m in the analysis to cover a wide range of bridge profiles. The bridges were designed elastically for a PGA of 0.42 g . Three soil conditions, i.e., firm, medium and soft, are considered.


Fig. 5.14 Bridge Analyzed for Wave Propagation Effect
In the analysis, the decks, piers and abutments are idealized by a linear beam model. A cracked stiffness, which is obtained from the uncracked stiffness divided by a factor of 2.5 , is assigned to the piers.

Special attention is paid to the idealization of the spatial (non-synchronous) ground motions. The cross power spectral density $S_{i j}(x, \omega)$, which represents the correlation between point i and j separated by a distance x , is defined as

$$
\begin{equation*}
\gamma(x, \omega)=S_{i j}(x, \omega) / S(\omega) \tag{5.9}
\end{equation*}
$$

in which $\gamma(x, \omega)$ is called a coherency function expressed as

$$
\begin{equation*}
\gamma(x, \omega)=\exp \left\{-\left(\frac{\alpha \omega x}{V_{S}}\right)^{2}\right\} \exp \left\{i \frac{\omega x^{L}}{V_{\text {app }}}\right\} \tag{5.10}
\end{equation*}
$$

where, $V_{S}$ : shear wave velocity; $\omega$ : angular frequency; $V_{\text {app }}$ : surface apparent velocity; and $x^{L}$ : horizontal distance projected along the direction of wave propagation. The first term on the right hand side of Eq. (5.10) represents the geometric incoherence in terms of the distance and the frequency, and the second term represents the delay in arriving times of the waves due to the apparent velocity of $V_{a p p}$. It is noted that when $V_{S} / \alpha \rightarrow \infty$, the first term approaches to 1 and the incoherence is derived only from the wave traveling effect. On the other hand, when $V_{\text {app }} \rightarrow \infty$, the second term approaches to 1 . In this case the incoherence is a direct consequence of the geometric effect. The case of $V_{S} / \alpha=V_{\text {app }}=\infty$ coincides with a rigid (incoherent) support excitation. Ground motion accelerations at each pier are numerically generated.

Fig. 5.15 shows a distribution of the maximum shear forces and the shear forces corresponding to the quasi-static component (refer to Eq. (2.11)) computed at each pier for $V_{S} / \alpha=300 \mathrm{~m} / \mathrm{s}, 600 \mathrm{~m} / \mathrm{s}$, and $\infty . V_{\text {app }}$ is varied $300 \mathrm{~m} / \mathrm{s}, 600 \mathrm{~m} / \mathrm{s}, 1200 \mathrm{~m} / \mathrm{s}$ and $\infty$. The pier height is 10 m and moderate soil condition is assumed. The solid line in Fig. 5.15 (c) shows the response of the bridges subjected to a rigid ground motion.

The distribution of the shear force at each pier depends on the coherency of ground motion. Under a rigid support excitation, the shear force at the intermediate piers is larger than that at the end piers. This is because the distribution of total shear force essentially depends on the first mode shape of the bridge system. On the other hand, as the incoherence in terms of $V_{S} / \alpha$ and $V_{a p p}$ increases, the distribution of shear force becomes more flattened, because higher modes are excited by the multiple support excitation.

It is important that the shear force under a rigid support excitation is systematically larger that the shear force under a multiple support excitation. This is explained by the multiple
excitation response spectrum (refer to 1.9 ).


Fig. 5.15 Total and Pseudo-Static Forces Induced in a Bridge with a Pier Height of 10 m on Medium Soil Site

### 5.4 Effect of Pounding of Decks

## 1) Importance of Pounding

Pounding occurs between two adjacent decks or between a deck and an abutment in a bridge under an extreme ground excitation. The importance of pounding effects on the total response of a bridge system is becoming widely known. Pounding provides a limited damage at the contact faces, however it develops a transfer of large seismic lateral force from one deck to the other, which results in a large change of seismic response in an entire bridge system. Pounding or transfer of lateral force in wider meaning through unseating prevention devices also affects the total response of a bridge system. A good example for such an effect is the collapse of an approach span of the Nishinomiya Bridge, Hanshin Expressway in the 1995 Kobe, Japan, earthquake. The main bridge (Nishinomiya Bridge) consisted of a Lohse bridge with a mass of $12,000 \mathrm{t}$, while the approach span consisted of a plate girder with a mass of $1,900 \mathrm{t}$. They were tied together by plate-type restrainers. The damage was initiated from a failure of two fixed steel bearings of main bridge. Since it allowed large response displacement of the main girder to take place, the main girder pulled the approach span, which resulted in failure of fixed steel bearings in the approaching span. As a consequence, the approaching span dislodged from its support when the deck moved in the other direction. The unseating prevention devices were not strong enough to support the approach span once it dislodged from the support. Since there was no evidence that the main bridge and the approach span collided, the transfer of seismic lateral force from the main bridge to the approach span was the main reason for collapse. Based on the damage in the recent earthquakes in USA, Japan, Taiwan and Turkey, extensive analyses and experiments are being conducted worldwide on the effect of pounding and unseating prevention devices. Some unique findings are presented below.

## 2) Idealization of Longitudinal Collisions of Two Elastic Bars using Impact Spring

Longitudinal collisions of two elastic rods as shown in Fig. 5.16 are defined as taking place at a contact point when the relative displacement between the two ends defined as

$$
\begin{equation*}
\Delta u=u_{R}-u_{L} \tag{5.11}
\end{equation*}
$$

becomes zero with a no-zero velocity, in which $u_{R}$ and $u_{L}$ represent the displacements of the contact surface of the right and left bars, respectively. A positive value of $\Delta u$ represents a separation of the contact surfaces while a negative value of $\Delta u$ represents an overlap of the contact surface. A negative value of $\Delta u$ is, of course, not possible in reality as given by the exact solution.


Fig. 5.16 Idealization of Longitudinal Impact of Bars using Impact Spring
Tseng and Penzien $(1973,1975)$ proposed an impact spring to idealize pounding effects of two bridges. Since the idealization of collisions by Tseng and Penzien was for inelastic collisions, it was later modified by Kawashima and Penzien $(1976,1979)$ so that it represents elastic collisions. The impact spring with a large stiffness $k_{I}$, which is attached to the end of one rod, starts to resist motion when $\Delta u \leq 0$. A collision is completed when rebound occurs and the relative displacement between the two rods becomes equal to zero. The impact spring completes to resist the motion when $\Delta u>0$. The stiffness of an impact spring and the force resisted by the impact spring are written as

$$
\begin{align*}
& k_{I}=\left\{\begin{array}{cc}
\widetilde{k}_{I} & \Delta u \leq 0 \\
0 & \Delta u>0
\end{array}\right.  \tag{5.12}\\
& P_{I}=\left\{\begin{array}{cc}
\widetilde{k}_{I} \Delta u & \Delta u \leq 0 \\
0 & \Delta u>0
\end{array}\right. \tag{5.13}
\end{align*}
$$

The impact spring elements can be included in an analytical model based on the analytical procedure in Chapter 2. Since a sudden change of the impact spring stiffness results in an unbalance force in the equilibrium of the equations of motion, the iteration of equilibrium by Eq. (2.64) may be required. In this idealization, a zero value of $\Delta u$ represents the position of the bars when initial contact occurs on both sides with the impact spring. Consequently a negative value of $\Delta u$ is possible which corresponds to the shortening of the impact spring. In modeling poundings using the impact springs, the impact spring stiffness $k_{I}$, the numerical time interval of integration $\Delta t$, and the number of beam elements $n$ representing each bar are factors influencing the required dynamic response following impact.

Using the idealization of the impact spring, collisions of two bars are analyzed for two examples (Kawashima and Penzien 1976, Kawashima and Watanabe 2001). The first example is a collision of two uniform elastic bars with the same length which are initially traveling in opposite directions with the same initial velocity $V_{0}\left(V_{1}=V_{0}\right.$ and $\left.V_{2}=-V_{0}\right)$ as shown in Fig. 5.17. The approximate solution of the post-pounding behavior obtained by the above
procedure can be compared to the exact solution obtained by the classical wave propagation theory (Goldsmith 1960).


Fig. 5.17 Longitudinal Collision of Two Bars with the Same Length
Assume both bars have the same properties as $E$ (modulus of elasticity)=100, $A$ (cross-sectional area) $=1, \rho$ (mass density) $=0.1, L$ (bar length) $=10$, and $V_{0}$ (initial bar velocity) $=+/-0.1$, in which any convenient units may be used. For this example problem, the impact contact duration $T_{I}$ is 0.2 units of time as given by the exact solution. This duration corresponds to the time required for a wave to propagate twice the length of the bar, i.e.,

$$
\begin{equation*}
T_{I}=\frac{2 L}{C_{0}} \tag{5.14}
\end{equation*}
$$

where $C_{0}$ is the longitudinal wave velocity given by

$$
\begin{equation*}
C_{0}=\sqrt{\frac{E}{\rho}} \tag{5.15}
\end{equation*}
$$

Stress $\sigma_{0}$ induced at the contact surface by wave propagating inside bars is

$$
\begin{equation*}
\sigma_{0}=\frac{-V_{0}}{C_{0}} \tag{5.16}
\end{equation*}
$$

where a negative value of $\sigma_{0}$ represents a compression stress.
In both the exact and approximate solutions, it is convenient to monitor the separation between the two bars $\Delta u$, and the velocity and the impact force at the contact surface of one of the bars (left bar) $\dot{u}_{L}$ and $P_{I L}$, respectively.

Response $\Delta u, \dot{u}_{L}$, and $P_{I L}$ as predicted by exact wave propagation theory are shown in Fig. 5.18. Before collision, the relative separation velocity $\dot{u}_{L}$ equals +0.1 which instantaneously changes to zero upon collision. This velocity stays at a value of zero during the contact duration equal to 0.2 unit time and then instantaneously changes to a value of -0.1 . It is noted that the instantaneous change in velocity represents a Dirac delta change in the acceleration, i.e., a pure acceleration pulse of duration and amplitude which tend to zero and infinity, respectively.


Fig. 5.18 Exact Solution of Relative Displacement between Two Bars $\Delta u$, and Velocity and Impact Force at the Contact Surfaces of the Left Bar $\dot{u}_{L}$ and $P_{I L}$

Turning now to the approximate solution, consider first the effect of the magnitude of the impact spring stiffness $\widetilde{k}_{I}$ upon response. Using a time interval of numerical integration $\Delta t$ equal to $\sqrt{10} / 2000 T_{I}$, a value of 10 for $n$, and six different values for $k$, responses $\Delta u$, $\dot{u}_{L}$, and $P_{I L}$ are obtained as shown in Figs. 5.19, 5.20 and 5.21 , in which $P_{I L}$ is normalized by the exact impact force $A \sigma_{0}$. A dimensionless parameter $\gamma$ is introduced to represent the magnitude of $\widetilde{k}_{I}$ as (Kawashima and Penzien 1976)

$$
\begin{equation*}
\gamma=\frac{\widetilde{k}_{I} \cdot L}{n \cdot E A} \tag{5.17}
\end{equation*}
$$

which represents the ratio of the impact spring stiffness $k$ to an individual bar element stiffness.


Fig. 5.19 Computed Relative Displacement between Two Bars $\Delta u$


Fig. 5.20 Velocity at the Contact Surface of the Left Bar $\dot{u}_{L}$


Fig. 5.21 Impact Force at the Contact Surface of the Left Bar $P_{I L}$
Comparing the computed relative displacements with the corresponding exact solutions shows that the exact and approximate solutions of $\Delta u$ are in good agreement for $\gamma$ between 0.5 and 10 , but they are significantly in error for the larger and smaller values of $\gamma$. Comparing the computed velocity $\dot{u}_{L}$ and the impact force $P_{I L}$ to the exact solutions shows considerable differences for all 6 values of $\gamma$ and it shows considerable vibrations of $\dot{u}_{L}$ and $P_{I L}$ taking place following unit time zero. These vibrations in velocity $\dot{u}_{L}$ and impact force $P_{I L}$ increase in amplitude with increasing values of $\lambda$. It is interesting to note that when larger values of $\gamma$ are used, contact and separation of the bar ends are repeated many times during period $T_{I}$. It appears that a value of unity for $\gamma$ yields the best overall correlation of the approximate results with exact solutions. This suggests that the numerical value of $k$ should be approximately equal to the stiffness of its neighboring beam elements.

Let us now consider the effect of the magnitude of $\Delta t$ upon response. Using $\widetilde{k}_{I}$ corresponding to $\gamma=1, n$ equal to 10 , and $\Delta t$ equal to $\sqrt{10} / 2000 T_{I}, 0.1 T_{I}, T_{I}$, and $5 T_{I}$, computed responses of $\Delta u$, velocity $\dot{u}_{L}$, and the impact force $P_{I L}$ are shown in Fig. 5.22. The computed relative displacement responses $\Delta u$ agree well with the exact solutions for $\Delta t / T_{I}$ less than 0.1. It is noted that the computed response of $\Delta u$ does not, of course, represent the exact solution during $T_{I}$, for $\Delta t / T_{I}$ equal to 1 and 5 , but overall responses with contact and separation can be well represented. The velocity $\dot{u}_{L}$ shows an erroneous oscillation. The oscillation becomes considerable as $\Delta t / T_{I}$ becomes small. However, if a smooth curve is drawn through these oscillations, it would be reasonably well with the exact relation shown in Fig. 5.18 (b).

Fig. 5.23 shows the effect of the number of beam element $n$ ( $n=5,10$, and 29) on the relative displacement response $\Delta u$ assuming $\widetilde{k}_{I}$ corresponding $\gamma=1$, and $\Delta t / T_{I}=\sqrt{10} / 2000$. The computed displacement response for all three values of $n$ agrees very closely with the exact response.


Fig. 5.22 Effect of Numerical Time Interval of Integration (1)


Fig. 5.22 Effect of Numerical Time Interval of Integration (2)


Fig. 5.23 Effect of the Number of Element on Relative Displacement $\Delta u$
The variations of particle velocity and longitudinal stress with position along both bars for the case of $n=10, \widetilde{k}_{I}$ corresponding to $\gamma=1$, and $\Delta t=\sqrt{10} / 2000 T_{I}$ were computed. The resulting variations along both bars for instantaneous times t equal to $0,0.25 T_{I}, 0.5 T_{I}$, $0.75 T_{I}, 1.0 T_{I}$, and $1.25 T_{I}$, which correspond to $0,0.05,0.1,0.15,0.2$, and 0.25 unit times, respectively, are shown in Fig. 5.24. The ordinates representing particle velocity and stress are normalized by dividing by the initial bar velocity $V_{0}(=0.1)$ and the exact intensity of the stress wave $\sigma_{0}=E V_{0} / C_{0}$, respectively. Comparing these computed variations with the exact solutions shows that reasonably accurate results can be obtained by the approximate method employing an impact spring.


Fig. 5.24 Variation of Stress and Particle Velocity

(b) Collision

(c) Separation

Fig. 5.25 Collision of Two Elastic Bars with Different Lengths
The second example is a collision of two elastic bars which are initially traveling in the same direction with initial velocities of $V_{1}=2 V_{0}$ and $V_{2}=V_{0}$ (refer to Fig. 5.25). The right bar is twice as long of the left bar, i.e., $L_{1}=L=10$ and $L_{2}=2 L=20$. The same non-dimensionalized properties with the first example are assumed in the second example. The duration of collision $T_{I}$ in this case is the time required for a wave to propagate twice the length of the right bar as

$$
\begin{equation*}
T_{I}=\frac{4 L}{C_{0}} \tag{5.18}
\end{equation*}
$$

which is 0.4 unit time. The exact solution for the relative displacement $\Delta u$ between the left and right bars, particle velocity at the contact surface in the left and right bars $\dot{u}_{L}$ and $\dot{u}_{R}$, respectively, and stresses at the contact surface in the left rod and right rod, $\sigma_{L}$ and $\sigma_{R}$, respectively, are shown in Fig. 5.26, in which approximate analytical results using an impact
spring are shown for comparison. Before collision, the particle velocity at the contact surface of the left bar $\dot{u}_{L}$ equals to $+2 V_{0}$ and the particle velocity at the contact surface of the left bar $\dot{u}_{R}$ equals $+V_{0}$. Both $\dot{u}_{L}$ and $\dot{u}_{R}$ instantaneously change to the same value of $+1.5 V_{0}$ upon collision. This velocity $\dot{u}_{L}$ stays at a value of $+1.5 V_{0}$ during 0.4 unit time and the velocity keeps this value even after the contact, while in the right bar this velocity $\dot{u}_{R}$ stays at a value of $+1.5 V_{0}$ during 0.2 unit time and then $\dot{u}_{R}$ instantaneously changes to a value of $+V_{0}$. Subsequently, in the right bar the velocity $\dot{u}_{R}$ repeats to change between $+2 V_{0}$ and $+V_{0}$ every 0.2 unit time after a separation occurs, which implies that an oscillation remains after collision.


Fig. 5.26 Relative Displacement, and Velocities and Stresses at the Contact surface of the Bars 1 and 2

Fig. 27 shows the variation of particle velocity and longitudinal stress with position along both bars, in which an approximate analytical result using an impact spring are also shown for comparison. It is noted that at $\Delta t$ equal to $T_{I}$ ( 0.2 unit time) the particle velocity in the entire position of the left and right bars becomes $V_{0}$ and $1.5 V_{0}$, respectively. However, because stress in the right bar is still $-\sigma_{0}$ (compression), the right bar stays in contact with
the left bar without separation. Separation finally occurs at $\Delta t$ equal to $2 T_{I}$ (0.4 unit time) when the particle velocity becomes $V_{0}$ and $1.5 V_{0}$ in the entire position of the left bar and right bar, respectively, and the stress in the right bar becomes $+\sigma_{0}$ (tension).

An approximate solution assuming $\widetilde{k}_{I}$ corresponding to $\gamma=1, n$ equal to 10 and 20 in the left and right bars, respectively, and $\Delta t$ equal to $\sqrt{10} / 2000 T_{I}$ are presented in Figs. 5.26 and 5.27. Reasonably accurate solution for the overall response during and after a collision can be obtained using an impact spring.


Fig. 5.27 Variation of Stress and Particle Velocity

## 3) Analysis of Seismic Response of a Straight Model Bridge with Pounding Effect

In order to verify the accuracy of predicting seismic response of bridges with pounding effects, a shake table test was conducted for a bridge consisting of two single span decks and columns as shown in Fig. 5.28 (Kawashima, Uehara, Shoji and Hoshi 2002). The decks and columns are made of steel. The columns are fixed to both the deck and the shake table. The fundamental natural period of the left and right decks (decks 1 and 2, respectively) is 0.357 and 0.422 seconds, respectively. Since energy dissipation is limited in a model bridge consisting of the steel deck and columns, two viscous dampers are set at two columns. As a consequence, the damping ratio of the decks 1 and 2 for the fundamental mode becomes 0.0478 and 0.0539 , respectively.


Fig. 5.28 Model Bridge for Shake Table Test
Two decks are tied together by a pair of restrainers at both sides as shown in Fig. 5.29. A shock absorber for tension is set in a restrainer. The restrainers start to resist opening between the two decks when the opening reaches an initial gap $\Delta u_{G 2}$. A pair of shock absorbers for compression is installed at the contact surface of the right deck. The shock absorbers start to resist closing between the two decks when the closing reaches an initial gap $\Delta u_{G 1}$. Two cases are analyzed here among a series of shake table tests.


Fig. 5.29 Model of Restrainers and Shock Absorbers
The first case is a shake table test for the model bridge without restrainers and shock absorbers. Consequently, collisions and separations of two decks take place without any restraints. The model is excited by the JMA Kobe record during the 1995 Kobe earthquake (refer to Fig. 1.1 (a)) with the intensity of acceleration being scaled-down. The initial gap between the two decks $u_{G 1}$ was 2.5 mm . Responses of accelerations and velocities of the two decks, and the relative displacement between the two decks $\Delta u$ (refer to Eq. (5.11)) are presented in Fig. 5.30, in which " $x$ " denotes the instance when collisions take place. Collisions take place nine times when the relative displacement $\Delta u$ reaches $u_{G 1}$ $(=-2.5 \mathrm{~mm})$ resulting in response accelerations with high spikes. The peak response acceleration is -2.82 g and 2.76 g in the decks 1 and 2, respectively.


Fig. 5.30 Response of Model Bridge without Restrainers and Shock Absorbers
It is noted here that collisions take place nine times when the response displacements of the decks 1 and 2 are negative, which shows that the collisions do not take place when the two decks collides resulting from the positive (right) movement of the deck 1 and the negative (left) movement of the deck 2 . Because the response displacement of the deck 2 is slightly larger than the response displacement of the deck 2 (the peak response displacement of the decks 1 and 2 is 11.95 mm and 14.83 mm , respectively), the all nine poundings occur when the deck 2 collides with the deck 1 which is at the position of nearly reaching its peak response displacements in the negative (left) side. The larger response displacement of the deck 2 than the deck 1 is resulted from the longer natural period of the deck 2 .

The other case is a shake table test for the model with restrainers and shock absorbers. The gaps for closing $u_{G 1}$ and the gap for opening $u_{G 2}$ are 1.0 mm and 4.0 mm , respectively. Responses of acceleration and displacement of the decks 1 and 2, the relative displacement $\Delta t$, and impact force $P_{I}$ are shown in Fig. 5.31, in which " x " and " o " denote the instances when collisions take place and the instances when the restrainers resist further openings, respectively. Response accelerations are mitigated by the shock absorbers, and the peak response acceleration is -0.62 g and 0.48 g in the decks 1 and 2, respectively. Collisions take place 16 times when the relative displacement $\Delta u$ reaches $u_{G 1}(=1 \mathrm{~mm})$. As is the above case, the 16 collisions take place when the deck 2 collides with the deck 1 which is at the position of nearly reaching its peak response displacements in the negative (left) side. The restrainers are effective to prevent excessive relative opening. The peak opening is 4.72 mm which is 0.72 mm longer than the initial gap $u_{G 2}$ of 4 mm , which shows that the shock absorbers deformed 0.72 mm resulting in tension with a magnitude of 29.5 N in the two restrainers.


Fig. 5.31 Response of Model Bridge with Restrainers and Shock Absorbers
The above shake table test results are analyzed using an analytical model incorporating an impact spring as shown in Fig. 5.32. When the restrainers and the shock absorbers are not installed, the stiffness and restoring force of the impact spring are given by Eqs. (5.12) and (5.13) as

$$
\begin{gather*}
k_{I}=\left\{\begin{array}{cc}
\widetilde{k}_{I} & \Delta u \leq-u_{G 1} \\
0 & \Delta u \geq-u_{G 1}
\end{array}\right.  \tag{5.19}\\
P_{I}=\left\{\begin{array}{cc}
\widetilde{k}_{I}\left(\Delta u-u_{G 1}\right) & \Delta u \leq-u_{G 1} \\
0 & \Delta u \geq-u_{G 1}
\end{array}\right. \tag{5.20}
\end{gather*}
$$

On the other hand, when the restrainers and the shock absorbers are installed, the stiffness and the restring force of the impact spring are given as

$$
\begin{gather*}
k_{I}=\left\{\begin{array}{lr}
\tilde{k}_{G 1} & \Delta u<-u_{G 1} \\
0 & -u_{G 1} \leq \Delta u \leq u_{G 2} \\
\widetilde{k}_{G 2} & u_{G 2}<\Delta u
\end{array}\right.  \tag{5.21}\\
P_{I}=\left\{\begin{array}{lr}
\widetilde{k}_{G 1}\left(\Delta u-u_{G 1}\right) & \Delta u<-u_{G 1} \\
0 & -u_{G 1} \leq \Delta u \leq u_{G 2} \\
\widetilde{k}_{G 2}\left(\Delta u-u_{G 2}\right) & u_{G 2}<\Delta u
\end{array}\right. \tag{5.22}
\end{gather*}
$$

where $\widetilde{k}_{G 1}$ : stiffness of shock absorber for compression, and $\widetilde{k}_{G 2}:$ stiffness of restrainers including shock absorbers for tension.


Fig. 5.32 Analytical Model
The impact spring stiffness $\widetilde{k}_{I}$ and the time interval of numerical integration $\Delta t$ are determined from the previous analysis so that the following relations are satisfied.

$$
\begin{gather*}
\gamma=\frac{k_{I} L}{n E A}=1.0  \tag{5.23}\\
\frac{\Delta t}{T_{I}}=1.0 \tag{5.24}
\end{gather*}
$$

Since $E A=5.56 \times 10^{5} \mathrm{kN}, L=1 \underset{\sim}{m}, n=11$, and $C_{0}=5120 \mathrm{~m} / \mathrm{s}$ when the restrainers and shock absorbers are not provided, $\widetilde{k}_{I}$ equals $5.6 \times 10^{6} \mathrm{kN} / \mathrm{m}$ and $\Delta t$ equals $3.9 \times 10^{-4} \mathrm{~s}$.

Fig. 5.33 shows the computed responses of the model bridge without the restrainers and shock absorbers. The experimental responses presented in Fig. 5.30 are also shown here for comparison. The computed responses agree well with the experimental responses except spikes of the deck response accelerations. The computed peak values of the accelerations of the decks 1 and 2 are 15.37 g and 10.88 g , respectively, which overestimates nearly four times the measured accelerations of the decks 1 and 2 . The predicted peak accelerations with spikes are considerably inaccurate in the analyses using the impact spring.


(e) Relative Displacement between Decks 1 and 2

Fig. 5.33 Computed and Experimental Responses for Model Bridge without Restrainers and Shock Absorbers

Fig. 5.34 shows the effect of varying the magnitude of the impact spring stiffness given by Eq. (5.23) as $k_{I} / 10$ and $10 k_{I}$ with $\Delta t$ stays at $3.9 \times 10^{-4} \mathrm{~s}$. The responses of the bridge is less sensitive to the magnitude of impact spring stiffness if it changes between $k_{I} / 10$ and $10 k_{I}$. The effect of varying the time interval of numerical integration $\Delta t$ given by Eq. (5.24) is also studied for $\Delta t / 10,2 \Delta t$, and $10 \Delta t$. Only minor changes from the responses presented in Fig. 5.33 occur for $\Delta t / 10$ and $2 \Delta t$. However unrealistic responses are obtained for $10 \Delta t$ as shown in Fig. 5.35. It is noted that $\Delta t$ is important in the evaluation of responses of bridges where direct collisions of two adjacent decks take place.


Fig. 5.34 Effect of Magnitude of Impact Spring Stiffness


Fig. 5.35 Responses of Deck 1 when Time Interval of Numerical Integration equal to 10 times $\Delta t$
Fig. 5.36 shows computed responses of the model bridge with the restrainers and shock absorbers. Fig. 5.37 shows a detailed comparison of the responses shown in Fig. 5.36 for the period of 0.5 s and 2.5 s . Overall responses of the model bridge are well predicted by the analysis including the force induced in the restrainers.


Fig. 5.36Response of Model Bridge with Restrainers and Shock Absorbers


Fig. 5.37 Detail of Responses in Fig. 5.19 (Between 0.5 and 2.5 s)

