2. DYNAMIC RESPONSE ANALYSIS OF BRIDGES

2.1 INTRODUCTION

Bridges are unique in their structural response. First, they are longitudinally lengthy, and consist of many structural components which contribute to the overall resistance capability of the system. Decks are often skewed and curved, and intermediate expansion joints divide a bridge system into several structural segments with different natural periods. Second, there are various structural types with complex geometries and dynamic response characteristics. Suspension bridges and cable stayed bridges generally display a very complex structural response with long natural periods, often exceeding 10 seconds. Many modes with closely spaced natural periods contribute to the complexity of the structural response. Third, bridges are generally constructed at soft soil sites such as rivers and bay areas. Because ground motions are amplified at these sites, greater attention should be paid to seismic design for large ground motion. Failure of foundations associated with the instability of surrounding ground is a common occurrence. Fourth, the degree of statical indeterminacy is smaller in bridges than in buildings, and therefore ductility of piers/ columns needs to be carefully examined to prevent failure during strong earthquakes.

Various analytical methods have been developed to predict the seismic response of bridges. This has enabled to construct bridges which were difficult to design when computer analysis was not available. For example, precise linear and nonlinear seismic response analysis is essential for long span bridges, bridges with complex geometric features, cable supported bridges, and tall bridges. Computers have also greatly assisted in the analysis of bridges which have failed during past earthquakes, and have greatly contributed to the improvement of seismic design methods.

2.2 ANALYTICAL MODELING OF BRIDGES

1) Structural System

Generally a bridge consists of a girder, piers/ columns, abutments, foundations, bearing supports, and expansion joints. In addition, special types of bridges such as arch bridges, suspensions bridges and cable stayed bridges have arch members, towers, anchorages, cables, hangers and links. Energy dissipating devices and active mass devices are used in passive and active control. Because the structural characteristics of bridges depend on their types, emphasis is placed here on showing analytical modeling for a girder bridge (Tseng and Penzien [34]).

The structural system of this type of bridge consists of a multiple-span continuous deck supported by bearings on or rigidly connected to reinforced concrete piers/ columns and abutments. The deck may be straight, curved or skewed, and is supported at discrete locations along its longitudinal axis. Intermediate expansion joints divide the deck into several segments. The entire structural system generally exhibits the characteristics of a continuous space frame. Its dynamic response to earthquake excitations is of a lower mode type; hence, a mathematical model of discrete form can be used to approximate the continuous system. This form of modeling leads to a system with a finite number of degrees of freedom. Following the standard finite element procedure, these degrees of freedom are chosen as the nodal displacements of the discrete finite element model. For a three dimensional model, each nodal point usually has 6 degrees of freedom, i.e., 3 translation components and 3 rotation components. Internal constraints may reduce this number at some nodal points.

For dynamic response analysis, the stiffness, mass and damping properties of each finite element must be realistically defined.

2) Stiffness Idealization

The finite element idealization of a complete bridge system results in a stiffness matrix which is an assemblage of the generalized stiffness matrices for individual elements as

$$\mathbf{K} = \sum_{i=1}^{N} \mathbf{k}_i \tag{2.1}$$

where, $\mathbf{K}$= total stiffness matrix for the entire bridge system, $\mathbf{k}_i$= stiffness matrix for element i, and N= total number of elements in the bridge system.

For small amplitude response, the bridge system may be modeled by a set of linear elements; however, when subjected to high amplitude response as occurs during severe earthquakes, certain critical regions of the system may undergo large cyclic inelastic deformations. Therefore, nonlinear finite elements for the mathematical model, which have realistic nonlinear hysteretic force - deformation characteristics, must be chosen. The stiffness of these elements are time dependent and are functions of element deformations and deformation histories. Usually, they are linearized for analysis in a piecewise fashion using tangent stiffnesses at discrete times. Thus, the total stiffness matrix for the entire structure may be written as

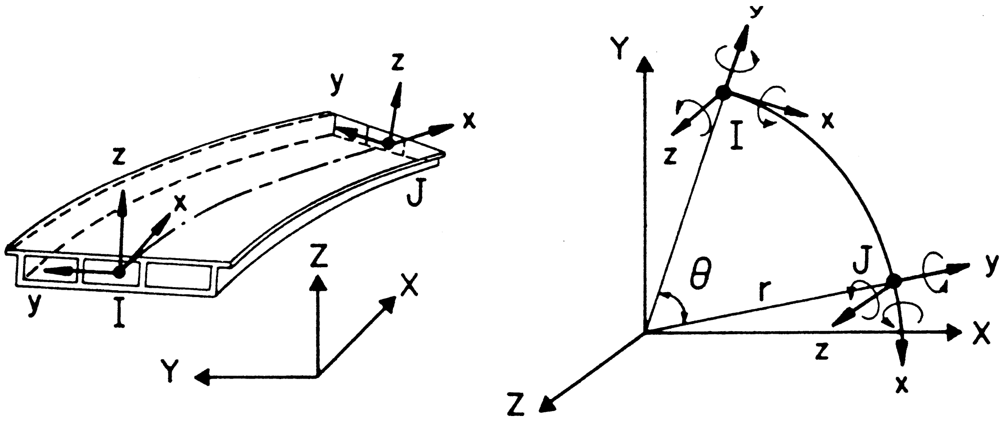
$$\mathbf{K}_t = \sum_{i=1}^{N} \mathbf{k}_{ti} \tag{2.2}$$

where, $\mathbf{K}_t$= total stiffness matrix at time t, and $\mathbf{k}_{ti}$ = stiffness matrix for element i at time t. Nonlinearity arising from large geometry changes is not generally included as it is negligible.

a) Decks

When determining internal stress distributions in the constituent flanges and webs of box girders under localized loadings, elaborate methods of analysis, such as finite element analysis, must be used. However, when the external loadings are relatively uniformly distributed, and when only resultant forces on transverse cross-section, i.e., 3 components of force and 3 components of moment, are required, a simple beam analysis as shown in Fig. 2.1 is usually sufficient to yield accurate results.

Since a typical deck is extremely stiff and strong in comparison with its supporting columns and abutments, the high amplitude bridge response produced during severe ground shaking will be caused primarily by deformations in the columns, abutments and expansion joints. The deck will remain elastic and, therefore, can be modeled by linear elastic elements. Nonlinear elements must, however, be used for columns, abutments and expansion joints.

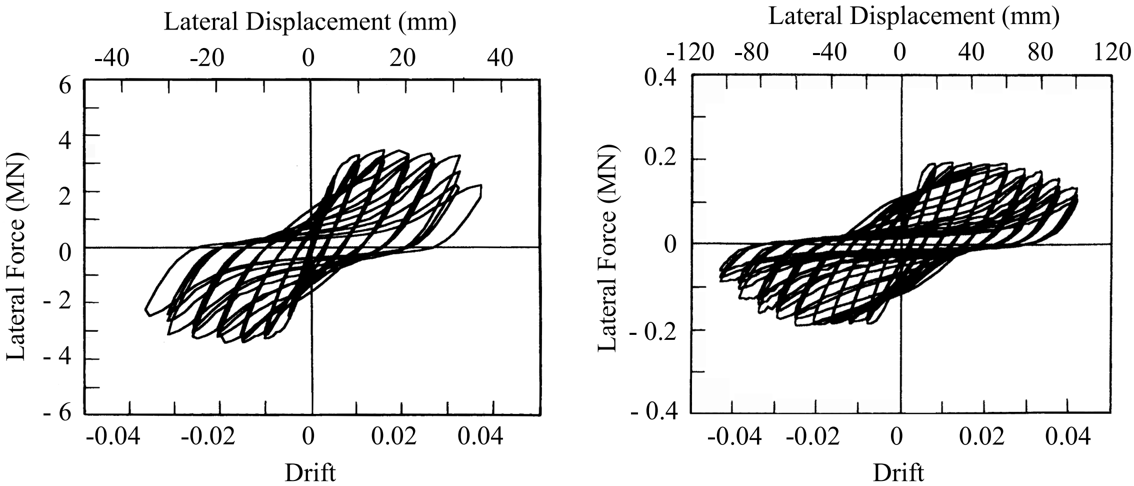


(a) Typical Box Girder Element (b) Idealized Model

Fig. 2.1 Analytical Model of Superstructures

b) Columns

The structural behavior of columns is generally adequately modeled using simple beam elements. Because of large amplitude response, coupled inelastic deformations may occur in these members. For example, Fig. 2.2 shows lateral force vs. lateral displacement hysteresis loops which were obtained by cyclic loading tests of circular reinforced concrete columns (Priestley, Seible and Chai [29]). Fig. 2.2 (a) shows the hysteretic behavior of an *as-built* column while Fig. 2.2 (b) shows the hysteretic behavior of a column strengthened with a steel cylinder jacket. Confinement of concrete by hoops is important to increase the ductility and energy dissipating capacity of reinforced concrete columns (Park [27], Priestley and Park [30]).



(a) *as-built* column (b) a column strengthened with a steel cylinder jacket

Fig. 2.2 hysteretic behavior of columns

Fig. 2.3 shows the lateral force vs. displacement hysteretic loops of steel columns (MacRae and Kawashima [18]). Degradation of stiffness after the maximum load is achieved generally larger in steel bridge columns than well confined reinforced concrete columns. Because this tends to cause large residual displacement when subjected to large ground motion, such a feature must be carefully idealized in analysis. In realistically modeling the hysteretic behavior of reinforced concrete columns, stiffness degradation, strength loss and pinching are the key issues (Williams and Sexsmith [38]). Therefore, nonlinear beam elements which realistically characterize the inelastic hysteretic behavior of columns must be used.

Fig. ? shows a more detailed idealization of reinforced concrete column by 3 dimensional finite elements. Concrete is modeled by solid elements while reinforcing bars are modeled by beam elements. Inelastic stress-strain hysteresis as well as failure criteria is idealized by an appropriate constitutive model.

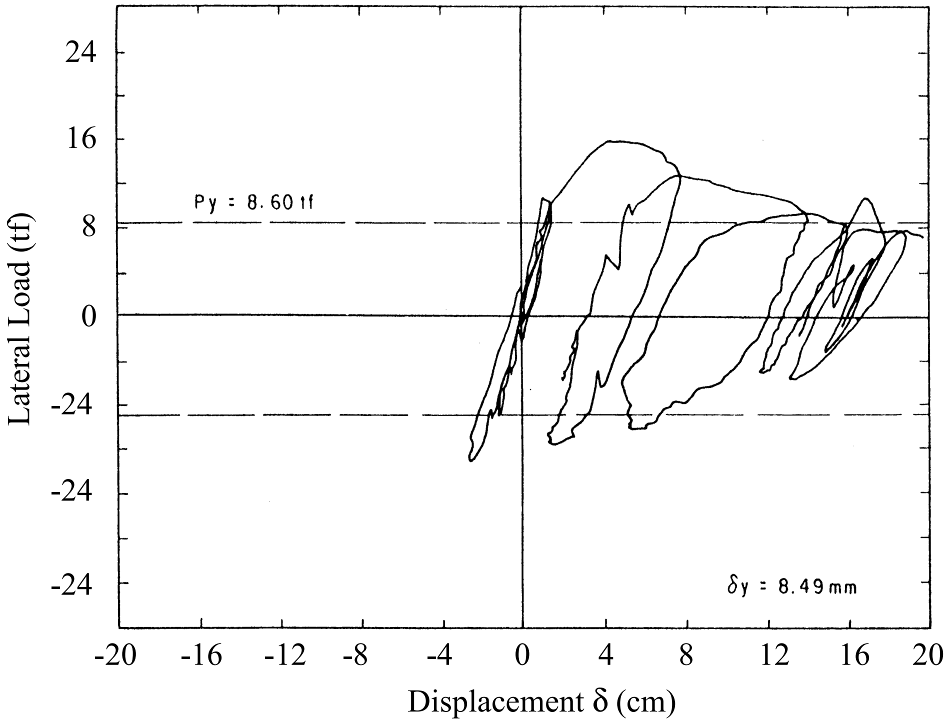


Fig. 2.3 The lateral force vs. displacement hysteretic loops of steel columns

c) Abutments

The force-displacement relationship of abutments is a highly complex nonlinear problem. Failures are likely to be of the shear type causing excessive damage. In idealizing abutments by beam elements, it is usual to assume equivalent linear springs in longitudinal and transverse directions to simulate the restraints on the superstructure provided by any abutment. It is important to select the spring stiffness accurately so as to allow correct distribution of seismic loads throughout the structural systems. For this purpose, the spring stiffness must reflect the dynamic behavior of the soil behind the abutment, the structural components of the abutment, and the interaction between the soil and the structural components of the abutment. Substantial nonlinear behavior is expected in the abutment because some of the elements constituting the abutment may be subjected to significant yielding (Maroney and Chai [19]).

d) Foundations

Various idealizations have been developed for foundations. Complex idealization uses nonlinear finite element models. A simpler and more appropriate model consists of 3 translational and 3 rotational soil springs, as shown in Fig. 2.4, to connect the base of each column and abutment to a rigid foundation where the seismic excitation is fully prescribed. For linear analysis, the stiffness of these soil springs may be evaluated using linear elastic half-space theory (Penzien [28]).

For large amplitude response, the foundation soils may undergo inelastic deformation of the hysteretic type. In this case, the six soil springs should be nonlinear hysteretic springs.

Their stiffness can only be established through extensive experimental studies on the dynamic properties of foundation soils (Lam [17]).

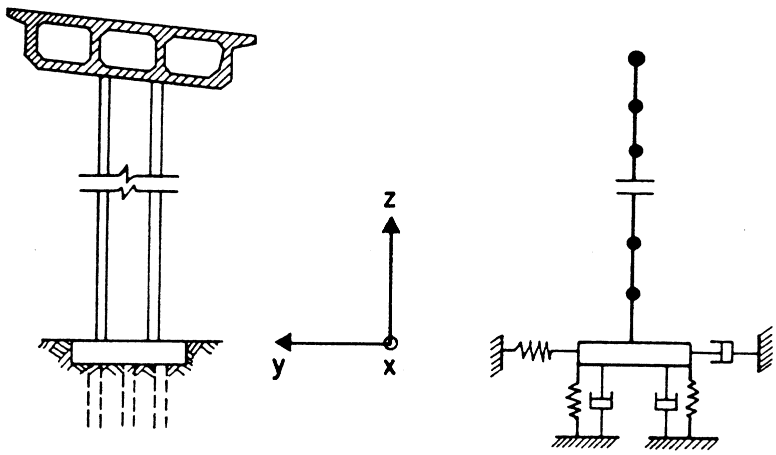


Fig2.4 Analytical Model of Foundations

3) Mass Idealization

The continuous mass of the bridge structural system is modeled in discrete form by lumping element masses at their end nodal points. Since inertia forces are associated with each of the six degrees of freedom at a nodal point, each lumped mass should be assigned an appropriate moment of inertia about its own coordinate axes. It should also be noted that when conducting nonlinear dynamic analysis, the instantaneous stiffness matrix can become singular, in which case it is required that a mass moment of inertia be assigned to each rotational degree of freedom. Following this procedure, a diagonal mass matrix $\mathbf{m}_i$ is established for each element i $(i = 1, 2, \cdots\cdots, N)$. The diagonal mass matrix for the complete bridge system can then be assembled and expressed as

$$\mathbf{M} = \sum_{i=1}^{N} \mathbf{m}_i \tag{2.3}$$

In determining the overall dynamic response of bridges, this lumped mass method has been found to be quite adequate for analytical purposes.

4) Damping Idealization

Velocity dependent damping in a bridge structural system is represented by a generalized damping matrix associated with the finite degree of freedom permitted in the analytical model. This matrix can be derived by consistent procedure similar to those used in deriving the stiffness matrix, provided the internal damping mechanism within each element is specified. The structural damping matrix for the complete bridge system would then be evaluated as

$$\mathbf{C} = \sum_{i=1}^{N} \mathbf{c}_i \tag{2.4}$$

where $\mathbf{c}_i$ is the damping matrix for the i-th element.

In practice, however, it is difficult to establish the basic characteristics of damping in individual elements. It is therefore often assumed that the damping force consists of one set which is proportional to the velocities of each mass point and a set which is proportional to the rate of deformation. Thus, the structural damping matrix becomes

$$\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K} \tag{2.5}$$

where α and β are the scalar proportionality constants. They are determined after assigning damping ratios to the first two natural modes of vibration (Clough and Penzien [7]).

In the mode superposition method, modal damping ratios ξ_k for each mode are required. Because energy dissipation occurs by specific mechanism, such as friction, the hysteretic behavior of structural components and radiation of energy from structures to soils, it may be possible to evaluate damping ratio at each element. Then, the modal damping ratios are approximated as (JRA [9])

$$\xi_k = \frac{\sum_{m=1}^{n} \xi_{km} \cdot \phi_{km}^T \cdot \mathbf{k}_m \cdot \phi_{km}}{\sum_{m=1}^{n} \phi_{km}^T \cdot \mathbf{k}_m \cdot \phi_{km}} \tag{2.6}$$

or

$$\xi_k = \frac{\sum_{m=1}^{n} \xi_{km} \cdot \phi_{km}^T \cdot \mathbf{m}_m \cdot \phi_{km}}{\sum_{m=1}^{n} \phi_{km}^T \cdot \mathbf{m}_m \cdot \phi_{km}} \tag{2.7}$$

where ξ_{km} = damping ratio of m-th element for k-th mode; ϕ_{km} = mode vector of m-th element for k-th mode; $\mathbf{k}_m$ = stiffness matrix of m-th element; $\mathbf{m}_m$ = mass matrix of m-th element, and n = number of element.

Eq. (2.6) assumes that modal damping ratios are proportional to the strain energy in a structural system, and a modal damping ratio is obtained from the weighted sum of the damping of each structural segment in a bridge system with the weight being the normalized elastic energy stored in each segment for a deformed shape corresponding to k-th mode shape. On the other hand, Eq. (2.7) assumes that modal damping ratios are proportional to the kinematic energy of a structural system, and a modal damping ratio is obtained from the weighted sum of the damping of each structural segment in a bridge system with the weight being the normalized kinematic energy of each segment for a deformed shape corresponding to k-th mode shape. Eq. (2.6) is appropriate to bridges in which the hysteretic-type energy dissipation is predominant. It has been found that Eq. (2.6) provides an accurate estimation for the modal damping ratios of seismic isolated bridges based on a model test (Kawashima, Hasegawa and Nagashima [13]).

For a nonlinear dynamic response analysis, the viscous damping properties of a bridge are more difficult to evaluate. As with the elastic case, it is often assumed as

$$\mathbf{C}_t = \alpha\mathbf{M} + \beta\mathbf{K}_t \tag{2.8}$$

In this relation, if degradation of the stiffness occurs, the damping matrix decreases. However damping generally increases under such a condition resulting from hysteretic energy dissipation. From such a point of view, Eq.(2.8) is only an assumption to make the analytical treatment easy.

2.3 ANALYTICAL PROCEDURE FOR SEISMIC RESPONSE OF BRIDGES

1) Equations of Motion

Equations of motion for a n degree of freedom bridge system expressing dynamic equilibrium at time t can be expressed as

$$\mathbf{M}\ddot{\mathbf{u}}_t + \mathbf{C}_t\dot{\mathbf{u}}_t + \mathbf{K}_t\mathbf{u}_t = \mathbf{R}(t) \tag{2.9}$$

where, $\mathbf{M}$, $\mathbf{C}_t$ and $\mathbf{K}_t$ are the mass, damping and stiffness matrices, respectively, and where $\mathbf{R}(t)$ is the applied dynamic load vectors. Vectors $\ddot{\mathbf{u}}_t$, $\dot{\mathbf{u}}_t$ and $\mathbf{u}_t$ are the absolute acceleration, absolute velocity and absolute displacement, respectively.

If a bridge is subjected to prescribed support excitations, a complete set of nodal displacements $\mathbf{u}_t^c$ should be considered which include, in addition to the n free nodal point displacements, the n^b prescribed non-zero support displacements. Consequently, the complete nodal displacement vector can be expressed as

$$\mathbf{u}_t^c = \begin{Bmatrix} \mathbf{u}_t \\ \mathbf{u}_t^b \end{Bmatrix} \tag{2.10}$$

where $\mathbf{u}_t^b$ is a vector containing the n^b non-zero support displacements. Vector $\mathbf{u}_t^c$ can be conveniently decomposed into a quasi-static displacement vector $\mathbf{u}_s^c$ and a dynamic displacement vector $\mathbf{u}^c$, i.e.,

$$\mathbf{u}_t^c = \begin{Bmatrix} \mathbf{u}_t \\ \mathbf{u}_t^b \end{Bmatrix} = \begin{Bmatrix} \mathbf{u}_s \\ \mathbf{u}_s^b \end{Bmatrix} + \begin{Bmatrix} \mathbf{u} \\ \mathbf{u}^b \end{Bmatrix} \equiv \mathbf{u}_s^c + \mathbf{u}^c \tag{2.11}$$

where by definition $\mathbf{u}^b = \mathbf{0}$. Enlarging the masses, damping and stiffness matrices as well as the dynamic load vector in Eq. (2.9) to account for the n^b support displacements, the equation of motion for the complete bridge system can be expressed as

$$\begin{bmatrix} \mathbf{M} & \mathbf{M}^b \\ (\mathbf{M}^b)^T & \mathbf{M}^{bb} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_t \\ \ddot{\mathbf{u}}_t^b \end{Bmatrix} + \begin{bmatrix} \mathbf{C}_t & \mathbf{C}_t^b \\ (\mathbf{C}_t^b)^T & \mathbf{C}_t^{bb} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}}_t \\ \dot{\mathbf{u}}_t^b \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_t & \mathbf{K}_t^b \\ (\mathbf{K}_t^b)^T & \mathbf{K}_t^{bb} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_t \\ \mathbf{u}_t^b \end{Bmatrix} = \begin{Bmatrix} \mathbf{R}(t) \\ \mathbf{R}^b(t) \end{Bmatrix} \tag{2.12}$$

The equations of motion associated with the n free nodal point displacements now become

$$\begin{bmatrix} \mathbf{M} & \mathbf{M}^b \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_b \\ \ddot{\mathbf{u}}_t^b \end{Bmatrix} + \begin{bmatrix} \mathbf{C}_t & \mathbf{C}_t^b \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}}_t \\ \dot{\mathbf{u}}_t^b \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_t & \mathbf{K}_t^b \end{bmatrix} \begin{Bmatrix} \mathbf{u}_t \\ \mathbf{u}_t^b \end{Bmatrix} = \mathbf{R}(t) \tag{2.13}$$

Substituting Eq. (2.11) to Eq. (2.13), one obtains

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}_t\dot{\mathbf{u}} + \mathbf{K}_t\mathbf{u} = \mathbf{R}(t) - \begin{bmatrix}\mathbf{M} & \mathbf{M}^b\end{bmatrix}\begin{Bmatrix}\ddot{\mathbf{u}}_s \\ \ddot{\mathbf{u}}_s^b\end{Bmatrix} - \begin{bmatrix}\mathbf{C}_t & \mathbf{C}_t^b\end{bmatrix}\begin{Bmatrix}\dot{\mathbf{u}}_s \\ \dot{\mathbf{u}}_s^s\end{Bmatrix} \tag{2.14}$$

By the definition of the quasi-static vector $\mathbf{u}_s$,

$$\mathbf{K}_t\mathbf{u}_s + \mathbf{K}_t^b\mathbf{u}_s^b = \mathbf{0} \tag{2.15}$$

Consequently, $\mathbf{u}_s$ can be directly obtained as

$$\mathbf{u}_s = -\mathbf{K}_t^{-1}\mathbf{K}_t^b\mathbf{u}_s^b \equiv -\mathbf{B}_t\mathbf{u}_s^b \tag{2.16}$$

where $\mathbf{B}_t \equiv \mathbf{K}_t^{-1}\mathbf{K}_t^b$ is a matrix of quasi-static influence coefficients resulting from the n^b non-zero support displacements. If the system is linear, all coefficients in $\mathbf{B}_t$ are invariant with time.

Usually the damping term on the right hand side of Eq. (2.14) is small than the inertia terms and therefore may be dropped from the equation without introducing a significant error. In addition, the coefficients in $\mathbf{M}^b$ can be set equal to zero since mass coupling vanishes for a lumped mass model. Therefore, after substituting Eq. (2.16) into Eq. (2.14), the equations of motion reduce to the form

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}_t\dot{\mathbf{u}} + \mathbf{K}_t\mathbf{u} = \mathbf{R}(t) + \mathbf{M}\mathbf{B}_t\ddot{\mathbf{u}}_s^b \tag{2.17}$$

where $\ddot{\mathbf{u}}_s^b$ is a vector containing the prescribed support excitations.

a) Multiple Support Excitation

Because a bridge has a substantial longitudinal axis, ground motion at each support will not be identical. When ground excitations corresponding to each of the n^b support displacements are prescribed by a vector $\ddot{\mathbf{u}}_g^m(t)$, the vector in Eq. (2.17) can be expressed as

$$\ddot{\mathbf{u}}_s^b = \ddot{\mathbf{u}}_g^m(t) \tag{2.18}$$

and the equations of motion become

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}_t\dot{\mathbf{u}} + \mathbf{K}_t\mathbf{u} = \mathbf{R}(t) + \mathbf{M}\mathbf{B}_t\ddot{\mathbf{u}}_g^m(t) \tag{2.19}$$

b) Rigid Support Excitation

When ground excitation at all supports along the bridge system are identical and are prescribed by a rigid acceleration vector $\ddot{\mathbf{u}}_g^r$ consisting of three translational components $\ddot{u}_{gX}$, $\ddot{u}_{gY}$ and $\ddot{u}_{gZ}$, measured along their corresponding global axes X, Y and Z, i.e.,

$$\ddot{\mathbf{u}}_g^r = \begin{Bmatrix} \ddot{u}_{gX} \\ \ddot{u}_{gY} \\ \ddot{u}_{gZ} \end{Bmatrix} \tag{2.20}$$

the equations of motion become

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}_t\dot{\mathbf{u}} + \mathbf{K}_t\mathbf{u} = \mathbf{R}(t) + \mathbf{M}\mathbf{B}^r\ddot{\mathbf{u}}_g^r(t) \tag{2.21}$$

In this equation, matrix $\mathbf{B}^r$ is defined by the relation

$$\mathbf{B}^r \equiv \begin{bmatrix} \mathbf{b}_X^r & \mathbf{b}_Y^r & \mathbf{b}_Z^r \end{bmatrix} \tag{2.22}$$

where vectors $\mathbf{b}_X^r$, $\mathbf{b}_Y^r$ and $\mathbf{b}_Z^r$ each have n components. Those components representing nodal displacements corresponding to the rigid base translation in the global X, Y and Z directions, respectively, equal unity while all other components equal zero.

2) Linear Analysis Procedure

When a bridge system is linear, the stiffness and damping matrices are invariant with time, i.e., $\mathbf{K}_t = \mathbf{K}$ and $\mathbf{C}_t = \mathbf{C}$. Consequently, Eq. (2.21) can be written as

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{R}(t) + \mathbf{M}\mathbf{B}\ddot{\mathbf{u}}_g(t) \tag{2.23}$$

A standard procedure can be followed in obtaining dynamic response, namely, solving the generalized eigenvalue problem for mode shapes and frequencies, solving a decoupled set of normal equations of motion, and using mode superposition to obtain time histories of the response.

a) Mode Shapes and Frequencies

The desired undamped free vibration mode shapes and corresponding frequencies can be obtained by solving the equation

$$\mathbf{K}\phi_i = \omega_i^2\mathbf{M}\phi_i \qquad (i=1, 2, \ldots, r);\ r<n \tag{2.24}$$

where ω_i and ϕ_i are the frequency and shape vector, respectively, for the i-th mode and where r is the lowest number of modes required for the accuracy of solution desired. Usually, r is much less than n.

The modal matrix $\Phi \equiv \begin{bmatrix} \phi_1 & \phi_2 & \cdots\cdots & \phi_q \end{bmatrix}$ must satisfy the orthogonality condition

$$\Phi^T\mathbf{K}\Phi = \Omega^2 \tag{2.25}$$

where Ω^2 is a diagonal matrix containing the squared frequencies ω_1^2, ω_2^2, …, ω_r^2. It is convenient to normalize the modal matrix so that it satisfies the condition

$$\Phi^T \mathbf{M} \Phi = \mathbf{I} \tag{2.26}$$

where $\mathbf{I}$ is the unit matrix.

Vectors $\mathbf{u}(t)$, $\dot{\mathbf{u}}(t)$ and $\ddot{\mathbf{u}}(t)$ can be expressed in terms of the modal matrix and the normal coordinate vector $\mathbf{q}(t)$ as

$$\mathbf{u} = \Phi \mathbf{q}; \quad \dot{\mathbf{u}} = \Phi \dot{\mathbf{q}}; \quad \ddot{\mathbf{u}} = \Phi \ddot{\mathbf{q}} \tag{2.27}$$

Substituting Eq. (2.26) into Eq. (2.23), premultiplying the resulting equation by Φ^T, and making use of Eqs. (2.24) and (2.25), one obtains

$$\ddot{\mathbf{q}} + \Lambda \dot{\mathbf{q}} + \Omega^2 \mathbf{q} = \mathbf{R}^*(t) \tag{2.28}$$

where Λ is a diagonal matrix containing the terms $2\xi_i \omega_i$ for i=1,2,…,r and where $\mathbf{R}^*(t)$ is a normal local vector defined by

$$\mathbf{R}^*(t) = \Phi^T \left[\mathbf{R}(t) + \mathbf{MB}\ddot{\mathbf{u}}_g(t) \right] \tag{2.29}$$

b) Response Time History

The solution of Eq. (2.28) can be carried out in the time domain using the convolution integration as

$$q_i(t) = \frac{1}{\omega_{Di}} \int_0^t R_i^*(\tau) e^{-\xi_i \omega_i (t-\tau)} \sin \omega_{Di}(t-\tau) d\tau \qquad \text{(i=1, 2,…, r)} \tag{2.30}$$

where ω_{Di} is the damped frequency and is given as $\omega_{Di} = \omega_i (1-\xi_i)^{1/2}$.

Thus, the response time histories $\mathbf{u}(t)$, $\dot{\mathbf{u}}(t)$ and $\ddot{\mathbf{u}}(t)$ can now be obtained using Eq. (2.27).

c) Spectral Response

When $\mathbf{R}(t) = \mathbf{0}$ in Eq. (2.29), the maximum response of $q_i(t)$ can be obtained as

$$|q_i(t)|_{\max} = \phi_i^T \mathbf{MB} \left[S_{DX}(T_i, h_i) \quad S_{DY}(T_i, h_i) \quad S_{DZ}(T_i, h_i) \right] \qquad \text{(i=1, 2, …, r)} \tag{2.31}$$

where $S_{DK}(T_i, h_i)$ (K=X,Y and Z) is the response displacement response spectra for the earthquake ground accelerations in X, Y and Z components, and where T_i is the natural period as given by $T_i = 2\pi / \omega_i$.

The maximum displacement response $|u_i(t)|_{\max}$ can then be determined by

$$|u_i(t)|_{\max} = \phi_i |q_i(t)|_{\max} \qquad \text{(i=1, 2, …, r)} \tag{2.32}$$

Consequently, the maximum displacement can be estimated by the square root of the sum of the squares (SRSS) of the individual modal response as

$$|\mathbf{u}(t)|_{SRSS} = \left\{ \sum_{i=1}^{r} |u_i(t)|_{\max}^2 \right\}^{1/2} \tag{2.33}$$

The SRSS method is generally applicable to most bridges. However, there are some bridges with unusual geometric features which cause some of the individual modes to have closely spaced frequencies, and this method may not be applicable for such bridges.

3) Single Mode Spectral Analysis

When the first mode is predominant compared to other higher modes, Eq. (2.33) can be approximated as

$$|\mathbf{u}(t)|_{\max} \approx \phi_1 |q_1(t)|_{\max} \tag{2.34}$$

Eq. (2.34) provides good approximation for most bridges with standard geometric features. Based on this characteristic, a *single mode spectral analysis method* (AASHTO, [1]) is often used to calculate the seismic design forces for bridges that respond predominantly in the first mode of vibration. The method, although completely rigorous from a structural dynamics point of view, reduces to a problem in statics after the introduction of inertia forces.

The equation of motion for a continuous bridge system is conveniently formulated using energy principles. The principle of virtual displacement may be used to formulate a generalized parameter model of the continuous bridge system in a manner which approximates the overall behavior of the system. To obtain an approximation to the mode shape, a uniform static loading p_0 is applied to the superstructure and the resulting deflection $v_s(x)$ is obtained. The dynamic deflection $v(x,t)$ of the superstructure under seismic excitation as shown in Fig. 2.5 is then approximated by the shape function multiplied by a generalized amplitude function $v(t)$ as

$$v(x,t) = v_s(x)v(t) \tag{35}$$

The strain energy stored internally by the uniformly applied loading in deforming the elastic structure $U_{\max}$ is

$$U_{\max} = \frac{p_0}{2} \int_0^L v_s(x)dx = \frac{p_0}{2}\alpha \tag{2.36}$$

where,

$$\alpha \equiv \int_0^L v_s(x)dx \tag{2.37}$$

The maximum kinetic energy of the system is given by

$$T_{\max} = \frac{\omega^2}{2g} \int_0^L w(x) v_s(x)^2 \, dx = \frac{\omega^2 \gamma}{2g} \tag{2.38}$$

where,

$$\gamma \equiv \int_0^L w(x) v_s(x)^2 \, dx \tag{2.39}$$

If the uniform loading p_0 is removed and the effects of damping are neglected, the structure will vibrate in the assumed mode shape at a natural frequency determined by equating maximum kinematic energy to maximum strain energy (Rayleigh method); i.e.,

$$T_{\max} = U_{\max} \tag{2.40}$$

Substituting Eqs. (2.36) and (2.38) into Eq. 2.40), one obtains

$$T = 2\pi \sqrt{\gamma / (p_0 g \alpha)} \tag{2.41}$$

The generalized equation of motion for the single degree-of-freedom system subjected to a ground acceleration $\ddot{v}_g(t)$ can be written

$$\ddot{v}(t) + 2\xi\omega\dot{v}(t) + \omega^2 v(t) = -\frac{\beta \ddot{v}_g(t)}{\gamma} \tag{2.42}$$

where

$$\beta \equiv \int_0^L w(x) v_s(x) dx \tag{2.43}$$

and ξ is the damping ratio.

Representing the displacement response spectra for $\ddot{v}_g(t)$ as $S_D(T,h)$, the maximum response of Eq. (2.42) may be written as

$$|v(t)|_{\max} = S_D(T,\xi) \frac{\beta}{\gamma} \tag{2.44}$$

Denoting $S_D(T,\xi) \approx (T/2\pi)^2 S_A(T,\xi)$ and using the standard acceleration response spectral value C_s in its dimensionless form,

$$C_s = S_A(T,\xi)/g \tag{2.45}$$

The maximum response of the bridge system is obtained as

$$|v(x,t)|_{\max} \approx \frac{C_s g \beta T^2}{4\pi^2 \lambda} \tag{2.46}$$

4) Nonlinear Dynamic Response Analysis

When the structural system is nonlinear, the coupled equations of motion, Eq. (2.9), must be solved using step-by-step integration method. Considering a time interval Δt starting at time t and assuming that the stiffness and damping matrices at time t, i.e., $\mathbf{K}_t$ and $\mathbf{C}_t$, can be applied over the full time interval, one obtains the equations of motion in the incremental form

$$\mathbf{M}\Delta\ddot{\mathbf{u}}(t) + \mathbf{C}_t\Delta\dot{\mathbf{u}}(t) + \mathbf{K}_t\Delta\mathbf{u}(t) = \Delta\mathbf{R}(t) + \mathbf{MB}\Delta\ddot{\mathbf{u}}_g(t) \tag{2.47}$$

where

$$\begin{aligned} \Delta\ddot{\mathbf{u}}(t) &= \ddot{\mathbf{u}}(t+\Delta t) - \ddot{\mathbf{u}}(t) \\ \Delta\dot{\mathbf{u}}(t) &= \dot{\mathbf{u}}(t+\Delta t) - \dot{\mathbf{u}}(t) \\ \Delta\mathbf{u}(t) &= \mathbf{u}(t+\Delta t) - \mathbf{u}(t) \end{aligned} \tag{2.48}$$

and where

$$\begin{aligned} \Delta\mathbf{R}(t) &= \mathbf{R}(t+\Delta t) - \mathbf{R}(t) \\ \Delta\ddot{\mathbf{u}}_g(t) &= \ddot{\mathbf{u}}_g(t+\Delta t) - \ddot{\mathbf{u}}_g(t) \end{aligned} \tag{2.49}$$

In the Newmark generalized acceleration method (Newmark [26]), the following approximations for nodal velocities and displacements are assumed,

$$\begin{aligned} \dot{\mathbf{u}}(t+\Delta t) &= \dot{\mathbf{u}}(t) + \left[(1-\delta)\ddot{\mathbf{u}}(t) + \delta\ddot{\mathbf{u}}(t+\Delta t)\right] \\ \mathbf{u}(t+\Delta t) &= \mathbf{u}(t) + \dot{\mathbf{u}}(t)\Delta t + \left[(1/2-\sigma)\ddot{\mathbf{u}}(t) + \sigma\ddot{\mathbf{u}}(t+\Delta t)\right] \end{aligned} \tag{2.50}$$

where parameters δ and δ can be chosen to give the required integration stability and accuracy. When $\delta = 1/2$ and $\sigma = 1/6$, the approximations correspond to the linear acceleration method, and when $\delta = 1/2$ and $\sigma = 1/4$ they correspond to the constant acceleration method. While the linear acceleration method is conditionally stable depending on the magnitude of Δt, the constant acceleration method is unconditionally stable for any magnitude of Δt (Bathe and Wilson [5]).

The approximation given by Eq. (2.49) can be expressed in the incremental form

$$\begin{aligned} \Delta\ddot{\mathbf{u}}(t) &= C_1\Delta\mathbf{u}(t) - C_3\dot{\mathbf{u}}(t) - C_4\ddot{\mathbf{u}}(t) \\ \Delta\dot{\mathbf{u}}(t) &= C_2\Delta\mathbf{u}(t) - C_4\dot{\mathbf{u}}(t) - C_5\ddot{\mathbf{u}}(t) \end{aligned} \tag{2.51}$$

where $C_1 = 4/\Delta t^2$, $C_2 = 2/\Delta t$, $C_3 = 4/\Delta t$, $C_4 = 2$ and $C_5 = 0$ for the constant acceleration method and where $C_1 = 6/\Delta t^2$, $C_2 = 3/\Delta t$, $C_3 = 6/\Delta t$, $C_4 = 3$ and $C_5 = \Delta t/2$ for the linear acceleration method.

Substituting Eq. (2.51) into Eq. (2.47), one obtains

$$\widetilde{\mathbf{K}}_t\Delta\mathbf{u}(t) = \Delta\widetilde{\mathbf{R}}(t) \tag{2.52}$$

where

$$\widetilde{\mathbf{K}}_t = C_1\mathbf{M} + C_2\mathbf{C}_t + \mathbf{K}_t \tag{2.53}$$

$$\Delta\widetilde{\mathbf{R}}(t) = \Delta\mathbf{R}(t) + \mathbf{MB}\Delta\ddot{\mathbf{u}}_g(t) + \left\{C_3\mathbf{M} + C_4\mathbf{C}_t\right\}\dot{\mathbf{u}}(t) + \left\{C_4\mathbf{M} + C_5\mathbf{C}_t\right\}\ddot{\mathbf{u}}(t) \tag{2.54}$$

Eq. (2.52) can be solved for $\Delta\mathbf{u}(t)$, and Eq. (2.51) can be used to obtain $\Delta\dot{\mathbf{u}}(t)$ and $\Delta\ddot{\mathbf{u}}(t)$. The displacements, velocities and accelerations at time $t+\Delta t$ can be obtained by Eq. (2.48). The displacements $\mathbf{u}(t+\Delta t)$ can be used to calculate the internal force vectors and the new tangent stiffness matrix $\mathbf{k}_{t+\Delta t}$ for each nonlinear element in the bridge system. The new total tangent stiffness matrix $\mathbf{K}_{t+\Delta t}$ is then obtained by the standard assemblage procedure.

5) Evaluation of Computed Solution

A measure of how well the dynamic equilibrium at time $t+\Delta t$ is being satisfied by the approximate solution of Eq. (2.47) may be expressed by the residual or unbalanced force $\delta\mathbf{R}_{t+\Delta t}$. The corrections of solution using Eq. (2.47) may then be given by comparing the ratio of the Euclidean norm of the residual forces and the external forces Δ_P with a specific tolerance Δ_{PS} using the relation

$$\Delta_P = \frac{\left\|\delta\mathbf{R}_{t+\Delta t}\right\|}{\left\|\mathbf{R}_{t+\Delta t}\right\| + \left\|\mathbf{R}_{t+\Delta t} - \delta\mathbf{R}_{t+\Delta t}\right\|} \le \Delta_{PS} \tag{2.62}$$

where

$$\delta\mathbf{R}_{t+\Delta t} = \mathbf{R}_{t+\Delta t} - \mathbf{M}\ddot{\mathbf{u}}_{t+\Delta t} - \mathbf{C}\dot{\mathbf{u}}_{t+\Delta t} - \mathbf{F}^S_{t+\Delta t} \tag{2.63}$$

and where $\mathbf{F}^S_{t+\Delta t}$ is the restoring force at $t+\Delta t$ of the total bridge system. When the accuracy of solution is unsatisfactory, it may be improved by using smaller time intervals or by applying an equilibrium correction through an iteration process.

The equations of motion for i-th equilibrium iteration at time t are expressed as

$$\mathbf{M}\delta\ddot{\mathbf{u}}_t^{(i)} + \mathbf{C}\delta\dot{\mathbf{u}}_t^{(i)} + \mathbf{K}_t^{(i)}\delta\mathbf{u}_t^{(i)} = \delta\mathbf{R}_t^{(i)} \tag{2.64}$$

where $\mathbf{M}$ and $\mathbf{C}$ = the constant mass and damping matrices, $\mathbf{K}_t^{(i)}$= tangential stiffness matrix at time t for i-th iteration, $\delta\ddot{\mathbf{u}}_t^{(i)}$, $\delta\dot{\mathbf{u}}_t^{(i)}$ and $\delta\mathbf{u}_t^{(i)}$ = corrective nodal accelerations, nodal velocities and nodal displacements for i-th iteration as defined by

$$\begin{aligned} \delta\ddot{\mathbf{u}}_t^{(i)} &= \ddot{\mathbf{u}}_t^{(i+1)} - \ddot{\mathbf{u}}_t^{(i)} \\ \delta\dot{\mathbf{u}}_t^{(i)} &= \dot{\mathbf{u}}_t^{(i+1)} - \dot{\mathbf{u}}_t^{(i)} \\ \delta\mathbf{u}_t^{(i)} &= \mathbf{u}_t^{(i+1)} - \mathbf{u}_t^{(i)} \end{aligned} \tag{2.65}$$

and $\delta\mathbf{R}_t^{(i)}$ representing the residual forces for the i-th iteration is given by

$$\delta\mathbf{R}_t^{(i)} = \mathbf{R}_t - \mathbf{M}\ddot{\mathbf{u}}_t^{(i)} - \mathbf{C}\dot{\mathbf{u}}_t^{(i)} - \mathbf{F}_t^{S(i)} \tag{2.66}$$

If convergence occurs, the iteration can be continued until the dynamic equilibrium of the motion is satisfied within the specific accuracy.