Engineering Characterization of Ground Motions 1.

1.1 Ground Motions

Fig. 1.1 shows accelerations measured at Sylmar Parking Lot in the 1994 Northridge, USA, Earthquake and Kobe Observatory of Japan Meteorological Agency (JMA Kobe Observatory) in the 1995 Kobe, Japan, Earthquake. Displacements computed by integrating the accelerations are also presented here. The peak accelerations and velocities at Sylmar are 8.3 m/sec², 0.32m and those at JMA Kobe Observatory are 8.2m/sec² and 0.21m, respectively. They were recorded near the faults. They include long period pulse accelerations. The durations are rather short; the durations with accelerations over 0.2g is 8 s and 5 s in the JMA Kobe record and the Sylmar Parking Lot record, respectively. They are typical examples of the destructive ground motions that resulted in the significant damage in the Northridge and Kobe Earthquakes.



(a) Japan Meteorological Agency Kobe Observatory

Ground Motions in the 1994 Northridge, USA, Earthquake and the 1995 Kobe, Fig. 1.1 Japan, Earthquake

Ground motions are generated by a rupture of a fault. Ground motion characteristics depend on the location where they are recorded. For example, Fig. 1.2 shows the ground accelerations recorded at JR-Takatori Station in the 1995 Kobe Earthquake. JR-Takatori Station was about 10 km apart from the JMA Kobe Observatory. The intensity, the predominant periods and the duration of the JR-Takatori Station record are very much different with those of the JMA Kobe record.



Fig. 1.2 Ground Acceleration at JR-Takatori Station the 1995 Kobe Earthquake

Ground motions recoded at the same location are not the same if they are generated by different earthquakes. For example, the ground accelerations presented in Fig. 1.3 were recoded at the ground surface in the vicinity of Kaihoku Bridge. They were recorded in the 1978 Miyagi-ken-oki Earthquake (M=7.4) and a M6.7 event in 1978. Although they are somewhat similar, they have different intensiiesy and periods. This is because ground motions depends on the source motions generated by a fault dislocation, the propagating path and the amplification in the subsurface ground.



(b) An Earthquake with Earthquake Magnitude of 6.7

Fig. 1.3 Ground Motions at Nearby Ground of the Kaihoku Bridge

The recent earthquake-damage to bridges and other structures located within a few kilometers from fault ruptures clearly indicates the importance to consider the near-field ground motions. Fig. 1.4 shows typical near-field ground accelerations recoded at Sylmar parking lot (NS-component) in the 1994 Northridge, USA, Earthquake, JMA Kobe Observatory in the 1995 Kobe, Japan, Earthquake, Shikhkang (TCU068) in the 1999 Chi-Chi, Taiwan, Earthquake, Bolu (L-component) and Duzce (L-component) in the Duzce, Turkey, Earthquake. The intensities of accelerations are very high, and they are characterized by single pulses with large accelerations and long predominant periods. Based on the response acceleration spectrum, the JMA Kobe record has the highest response acceleration at natural periods of 0.5-1 second. It is noted that the Shikhkang record has the higher response

accelerations at natural periods over 2 s. Such a record would develop extensive effect to structures with long natural periods.

As well as the strong intensity, the directivity of the near-field ground motion is important in seismic design. The pulses with large intensities are generally different in a direction parallel or perpendicular to the fault plane, and depend on the amount and distribution of slip developed on the fault rupture. This is important when the bilateral directional excitation is considered.





Fig.1.4 Comparison of Strong Motion Accelerations in the Duzce, Chi-Chi, Northridge, and Hyogo-ken Nanbu Earthquakes

Although the ground motion characteristics have been studied based on measured data, analytical simulations of the near-field ground motions have being developed. For example, Fig. 1.5 shows the predicted ground motion at Motoyama Elementary School that was located in the heavily damaged area in Kobe in the 1995 Kobe Earthquake (Kamae and Irikura 1998). They were synthesized from the after shock records based on the empirical Green's function. Measured records are presented in Fig. 1.5 for comparison. Quite good agreement is observed between the predicted and the measured ground motions. Since the empirical Green's function has a certain limitation that it can be applied only when appropriate records by small events are available in the source area, a new hybrid method for the evaluation of near-field ground motions was developed. The new method is successfully applied to predict the ground motions in Kobe in the 1995 Kobe Earthquake (Kamae et al. 1998b). Using this method, the near-field ground motions in the 1948 Fukui earthquake (M7.1), which developed destructive damage in Fukui City, are evaluated (Irikura and Kamae 1999). The predicted peak response velocity of 5% damping ratio reaches about 7 m/s at period of 1.7 s. It is noted that the ground motions in the Northridge and Kobe Earthquakes are not the strongest. It is important to recognize in seismic design of structures that there exists large uncertainty in the determination of design ground motions.



Fig. 1.5 Accuracy of Synthesized Ground Motion Estimated at Motoyama Elementary School, Kobe, in the 1995 Kobe, Japan, Earthquake [Kamae and Irikura 1998a]

1.2 Peak Ground Motions

Engineering characterization of ground motions is important in the determination of seismic design force. One of the most important engineering characteristics of ground motions is the peak values of acceleration, velocity and displacement. Fig. 1.6 shows how the peak ground accelerations attenuate from their epicenters. The rate of attenuations depend on the earthquake magnitude and the soil condition. A statistical analysis of ground motions for a data set consisting of 394 lateral and 107 vertical records provides valuable information for the attenuation (Kawashima, Aizawa and Takahashi 1984, Kawashima, Aizawa and Takahashi 1985, Kawashima and Aizawa 1986). They were recorded at 67 free field sites, and any data recorded at first floor and downstairs of buildings are excluded. Since the peak ground motions of horizontal components depend on the directions measured, the maximum peak ground motions on the horizontal plane are analyzed. They were generated by the earthquakes with the magnitude of 5.0 - 7.9 and the focal depth smaller than 60 km (shallow earthquakes). The soil conditions are classified into stiff (Group I), moderate (Group II) and soft (Group III), as shown in Table 1.1, in terms of the fundamental natural period of ground T_G (s) (JRA 2002). T_G is estimated from the shear wave velocity at small strain v_{si} and the thickness of ground h_i as

$$T_G = \sum_{i=1}^{n} \frac{4h_i}{V_{Si}}$$
(1.1)

in which subscript *i* denotes i-th soil layer, and *n* represents the total number of soil layer.



Fig. 1.6 Attenuation of Peak Ground Acceleration

Soil Conditions	Definition	Approximate Estimation
Group I	$T_G < 0.2s$	Tertiary or older
Group II	$0.2 \le T_G < 0.6s$	Alluvium and Diluvium
Group III	$0.6 \le T_{G}$	Soft Alluvium

Table 1.1 Classification of Ground Condition

Based on the analyses, the attenuations of peak ground accelerations at stiff sites (top), moderate sites (middle) and soft sites (bottom) are given as (Kawashima and Aizawa 1986)

$$a_{\max} = \begin{cases} 987.4 \times 10^{0.216M} \\ 232.5 \times 10^{0.313M} \\ 403.8 \times 10^{0.265M} \end{cases} \times (\Delta + 30)^{-1.218}$$
(1.2)

in which a_{max} is the peak ground acceleration in the horizontal component (cm/s²), *M* is the earthquake magnitude, and Δ is the epicentral distance (km). Fig. 1.7 shows the attenuation of the peak ground accelerations predicted by Eq. (1.2) for the earthquakes with magnitudes of 6 and 8.

In Eq. (1.2), the second and the third term represent the effect of earthquake magnitude and the attenuation with distance. For example, at the moderate sites, the second term implies that the peak acceleration at the same epicentral distance increases $10^{0.313} = 2.06$ times for a unit increase of earthquake magnitude. It is $10^{0.216} - 10^{0.313}$ (=1.64-2.06) in peak ground acceleration. The third term $(\Delta + 30)^{-1.218}$ represents the attenuation in terms of distance. At the moderate sites, it is 0.0048 and 0.0027 at $\Delta = 50$ km and 100 km, respectively. It implies that in an earthquake with the same magnitude the peak acceleration increases 0.0048/0.0027=1.8 times if the epicental distance decreases from 100 km to 50 km.

Using Eq. (1.2), the peak ground acceleration can be estimated for various combinations of an earthquake magnitude M and an epicentral distance Δ . Fig. 1.7 shows the attenuation of peak acceleration for earthquake magnitude M equal to 6 and 8. At the moderate sites, the peak ground acceleration a_{max} is 1.74 m/s and 3.57 m/s for an earthquake with magnitude of 7 and 8, respectively, at Δ =50km. As describe above, the peak acceleration increases 3.57/1.74=2.06 times for a unit increase of the earthquake magnitude.



Fig. 1.7 Attenuation of Peak Ground Acceleration for Earthquake Magnitude M = 6 and 8

The effect of soil condition on the peak ground acceleration is important. It sometimes affects damage degree of structures with short natural periods. For example, for a combination of M = 8 and $\Delta = 50$ km, a_{max} is 254, 357 and 256 cm/s², respectively. It is the highest at the moderate sites followed by soft and stiff sites.

An important point of Eq. (1.2) is the large scattering of measured values around the predicted value. If one defines a ratio of measured peak acceleration a_{max}^M and predicted peak accelerations a_{max}^P as

$$U_a = a_{\max}^M / a_{\max}^P \tag{1.3}$$

the $\log U_a$ distributes as shown in Fig. 1.8. It is idealized by the normal distribution. Consequently the probability p that $\log U_a$ does not exceed a certain value b is given as

$$p = 1 - \phi \left(\frac{b}{\sigma}\right) \tag{1.4}$$

in which ϕ is the normal distribution function and σ is the standard deviation of $\log U_a$. The standard deviation σ does not significantly depend on the earthquake magnitude and the epicentral distance, and it is 0.216, 0.224 and 0.197 at stiff, moderate and soft sites, respectively. The probability of not being exceeded p vs. $\log U_a$ relation is shown in Fig. 1.5 for σ =0.24. If one considers +/- one standard deviation from the mean values, the peak ground acceleration a_{max} by Eq. (1.2) must be multiplied by 1.7 and 1/1.7, respectively. For example, the peak ground acceleration a_{max} for a combination of M=8 and Δ =50 km is in the range of 357x1.7=607 cm/s and 357/1.7=210 cm/s when one standard deviation from the mean value is considered.



Fig. 1.8 Dependence of $\log U_a$ on Earthquake Magnitude M and Epicentral Distance



Fig. 1.9 Probability of Not Being Exceeded p vs. $\log U_a$

Similarly, the peak ground velocity and the peak ground displacement, v_{max} and d_{max} at stiff soil sites (top), moderate soil sites (middle) and soft soil sites (bottom) are obtained as

$$v_{\text{max}} = \begin{cases} 20.82 \times 10^{0.263M} \\ 2.805 \times 10^{0.430M} \\ 5.105 \times 10^{0.404M} \end{cases} \times (\Delta + 30)^{-1.222}$$
(1.5)

$$d_{\max} = \begin{cases} 0.626 \times 10^{0.372M} \\ 0.062 \times 10^{0.567M} \\ 0.070 \times 10^{0.584M} \end{cases} \times (\Delta + 30)^{-1.254}$$
(1.6)

The second term on the right hand side in Eqs. (1.5) and (1.6) is $10^{0.263} \cdot 10^{0.430}$ (=1.83-2.69) and $10^{0.372} \cdot 10^{0.584}$ (=2.36-3.84), respectively. Since it is 1.64-2.06 in the peak acceleration, the increasing rate of peak ground motion associated with a unit increase of earthquake magnitude becomes larger in the order of peak acceleration, peak velocity and peak displacement.



Figs. 1.10 and 1.11 show the attenuation of v_{max} and d_{max} for M = 6 and 8. Effect of the soil condition on the peak velocity and the peak displacement is significant. The peak velocity and displacement increase as the ground becomes softer. Under a combination of M = 8 and $\Delta = 50$ km, the peak ground velocity is in the range of 12.5 (stiff sites) – 41.2 cm/s (soft sites), while the peak ground displacement is in the range of 2.4 cm (stiff sites) – 13.5 cm (soft sites). Similar to the peak ground acceleration, the scattering around the mean peak velocity and displacement by Eqs. (1.5) and (1.6) is quite large. Defining ratios of the predicted peak values and the measured ones similar to Eq. (1.3), the standard deviations of $\log U_v$ and $\log U_d$ are 0.236 (stiff sites), 0.239 (moderate sites) and 0.243 (soft sites), and 0.262 (stiff sites), 0.258 (moderate sites) and 0.262 (soft sites), respectively. The provability of not being exceeded vs. $\log U$ relation presented in Fig. 1.9 can be used to consider the scattering of Eqs. (1.5) and (1.6).

The empirical attenuations for peak ground motions in vertical directions are evaluated as (Kawashima, Aizawa and Takahashi 1985)

$$a_{\max}^{V} = \begin{cases} 117.0 \times 10^{0.268M} \\ 88.2 \times 10^{0.297M} \\ 13.5 \times 10^{0.402M} \end{cases} \times (\Delta + 30)^{-1.190}$$
(1.7)

$$v_{\max}^{V} = \begin{cases} 1.02 \times 10^{0.311M} \\ 0.558 \times 10^{0.374M} \\ 0.0837 \times 10^{0.511M} \end{cases} \times (\Delta + 30)^{-0.968}$$
(1.8)

$$d_{\max}^{V} = \begin{cases} 0.0100 \times 10^{0.474M} \\ 0.0289 \times 10^{0.417M} \\ 0.00363 \times 10^{0.579M} \end{cases} \times (\Delta + 30)^{-0.879}$$
(1.9)

in which a_{max}^V , v_{max}^v and d_{max}^v are the peak ground acceleration (cm/s²), velocity (cm/s) and displacement (cm) in vertical direction, M is the earthquake magnitude, and Δ is the epicentral distance (km). The top, middle and bottom shows the attenuation at the stiff, moderate and soft soil sites, respectively.

A ratio of the horizontal and the vertical peak accelerations may be obtained from Eqs. (1.2) and (1.7) as

$$R_{a} = \frac{a_{\max}^{V}}{a_{\max}} = \begin{cases} 0.118 \times 10^{0.0523M} \\ 0.379 \times 10^{-0.0161M} \\ 0.00334 \times 10^{0.136M} \end{cases} \times (\Delta + 30)^{0.0282}$$
(1.10)

Fig. 1.12 shows R_a by Eq. (1.10). R_a is about 0.3-0.4, and it is less affected by the earthquake magnitude M and the epicentral distance Δ . However, it should be noted that this was derived from two empirical equations with larger scatterings. There exist several events that R_a was larger than 1.0, in particular at the near fault zone.



Fig. 1.12 Ratio of Horizontal and Vertical Peak Acceleration

1.3 Duration of Ground Accelerations

1.3 Duration of Ground Accelerations

How long a ground acceleration lasts is important in the evaluation of design ground motions a structure. Various means to define the duration of ground motion have been developed by many researchers. A direct estimation of ground acceleration based on two definitions is presented here. Time history of a ground acceleration may be characterized by Fig.1.13. An acceleration starts to increase to have a peak value of a_{max} (cm/s²) at t_{max} (s), and then decays to final zero value. The *bracketed duration* is defined as (Kawashima and Aizawa 1989)

$$t_a = t_{a1} - t_{a2} \tag{1.11}$$

in which t_{a1} and t_{a2} represent the time in second when the acceleration takes a given level of $a \text{ (cm/s}^2)$ in the first and in the last excursion.



Fig. 1.13 Bracketed Duration

Although Eq. (1.11) provides a direct evaluation of ground acceleration having an acceleration larger than a (cm/s²), number of records with high acceleration is limited at this moment. Consequently, a regression analysis provide only the estimation for t_{50} and t_{100} as

$$t_{50} = 0.0474 \times 10^{0.929M} \times (\Delta + 30)^{0.554} \tag{1.12}$$

$$t_{100} = 0.00452 \times 10^{0.93M} \times (\Delta + 30)^{0.469}$$
(1.13)

Fig. 1.14 shows t_{50} and t_{100} by Eqs.(1.12) and (1.13). Scattering is very large. Duration with acceleration over 50 cm/s² may be over 50 s for a ground acceleration induced by an earthquake with magnitude 8 at a distance of 50 km.



Fig. 1.14 Attenuation of Bracketed Duration

On the other hand, as shown in Fig. 1.15, one can define the duration with acceleration having larger than α times ($0 < \alpha < 1$) peak acceleration a_{max} as

$$T_{\alpha 1} = t_{\max} - t_{\alpha 1} \tag{1.14}$$

$$T_{\alpha 2} = t_{\alpha 2} - t_{\max} \tag{1.15}$$

$$T_{\alpha} = T_{\alpha 1} + T_{\alpha 2} \tag{1.16}$$

in which $t_{\alpha 1}$ and $t_{\alpha 2}$ represent the time when the ground acceleration takes a given level of α ($0 < \alpha < 1$), times peak ground acceleration a_{max} in the first and in the last excursion. They are called the *normalized durations*.



Fig. 1.15 Normarized Duration

A regression analysis for the 394 ground acceleration records provided T_{α} for α of 0.1, 0.2, ..., 0.9. For example, $T_{0.5}$ is given as

$$T_{0.5} = \begin{cases} 0.000443 \times 10^{0.292M} \times (\Delta + 30)^{1.041} \\ 0.00691 \times 10^{0.301M} \times (\Delta + 30)^{0.498} \\ 0.0149 \times 10^{0.207M} \times (\Delta + 30)^{0.691} \end{cases}$$
(1.17)

Fig. 1.16 shows how $T_{0.5}$ depends on the earthquake magnitude and the epicentral distance. $T_{0.5}$ is 15.7 s for M = 8 and $\Delta = 50$ km.



Fig. 1.16 Attenuation of Normalized Duration $T_{0.5}$

After determining the normalized duration T_{α} , one can estimate a shape of ground acceleration. By plotting the normalized duration for various α , a general shape of ground acceleration is obtained as shown in Fig. 1.17. The time t_{max} is taken as the origin of time axis..



Fig. 1.17 Shape of Ground Accelerations

From Figs. 1.14 and 1.17, one can evaluate the effect of earthquake magnitude and distance on the duration. Both bracketed and normalized durations increase as the earthquake magnitude increases.

1.4 Response Spectra

1) Horizontal Component

Response spectrum represents the peak response of a single degree of freedom oscillator with the natural periods T and damping ratios ξ . Acceleration response spectrum $S_A(T,\xi)$, velocity response spectrum $S_V(T,\xi)$ and displacement response spectrum $S_D(T,\xi)$ are generally used. There are following approximate relations among the three response spectra.

$$S_A(T,\xi) \approx \left(\frac{2\pi}{T}\right) \cdot S_V(T,\xi)$$
 (1.18)

$$S_D(T,\xi) \approx \left(\frac{T}{2\pi}\right) \cdot S_V(T,\xi)$$
 (1.19)

Fig. 1.18 shows the response spectra of $\xi = 0.05$ for the JMA Kobe record in the 1995 Kobe Earthquake (refer to Fig. 1.1(b)).

Based on a regression analysis for the 394 free-field ground accelerations, the attenuation of acceleration response spectrum with damping ratio of 0.05 of critical $S_A(T,0.05)$ (cm/s²) is provided as

$$S_A(T, 0.05) = a \times 10^{bM} \times (\Delta + 30)^{-1.178}$$
(1.20)

in which coefficients a and b are given in Table 1.2, and M and Δ are earthquake magnitude and epicentral distance (km), respectively.



Fig. 1.18 Response Spectra for Kobe Observatory of Japan Meteorological Agency in the 1995 Kobe Earthquake

Natural	Gro	oup I	Group II		Grou	p III
Period (s)	а	b	a	b	а	b
0.1	2420	0.211	848.0	0.262	1307	0.208
0.15	2407	0.216	629.1	0.288	948.2	0.238
0.2	1269	0.247	466.0	0.315	1128	0.228
0.3	574.8	0.273	266.8	0.345	1263	0.224
0.5	211.8	0.299	102.2	0.388	580.6	0.281
0.7	102.5	0.317	34.34	0.440	65.67	0.421
1	40.1	0.344	5.036	0.548	7.411	0.541
1.5	7.122	0.432	0.719	0.630	0.803	0.647
2	5.784	0.417	0.347	0.644	0.351	0.666
3	1.672	0.462	0.361	0.586	0.262	0.635

Table 1.2 Coefficients *a* and *b* in Eq. (1.20)

It is important to note in Eq.(1.20) that the coefficient b increases as the natural period T increases. It shows that the increasing rate of acceleration response spectra associated with a unit increase of earthquake magnitude is larger at longer natural period. If one determines the design ground motions based on ground motions by small magnitude events, this characteristic must be carefully taken into account.

Fig. 1.19 shows empirical acceleration response spectra for several combinations of an earthquake magnitude M and an epicentral distance Δ . Acceleration response depends on the soil condition and the natural periods. Acceleration response is much larger at the soft soil sites than the stiff soil sites at the natural periods longer than about 0.3 s. At an epicentral distance of 50 km, the acceleration response is 0.8 m/s^2 (stiff) – 3.5 m/s^2 (soft) for M = 7, while it is 2 m/s^2 (stiff) – 9 m/s^2 (soft) for M = 8. The effect of earthquake magnitude is significant.





Fig. 1.19 Acceleration Response Spectrum $S_A(T, 0.05)$

Similar to the peak ground motions, scattering of the acceleration response spectra around Eq. (1.20) is quite large. If one defines the ratio U_{SA} similar to Eq. (1.3), the standard deviation of $\log U_{SA}$ is 0.211-0.307 depending on the natural periods and the soil condition. Since the average of standard deviation of $\log U_{SA}$ for entire natural periods and the soil conditions is 0.26, Fig. 1.5 can be used to evaluate the scattering around Eq. (1.29).

Since Eq. (1.20) provides the acceleration response spectra for damping ratio $\xi = 0.05$, it is required to modify it when one wants to evaluate an acceleration response spectrum with damping ratio other than 0.05. For such purpose, a ratio $c_D(T,\xi)$ define as is often used.

$$c_D(T,\xi) = \frac{S_A(T,\xi)}{S_A(T,0.05)}$$
(1.21)

There are various expressions for $c_D(T,\xi)$. One of the relations which was derived from an analysis of 206 records (Kawashima and Aizawa 1986) is

$$c_D(T,\xi) = \left(\frac{1.5}{40\xi + 1} + 0.5\right) \times \left(\frac{S_A(T,0.05)}{a_{\max}}\right)^{\left(\frac{1}{300\xi + 6} - 0.8\xi\right)}$$
(1.22)

The second term on the right hand side represents the period dependency of c_D . $S_A(T,0.05)/a_{\text{max}}$ represents the amplification or de-amplification of response acceleration relative to the peak acceleration. Fig. 1.20 shows the first and the second terms in Eq. (1.22).

Since the second term is nearly 1.0 at damping ratio no larger than about 0.2, it may be dropped without introducing a serious error. In such a case, Eq. (1.22) can be simplified as

$$c_D(T,\xi) \approx c_D(\xi) = \frac{1}{40\xi + 1} + 0.5$$
 (1.23)



2) Vertical Components

Similar analysis for 107 vertical ground accelerations provides an attenuation of acceleration response spectrum for vertical component, S_A^V (cm/s²) as

$$S_A^V(T, 0.05) = a^V \times 10^{b^V M} \times (\Delta + 30)^{-1.015}$$
 (1.24)

in which coefficients a^V and b^V are given in Table 1.3.

Fig. 1.21 shows the vertical acceleration response spectra by Eq. (1.24) for various combinations of an earthquake magnitude and an epicentral distance.

Natural	Group I		Grou	p II	Group III	
Period (s)	а	b	а	b	а	b
0.1	246.4	0.230	224.8	0.232	114.5	0.251
0.15	207.2	0.235	168.4	0.255	107.8	0.258
0.2	124.1	0.263	105.5	0.288	155.7	0.232
0.3	95.72	0.267	31.92	0.351	171.1	0.228
0.5	31.86	0.304	10.44	0.410	13.82	0.388
0.7	5.869	0.388	4.039	0.451	1.939	0.509
1	2.185	0.428	1.386	0.495	0.352	0.596
1.5	0.441	0.506	0.758	0.486	0.0343	0.711
2	0.203	0.541	0.670	0.468	0.105	0.610
3	0.196	0.497	0.389	0.462	0.0886	0.584

Table 1.3 Coefficients a^V and b^V in Eq. (1.24)



Fig. 1.21 Accuracy of Synthesized Ground Motion Estimated at Motoyama Elementary School, Kobe, in the 1995 Kobe, Japan, Earthquake [Kamae and Irikura 1998a]

1.5 Acceleration Response Spectrum Taking Number of Response Cycle into Account

The acceleration response spectrum provides a peak acceleration response that occurs only once in an oscillator during an excitation. Although the peak response is usually important in the evaluation of seismic performance of structures, response accelerations that occur several times in a structure is important than the peak response acceleration depending on structural members. For example, the rupture of reinforcements in a reinforced concrete column resulting from low cycle fatigue depends on the number of inelastic load reversal. Strength of sandy soils for liquefaction depends on number of load reversal.

For clarifying the dependence of structural response on number of load reversals, the *acceleration response spectra taking account of number of response cycles* was developed (Kawashima and Aizawa 1986). When a linear SDOF oscillator with a natural period T and a damping ratio ξ is subjected to an earthquake ground motion, the acceleration response of the oscillator generally has a time history as shown in Fig. 1.22. One can count the peak acceleration amplitude associated with each response cycle from the largest to n_{max} -th largest nalues. One response cycle is defined as the interval of response between two adjacent zero crossing points.



(b) Acceleration Response

Fig. 1.22 Definition of Acceleration Response Spectrum Taking Account of Number of Response Reverse (EESD)

Consequently the number of response cycles n takes a number of 1 (the largest peak amplitude) to n_{max} (the smallest peak amplitude), where n_{max} depends on each record. The n-th largest peak amplitude represents the response acceleration over which amplitude an oscillator will experience n times response excursions. It is called as the *response spectra* taking account of number of response cycles $S_A(T,\xi,n)$. $S_A(T,\xi)$ is a special case of $S_A(T,\xi,n)$ as

$$S_A(T,\xi) = S_A(T,\xi,l)$$
 (1.25)

Because $S_A(T,\xi)$ may be estimated by an appropriate attenuation equation such as Eq. (1.20), a reduction factor $\eta(T,\xi,n)$ is defined as

$$\eta(T,\xi,n) = \frac{S_A(T,\xi,n)}{S_A(T,\xi)}$$
(1.26)

Based on an analysis for the 394 ground accelerations, $\eta(T,\xi,n)$ is given as

$$\mu(T,\xi,n) = \frac{1}{1 + e(T,\xi)(n-1)}$$
(1.27)

where,

$$e(T,\xi) = \frac{80\xi}{60\xi + 1} \times 0.0815 \times T^{0.349}$$
(1.28)

Fig. 1.23 shows $S_A(T,0.05,n)$ and $\eta(T,0.05,n)$ computed for the acceleration at Sylmer Parking Lot (Shakal et al 1994) in the 1994 Northridge, USA, Earthquake. If one assumes n =10, the reduction at T=1.0s is approximately 0.2. Consequently, the amplitude of response acceleration over which 10 acceleration reversals occur is only 20% of the peak acceleration.



Fig. 1.23 $S_A(T,0.05,n)$ and $\eta(T,0.05,n)$ computed for the acceleration at Sylmer Parking Lot (Shakal et al 1994) in the 1994 Northridge, USA, Earthquake

1.6 Force Reduction Factor Resulting from Nonlinear Response

In the force based seismic design, it is usual to estimate the demand from a linear response of a structure by dividing it by the force reduction factor. The force reduction factor or response modification factor, which is often called q-factor or R-factor, has an important role in the estimation of design force of a structure.

In a structure idealized by a single-degree-of-freedom (SDOF) oscillator with a nonlinear hysteretic behavior as shown in Fig. 1.24, the force reduction factor R_{μ} is defined as

$$R_{\mu}(T,\mu_{T},\xi_{EL},\xi_{NL}) = \frac{F_{R}^{EL}(T,\xi_{EL})}{F_{Y}^{NL}(T,\mu_{T},\xi_{NL})}$$
(1.29)

in which T: natural period, F_R^{EL} and F_Y^{NL} : the maximum restoring forces in an oscillator with a linear and a nonlinear hysteresis, respectively, μ_T : target ductility factors, and ξ_{EL} and ξ_{NL} : damping ratios assumed in the evaluation of the linear and nonlinear responses,

respectively. The natural period T is usually evaluated based on the initial stiffness of an oscillator. Representing u_y the yield displacement where the stiffness changes from the elastic stiffness k_e to the post-yield stiffness k_{py} , a target ductility factor μ_T is defined based on the yielding displacement u_y as

$$\mu_T = \frac{u_{\max T}}{u_y} \tag{1.30}$$

in which $u_{\max T}$ is a target maximum displacement of an oscillator.



Fig. 1.24 Definition of Force Reduction Factor

Since the damping controls structural response, it is important to carefully assume the damping ratios in the evaluation of the force reduction factor. A structure under a strong excitation generally exhibits strong hysteretic behavior, which results in an energy dissipation in a structure. For example, the inelastic flexural deformation of columns dissipate the energy in a bridge.

If one idealizes the energy dissipation in nonlinear structural components by incorporating nonlinear elements, the energy dissipation in the nonlinear structural components is automatically included in the analysis. On the other hand, if one idealizes the nonlinear structural components by elastic linear elements, the energy that is dissipated in the nonlinear structural components has to be included in the analysis by other means. The equivalent viscous damping ratio ξ_h defined as

$$\xi_h = \frac{1}{4\pi} \cdot \frac{\Delta W}{W} \tag{1.31}$$

is generally used for such a purpose, in which ΔW and W represent the energy dissipation in a hysteretic excursion and the elastic energy, respectively. For example, in an oscillator with an elasto-plastic bilinear hysteresis, the equivalent damping ratio ξ_h is

$$\xi_h = \frac{2}{\pi} \cdot \frac{(1-r) \cdot (\mu - 1)}{(1 + r(\mu - 1)) \cdot \mu}$$
(1.32)

where, r is the post-yield stiffness ratio, and is defined as

$$r = \frac{k_e}{k_{py}} \tag{1.33}$$

Fig. 1.25 shows the equivalent damping ratio by Eq. (1.33). The equivalent damping ratio is generally large such as 0.3-0.4 at the target ductility factor of 3-5 at r=0.



Fig. 1.25 Equivalent Dmaping Ratio by Eq. (1.33)

In addition to such hysteretic energy dissipation, there are other sources of energy dissipation (for example, Bleich and Teller 1852, Kawashima, Unjoh, Tsunomoto 1993). The radiation of energy from a foundation to the surround ground results in energy dissipation (Richard, Hall and Wood 1970). Structural damping such as friction at connections may be important in structures (for example, Kawashima and Unjoh 1989). Viscous damping due to friction with air is generally predominant in a structure with a long natural period. It is general to idealize those sources of energy dissipation in terms of the equivalent viscous damping by Eq. (1.31).

If one considers a structure in which the flexural hysteretic energy dissipation is predominant with other sources of energy dissipation being a secondary importance, the total damping ratio ξ_{eq} of a SDOF oscillator is provided as

$$\xi_{eq} = \xi_h + \xi_{oth} \tag{1.34}$$

in which ξ_h is the damping ratio that accounts the hysteretic energy dissipation by Eq. (1.31), and ξ_{oth} is the damping ratio that accounts the energy dissipation other than the hysteretic energy dissipation.

In the evaluation of force reduction factor R_{μ} based on Eq. (1.29), if one assumes the damping ratios as

$$\xi_{EL} = \xi_{eq} \quad \text{and} \quad \xi_{NL} = \xi_{oth} \tag{1.35}$$

the energy dissipation is essentially the same between the linear and the nonlinear responses. Hence, the force reduction factor by Eq. (1.29) represents the difference of restoring forces between the linear and the nonlinear responses. In other word, Eq. (1.29) reflects the effect of nonlinear response of an oscillator.

On the other hand, if one assumes the damping ratios as

$$\xi_{EL} = \xi_{NL} = \xi_{eq} \tag{1.36}$$

$$\xi_{EL} = \xi_{NL} = \xi_{oth} \tag{1.37}$$

the energy dissipation in the linear and the nonlinear responses is not the same. Eq. (1.36) double counts the hysteretic energy dissipation of the nonlinear structural components in the nonlinear analysis, while Eq. (1.37) neglects the hysteretic energy dissipation of the nonlinear structural components in the linear response analysis. Hence, the force reduction factor by Eq. (1.29) includes the effect of different energy dissipation between the linear and the nonlinear responses, in addition to the effect of nonlinear response.

Based on the definition inherent to the force reduction factor, the damping ratios need to be assigned by Eq. (1.35). However there are many past researches in which the damping ratios are assumed as $\xi_{EL} = \xi_{NL}$. The effect of damping ratios will be described later.

Newmark and Hall (Newmark and Hall 1973) proposed the following force reduction factors based on 10 ground motions assuming $\xi_{EL} = \xi_{NL} = 0.05$ as

$$R_{\mu} = \begin{cases} 1 \cdots (0 \le T \le T_{1} / 10) \\ \sqrt{2\mu - 1} \left(\frac{T}{4T_{1}} \right)^{2.513 \log(1 / \sqrt{2\mu - 1})} \\ \cdots (T_{1} / 10 \le T \le T_{1} / 4) \\ \sqrt{2\mu - 1} \cdots (T_{1} / 4 \le T \le T_{1} ') \\ T\mu / T_{1} \cdots (T_{1} / 4 \le T \le T_{1}) \\ \mu \cdots (T_{1} - (T_{1} -$$

where,

$$T_{1} = 2\pi \frac{\phi_{ev}V}{\phi_{ea}A}$$

$$T_{1}' = T_{1} \frac{\mu}{\sqrt{2\mu - 1}}$$

$$T_{1} = 2\pi \frac{\phi_{ed}D}{\phi_{ev}V}$$
(1.39)

in which, A, V and D represent peak ground acceleration, velocity and displacement, respectively, and ϕ_{ea} , ϕ_{ev} and ϕ_{ed} represent the amplification for acceleration, velocity and displacement, respectively. Eq. (1.39) gave strong influence on the seismic design practice worldwide.

Nassar and Krawinkler analyzed the force reduction factor based on 15 ground motions (Nassar and Krawinkler 1991). They assumed $\xi_{EL} = \xi_{NL} = 0.05$, and proposed the following formulation.

$$R_{\mu} = \{c(\mu - 1) + 1\}^{1/c}$$
(1.40)

where

or,

$$c(T,\alpha) = \frac{T^r}{1+T^r} + \frac{b}{T}$$
(1.41)

in which r represents the bilinear factor defined by Eq. (1.33), and a and b are coefficients depending on α . Nassar and Krawinkler clarified the effect of stiffness deterioration. The coefficients a and b are given depending on r.

Miranda and Bertero proposed the force reduction factor, assuming $\xi_{EL} = \xi_{NL} = 0.05$, based on 124 ground motions (Miranda and Bertero 1994) as

$$R_{\mu} = \frac{\mu - 1}{\Phi(T, T_g)} + 1 > 1 \tag{1.42}$$

where,

$$\Phi = \begin{cases} 1 + \frac{1}{10T - \mu T} - \frac{1}{2T} \exp\left\{-\frac{3}{2}\left(\ln T - \frac{3}{5}\right)^2\right\} & \dots & (\text{rock}) \\ 1 + \frac{1}{12T - \mu T} - \frac{2}{5T} \exp\left\{-2\left(\ln T - \frac{1}{5}\right)^2\right\} & \dots & (\text{alluvium}) \\ 1 + \frac{T_g}{3T} - \frac{3T_g}{4T} \exp\left\{-3\left(\ln \frac{T}{T_g} - \frac{1}{4}\right)^2\right\} & \dots & (\text{soft}) \end{cases}$$
(1.43)

in which T_g represents a most predominant period.

Based on the studies, it is known that that the maximum displacement of a nonlinear oscillator is nearly the same to the maximum displacement of the liner oscillator (refer to Fig. 1.26 (a)). This is often called the *equal displacement assumption*. Another assumption that is also used is the *equal energy assumption* that assumes the maximum displacement of a nonlinear oscillator develops so that the strain energy of the nonlinear oscillators is nearly equal to the kinematic energy of the linear oscillator (refer to Fig. 1.26 (b)). Both are empirical relations. The force reduction factors for the equal displacement and the equal energy assumptions are given as

$$R = \begin{cases} \mu & \text{Equal Displacement} \\ \sqrt{2\mu - 1} & \text{Equal Energy} \end{cases}$$
(1.44)



It is generally known that the equal displacement assumption provides good estimation at mid and long natural period range, while the equal energy assumption provides good estimation at short natural period range.

Watanabe an Kawashima recently analyzed the force reductions based on 70 free field ground motions assuming two damping ratios; 1) $\xi_{EL} = 0.05$ and $\xi_{NL} = 0.02$, and 2) $\xi_{EL} = \xi_{NL} = 0.05$ (Watanabe and Kawashima 2002). To compare this study to the above previous studies, the force reduction factors assuming $\xi_{EL} = \xi_{NL} = 0.05$ is first presented here.

Fig. 1.27 shows the force reduction factors for μ_T =4 and 6 at the moderate soil sites. The definition of soil condition follows Table 1.1. Scattering of the force reduction factors depending on ground motions is considerable. For example at natural period of 1 second, the force reduction factors varies from 1.9 to 10.3 depending on ground motions for μ_T =4. Such a large scattering of the force reduction factors result in a large change of sizing of a structure in seismic design. It suggests that using the mean force reduction factor underestimates the design force of a structure with the probability of 50%. Treatment of the scattering will be discussed later.



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Fig. 1.27 Force Reduction Factors for 70 Ground Motions



Fig. 1.28 Idealization of Force Reduction Factors

Watanabe and Kawashima developed a new formulation for the force reduction factors as (refer to Fig. 1.28)

$$R_{\mu} = (\mu - 1) \cdot \Psi(T) + 1 \tag{1.45}$$

where,

$$\Psi(T) = \frac{T-a}{ae^{bT}} + 1 \tag{1.46}$$

in which a and b are parameters given in Table 1.4.

Table 1.4	Parameters	a	and	b	$(\xi_{NL} =$	$\xi_{EL} = 0.$.05)
-----------	------------	---	-----	---	---------------	-----------------	------

μ_T	a and	Soil Conditions			
	b	Type-I	Type-II	Type-III	
2	а	0.226	0.344	0.521	
	b	4.14	1.94	1.34	
4	а	0.778	0.572	0.976	
	b	3.50	1.35	0.994	
6	а	0.981	0.725	1.23	
	b	2.93	1.15	0.757	
8	а	1.23	0.807	1.28	
	b	2.57	0.983	0.569	

Eqs. (1.45) and (1.46) are featured by its easy interpretation of the parameters a and b. The parameter a and a+1/b represent the period where R_{μ} is equal to μ (Point P) and the period where R_{μ} takes the peak value. Fig. 1.29 shows the parameters a and a+1/b. Since $R_{\mu} = \mu$ at a, a represents the period where the equal displacement assumption by Eq. (1.44) provides the best estimation. Parameter a is in the range of 1.0-1.4 s at stiff and moderate sites, and 1.5-2.4 s at soft sites. They are less sensitive to the target ductility factor μ_T ranging from 2 to 8.



Fig. 1.29 Parameters a and a+1/b

On the other hand, a+1/b represents the natural period where R_{μ} takes the peak value. It is 1.5-2 s at the stiff and the moderate sites, and 2.5-3.5 s at the soft sites.

Fig. 1.30 compares the force reduction factors among Nassar and Krawinkler by Eq. (1.40), Miranda and Bertero by Eq. (1.42) and Watanabe and Kawashima by Eq. (1.45). The mean force reduction factors computed based on the 70 ground motions are also presented for comparison. Note that the definition of soil conditions is not necessarily the same among three studies. In the Miranda and Bertero formulation, T_g was assumed 1.5 s at soft (alluvial) site in Eq. (1.43). The three formulations provide quire close force reduction factors.





Fig. 1.30 Comparison of Force Reduction Factors

In the above estimation, the damping ratios were assumed as $\xi_{NL} = \xi_{EL} = 0.05$, and only the mean values of force reduction factors were used. However, as described previously, the damping ratios in the linear and nonlinear responses have to satisfy Eq. (1.35). Since damping ratio of 0.05 is often used in nonlinear analyses, it is assumed here that ξ_{NL} is 0.02 and ξ_{EL} is 0.05. In the following, the effect of damping ratios and the scattering of the force reduction factors are presented.

When one assumes $\xi_{NL} = 0.02$ and $\xi_{EL} = 0.05$, the parameters *a* and *b* in Eq. (1.46) become as shown in Table 1.5. Fig. 1.31 compares the mean force reduction factors obtained under these damping ratios with those obtained under $\xi_{NL} = \xi_{EL} = 0.05$. The combination of $\xi_{EL} = 0.05$ and $\xi_{NL} = 0.02$ provides smaller force reduction factors than the combination of $\xi_{EL} = \xi_{NL} = 0.05$. It is important to careful evaluate the damping ratios in the linear band nonlinear analyses.

η_T	<i>a</i> , <i>b</i>	Soil Conditions				
	and R	Type-I	Type-II	Type-III		
	а	1.29	1.12	2.35		
2	b	2.77	2.18	1.69		
	R	0.379	0.701	0.851		
	а	1.24	0.989	1.52		
4	b	2.39	1.62	1.05		
	R	0.673	0.842	0.886		
	а	1.34	1.03	1.85		
6	b	2.15	1.24	0.821		
	R	0.717	0.869	0.878		
	а	1.36	1.20	1.74		
8	b	1.67	1.11	0.611		
	R	0.776	0.899	0.895		

Table 1.5 Parameters *a* and *b* and Regression Coefficients *R* (ξ_{NL} =0.02 and ξ_{EL} =0.05)



Fig. 1.31 Effect of Damping Ratios on the Force Reduction Factors

Since the scattering of force reduction factors around the mean values are extensive, it is not sufficient to evaluate only the mean values. One way to consider the significant scattering around the mean values may be to include a certain level of redundancy in the evaluation of the force reduction factors. The standard deviations of the force reduction factors $\sigma(R_{\mu})$ are

$$\sigma(R_{\mu}) \approx -0.3 + 0.4\mu_T \tag{1.47}$$

The force reduction factors corresponding to the mean values m substituted by a standard deviation $\sigma(R_{\mu})$ are evaluated for $\mu = 4$ and 8 as shown in Fig. 1.32. The force reduction factors estimated by the equal displacement and equal energy assumptions by Eq. (1.44) are presented in Fig. 1.32 for comparison. Although it has been pointed out that the equal displacement assumption provides good estimation to the force reduction factors, this is valid for the mean values. However the equal displacement assumption considerably underestimates the force reduction factors corresponding to the mean minus one standard deviation.





Fig. 1.32 Force Reduction Factors corresponding to Means minus One Standard Deviations for $\mu_T = 4$ and $\mu_T = 8$

On the other hand, the equal energy assumption provides a better estimation to the force reduction factors corresponding to the mean minus one standard deviation, although it provides too conservative estimation to the mean values. Taking account of the considerable scattering of the force reduction factors depending on ground motions, it is more conservative to assume the equal energy assumption instead of the equal displacement assumption.

1.7 Relative Displacement Response Spectrum

In the past earthquakes, many bridges experienced unseating of decks from their piers and abutments. To prevent the unseating, one of the approaches is to provide an adequate seat width on a pier to support a deck. The key parameter to determine the seat width is the maximum relative displacement developed between two adjacent bridge segments. If the provided seat width is longer than the maximum relative displacement developed during a strong excitation, the deck will not unseat from the piers.

Kawashima and Sato proposed a method to predict the maximum relative displacement between two structural segments with different natural periods (Kawashima and Sato 1996).

To analyze the relative displacements between two segments, a simplified model as shown in Fig. 1.33 is used. Two bridge segments connected at a joint are idealized by two linear SDOF systems. Two systems are subjected to the same ground motion \ddot{u}_g , neglecting the spatial variation of ground motions along the longitudinal direction of bridge axis. The natural periods of systems 1 and 2 are T_1 and T_2 , respectively. The damping ratios of systems 1 and 2 are ξ_1 and ξ_2 , respectively. Since most standard bridges have the damping ratios of about 0.05, it is assumed here that $\xi_1 = \xi_2 = 0.05$. For generality, the procedure presented here can be extended to any combinations of ξ_1 and ξ_2 . The lumped mass of system 1 is m_1 and that of system 2 is m_2 . The mass ratio r_M is defined as

$$r_M = \frac{m_2}{m_1}$$
(1.49)

By representing the gap between two segments Δ_G , one can define the gap ratio r_G as

$$r_G = \frac{\Delta_G}{\max\{u_1(t) - u_2(t)\}}$$
(1.50)



Fig. 1.33 Definition of Relative Displacement Response Spectrum

If the gap ratio r_G is large than 1.0, pounding will not occur between two segments. On the other hand, if the gap ratio r_G is smaller than or equal to 1.0, pounding will occurred between two segments. The effect of pounding on the relative displacement between two segments will be shown later.

Plot of the maximum relative displacement, max $(u_1(t) - u_2(t))$, for a ground motion, as a parameter of T_1 and T_2 is called the *relative displacement response spectrum* $\Delta S_D(T_1T_2)$.

Fig. 1.34 (a) shows an example of the displacement responses and relative displacement for the JMA Kobe record (refer to Fig. 1.1 (a)) under the conditions of $T_1 = 0.8$ s, $T_2 = 2$ s, and $r_M = 1$. The maximum relative displacement is 0.41 m and 0.46 m in the separating and closing directions, respectively. Computing the maximum relative displacement for various combinations of T_1 and T_2 , one obtains the relative displacement response spectrum $\Delta S_D(T_1,T_2)$ for the JMA Kobe record as shown in Fig. 1.35. The relative displacement does not occur at $T_1 = T_2$. It increases as the difference of natural period $\Delta T \equiv T_2 - T_1$ increases. The relative displacement is almost symmetrical around the line of $T_1 = T_2$. The relative displacement is equal to 0.65 m at $T_1 = 3$ s and $T_2 = 1.5$ s.



Fig. 1.34 Responses for the JMA Kobe ground motion, $T_1 = 0.8$ s, $T_2 = 2$ s, $\xi_1 = \xi_2 = 0.05$, and $r_M = 1$



Fig. 1.35 Relative Displacement Response Spectrum for the JMA Kobe record, $\xi_1 = \xi_2 = 0.05$, and $r_M = 1$

Since the displacement response spectrum $S_D(T,\xi)$ may be estimated by an attenuation equation of displacement response spectrum, the ratio of ΔS_D to S_D is defined as

$$r_D = \frac{\Delta S_D(T_1, T_2)}{S_D(T_1)}$$
(1.50)

in which r_D is called *relative displacement ratio*. Fig. 1.36 shows the relative displacement ratio r_D vs. normalized natural period difference $\Delta T/T_1(=T_2/T_1-1)$ relation of the JMA Kobe record. At $\Delta T/T_1=0$, r_D is equal to 0, and r_D increases as $\Delta T/T_1$ increases or decreases. If one assumes that T_1 is longer than T_2 , $S_D(T_1)$ is generally larger than $S_D(T_2)$. Hence, Eq. (1.50) represents the amplification or deamplification of the relative displacement compared to the larger value of relative displacement between two segments. Thus, the negative region of $\Delta T/T_1$ is important in Fig. 1.36 to estimate how large the relative displacement is. Noted that r_D at the positive $\Delta T/T_1$ is almost the same with r_D at the positive $\Delta T/T_1$ if r_D is defined $r_D = \Delta S(T_1,T_2)/S_D(T_2)$ in Eq. (1.50). Depending on natural periods, r_D is in the range of 1-2 at $\Delta T/T_1 < 0$.



Fig. 1.36 Relative Displacement Ratio r_D for the JMA Kobe record, $\xi_1 = \xi_2 = 0.05$, and $r_M = 1$

The relative displacement ratio r_D was evaluated for 63 free field records. Fig. 1.37 shows the means and means +/- one standard deviations for 63 records. Since the r_D vs. $\Delta T/T_1$ relation does not significantly depend on the natural periods and the soil conditions, they are averaged for the entire records and the natural periods. At $\Delta T/T_1 < 0$, r_D starts to increases as $\Delta T/T_1$ increases, and takes the maximum value of 1.4 at $\Delta T/T_1=0.2-0.4$.



Fig. 1.37 Means and Means +/- One Standard Deviations of Relative Displacement Ratio r_D for 63 Records

Using Fig. 1.37, one can estimate the relative displacement response spectrum $\Delta S_D(T_1,T_2)$ from Eq. (1.50) for any combinations of T_1 and T_2 .

When the gap ratio $r_G \leq 1.0$, pounding will occur between two segments. Fig. 1.34 (b) shows the relative displacement and velocity responses for two masses when pounding occurs $(r_M = 1.0)$. The gap Δ_G was set equal to 0.23m, corresponding to the gap ratio r_G of 0.5. Due to the pounding effect, the relative displacement in the closing direction is limited to 0.23 m. Pounding occurs twice at about 6 s. Since the natural period of System 1 is much shorter than that of System 2, after the first pounding System 1 rebounds and returns to collide with System 2 again. Comparing the relative displacement without pounding and that with pounding, it is obvious that the pounding causes the increase in the relative displacement in the separating direction is equal to 0.85 m, which is 2.1 times larger than that without pounding. The exchange of velocity occurs at the moment of pounding, and this results in the increase of the relative displacement.

Similar to the relative displacement response spectrum, the plot of max $(u_1(t) - u_2(t))$ that is computed taking the pounding into account for a ground motion as a parameter of T_1 and T_2 , ξ_1 and ξ_2 , r_M and r_G is called the *relative displacement response spectrum with pounding effect* $\Delta S_D^P(T_1, T_2)$. Fig. 1.38 shows an example of the relative displacement response spectrum with pounding effect for the JMA Kobe record for $r_G=0.5$ and $r_M=1.0$. Similar to the relative displacement response spectrum presented in Fig. 1.37, the relative displacement with pounding effect does not occur at $T_1=T_2$, and it is almost symmetrical along the line of $T_1=T_2$. The relative displacement with pounding effect ΔS_D^P increase as the natural period difference ΔT increases, and it takes the maximum value of 1.15 m at $T_1=3$ s and $T_2=1$ s. This is much larger than the relative displacement without pounding ΔS_D presented in Fig. 1.37.



Fig. 1.38 Relative Displacement Response Spectrum with Pounding Effect ΔS_D^P of the JMA Kobe Record for $r_G = 0.5$, $r_M = 1.0$ and $\xi_1 = \xi_2 = 0.05$

To represents the amplification or deamplification of the maximum relative displacement due to the pounding effect, the relative displacement response spectrum with pounding effect ΔS_D^P is normalized by ΔS_D as

$$r_D^P = \frac{\Delta S_D^P}{\Delta S_D} \tag{1.51}$$

where r_D^P is called the pounding effect ratio. If r_G is larger than 1.0, r_D^P is equal to 1.0 because of no pounding. Fig. 1.39 shows the pounding effect ratio r_D^P of the JMA Kobe record for $r_G=0.5$ and $r_M=1.0$. Since deamplification and amplification occur at small and large ΔT , respectively.



Fig. 1.39 Pounding Effect Ratio r_D^P of the JMA Kobe Record for $r_G = 0.5$, $r_M = 1.0$ and $\xi_1 = \xi_2 = 0.05$

Based on an analysis of 80 ground motions, Ruangrassamee and Kawashima (2001) proposed the pounding effect ratio r_D^P as

$$r_D^P = c_G \left\{ c_M \left(2.4 - 2.1 \frac{T_1}{T_2} \right) - 1 \right\} \frac{T_1}{3} + 1$$
(1.52)

where c_G is the modification factor for the gap ratio and c_M is the modification factor for the mass ratio, and they are given as

$$c_{G} = \begin{cases} 1.0 \cdots 0.6 \le r_{G} < 0.6 \\ 1.0 - 5.3(r_{G} - 0.6)^{1.82} \cdots 0.6 \le r_{G} < 1.0 \\ 0 \cdots r_{G} \ge 1.0 \end{cases}$$
(1.53)

$$c_{M} = \begin{cases} 1 + 6(c_{M1} - 1)\frac{T_{2}}{T_{1}} \cdots 0 \leq \frac{3T_{2}}{T_{1}} < 0.5 \\ c_{M1} + (c_{M2} - c_{M1})\left(\frac{3T_{2}}{T_{1}} - 0.5\right) \cdots 0.5 \leq \frac{3T_{2}}{T_{1}} < 1.5 \\ 1 + \frac{c_{M2} - 1}{2.25}\left(\frac{3T_{2}}{T_{1}} - 3\right)^{2} \cdots 1.5 \leq \frac{3T_{2}}{T_{1}} < 3.0 \end{cases}$$
(1.54)

where,

1

$$c_{M1} = \frac{2}{2 - \log_{10} r_M} \tag{1.55}$$

$$c_{M2} = \frac{2}{2 - \log_{10} r_M - 0.17 (\log_{10} r_M)^2}$$
(1.56)

Fig. 1.40 shows the pounding effect ratio r_D^P evaluated by Eq. (1.52) for $r_G = 0.5$, and $r_M = 1.0$ and 10.0. Mean values of r_D^P computed by 80 ground motions are presented in Fig. 1.40 for comparison. Eq. (1.40) provides a good estimation for the computed mean values of r_D^P . At $r_M = 10$, the pounding effect ratio r_D^P increases as T_1 becomes longer than T_2 .

Application of the relative displacement response spectrum ΔS_D and the relative displacement response spectrum with pounding effect ΔS_D^P will be shown in Chapters 7 and 6, respectively.



(1) Mean Pounding Effect Ratio r_D^P for $r_G = 0-0.6$



Fig. 1.40 Pounding Effect Ratio r_D^P for $r_M = 1$ and 10

1.8 Residual Displacement Response Spectrum

Fig. 1.41 shows the response displacement of an oscillator with bilinear hysteresis subjected to the free-field ground acceleration recorded in the vicinity of Itajima Bridge in a M7.2 event. The bilinear factor r that is defined by Eq. (1.33) is -0.05, 0.0 and 0.1. The natural period computed from the initial elastic stiffness is 1 s. To represent energy dissipation from sources other than hysteretic energy dissipation, a viscous damping ratio ξ_{oth} of 5% was assumed.

A target ductility μ_T that is defined by Eq. (1.30) is 4. The residual displacement starts to occur at about 16 seconds when the ground acceleration increases. The magnitude of u_r increases as the bilinear factor r decreases. In particular, the residual displacement u_r is quite large when r=-0.05. Accumulation of residual displacements is likely to occur by a single drift over a short period of time as shown in Fig. 1.41(b) and (c), while in Fig. 1.41(d) the displacement around which the oscillation occurs more gradually accumulates and even changes direction.



Fig. 1.41 Residual Displacement for Itajima Record (*T*=1s): (a) Ground Acceleration; (b) *r*=-0.05; (c) *r*=0; (d) *r*=0.1

Residual displacements of structures, which make them difficult to repair, have generally

been considered to be of secondary importance compared to maximum ductility demand in the seismic design of structures. Residual displacement should be considered independently from maximum ductility demand, because a wide range of residual displacement may occur for the same ductility demand. It is essential that residual displacements after an earthquake be smaller than an acceptable value so that the structures can be repaired. The acceptable residual displacement depends on various considerations including the type of structures. Methods of assessment of the likely value of residual displacement are therefore required for different structural systems. Kawashima and Sato developed the residual displacement response spectrum for such a purpose (Kawashima and Sato 1996).

The residual displacement u_r is plotted against natural period T, damping ratio ξ , bilinear factor r and ductility factor μ , and this is called as the *residual displacement response spectrum* S_{RD} by analogy to the original definition of response spectrum. For any hysteretic loop, the maximum possible value of residual displacement $u_{r,\max}$ may be calculate from the peak displacement u_{\max} . It occurs when the oscillator unloads from the peak displacement u_{\max} to the line of zero lateral force. For a bilinear loop, the maximum possible value of residual displacement $u_{r,\max}$ is

$$u_{r,\max} = \begin{cases} (\mu-1)(1-r)u_y & \text{for } r(\mu-1) < 1\\ \{(1-r)/r\}u_y & \text{for } r(\mu-1) \ge 1 \end{cases}$$
(1.57)

It is convenient to normalize the residual displacement u_r by $u_{r,max}$ to get the residual displacement ratio r_{RD} as (MacRae and Kawashima 1997)

$$S_{RDS} = \left| u_r / u_{r,\max} \right| \tag{1.58}$$

Plots of S_{RDR} against natural period T, damping ratio ξ , bilinear factor r, and ductility factor μ are called the *residual displacement ratio response spectra* (Kawashima, MacRae, Hoshikuma and Nagaya 1998). The residual displacement was computed in the same manner for oscillators with various periods. The residual displacement ratio response spectra S_{RDR} defined by Eq. (1.58) was then evaluated for the Itajima record as shown in Fig. 1.42. The absolute acceleration response spectrum S_A with damping ratio of 0.05 is also presented in Fig. 1.42. The residual displacement ratio response spectra S_{RDR} depends significantly on r and the natural period, and have peak values at natural periods close to the predominant periods of ground motion.



Fig. 1.42 Residual Displacement Ratio Response Spectra S_{RDR} for Itajima Record: (a) Acceleration Response Spectra S_A (h=0.05); (b) r=-0.05; (c) r=0; (d) r=0.1

The residual displacement ratio response spectra S_{RDR} for the target ductility factors μ_T of 2, 4 and 6 from the 63 components of free field ground accelerations are shown in Fig. 1.43 (a). Because the influence of natural periods, soil conditions and target ductility factors do not dominate the trends of the behavior, only the effect of the bilinear factor r is used to evaluate S_{RDR} . As the scatters in the residual displacements about the means are considerable, one standard deviations from the means, as shown in Fig. 1.43 (b), are used to evaluate S_{RDR} for seismic design.

The residual displacement response spectra in Fig. 43 (b) was based on the analyses of oscillators with the bilinear hystereseis. AS a consequence, the spectra presented in Fig. 1.43 (b) are applicable to any structures where the hysteretic behavior can be reasonably approximated by a bilinear model.



(a) Dependence of S_{RDS} on the Target Ductility Factors



(b) Averaged S_{RDS}

Fig. 1.43 Residual Displacement Ratio Response Spectra S_{RDR} vs. Bilinear Factor r

1.9 Multiple Excitation Response Spectrum

Ground motions subjected to a structure at each support may be different, and this results in some additional response of a structure. To idealize the effect, a single-degree-of-freedom oscillator supported by two springs is considered as shown in Fig. 1.44. It is subjected to two different ground motions \ddot{u}_{gA} and \ddot{u}_{gB} .



Fig. 1.44 Definition of Multiple Excitation Response Spectrum

The equation of motion of the mass is written as

$$m\ddot{u}_t + c\dot{u}_t + (k_A + k_B)u_t - k_A u_{gA} - k_B u_{gB} = 0$$
(1.59)

in which \ddot{u}_t , \dot{u}_t and u_t represent absolute acceleration, velocity and displacement, respectively, of the mass, and *c* represents viscous damping coefficient. The absolute displacement u_t may be decomposed into a quasi-static displacement u_s and a dynamic displacement *u* as

$$u_t = u_s + u \tag{1.60}$$

From the definition of the quasi-static displacement, u_s can be evaluated as

$$u_{s} = \frac{1}{k_{A} + k_{B}} (k_{A} u_{gA} + k_{B} u_{gB})$$
(1.61)

Substituting Eqs. (1.28) and (1.29) into Eq. (1.27), one obtains

$$m\ddot{u} + c\dot{u} + (k_A + k_B)u = -\frac{m}{k_A + k_B}(k_A\ddot{u}_{gA} + k_B\ddot{u}_{gB})$$
(1.62)

When the mass is excited by either \ddot{u}_{gA} or \ddot{u}_{gB} at both supports (rigid excitation), it is easy to show that the equation of motion by Eq. (1.62) is written as

$$m\ddot{u} + c\dot{u} + (k_A + k_B)u = -m\ddot{u}_{gA} \tag{1.63}$$

$$m\ddot{u} + c\dot{u} + (k_{gA} + k_{gB})u = -m\ddot{u}_{gB}$$
(1.64)

Introducing a coefficient γ as

$$\gamma = k_B / k_A \tag{1.65}$$

and denoting

$$\omega_0 = \sqrt{(k_A + k_B)/m} \tag{1.66}$$

Eqs. (1.62), (1.63) and (1.64) can be written as

$$\ddot{u} + 2\xi\omega\dot{u} + \omega^2 u = -\ddot{u}_g \tag{1.67}$$

where,

$$\ddot{u}_{g} = \begin{cases} \frac{\ddot{u}_{gA} + \gamma \cdot \ddot{u}_{gB}}{1 + \gamma} \\ \ddot{u}_{gA} \\ \ddot{u}_{gB} \end{cases}$$
(1.68)

in which h is damping ratio. The top, center and bottom lines of Eq. (1.68) represent the multiple excitation by \ddot{u}_{gA} and \ddot{u}_{gB} , the single excitation by \ddot{u}_{gA} and the single excitation by \ddot{u}_{gB} , respectively.

By solving Eq. (1.67), one obtains the response acceleration spectra S^{AB} , S^{A} and S^{B} . S^{AB} represents the response acceleration of the mass subjected to \ddot{u}_{gA} at support A and \ddot{u}_{gB} at support B, and is defined as *multiple response spectra*. S^{A} and S^{B} are the response acceleration of the mass subjected to \ddot{u}_{gA} and \ddot{u}_{gB} at both supports, respectively.

For expressing the effect of multiple excitation, one defines

$$\eta^A_{\ P} = S^{AB}_{\ AP} / S^A_{\ P} \tag{1.69}$$

$$\eta^{\scriptscriptstyle B} = S^{\scriptscriptstyle AB} / S^{\scriptscriptstyle B} \tag{1.70}$$

The shear force developed at supports A and B when the mass is subjected to different ground motions can be written

$$F_A = k_A \left\{ u - \frac{\gamma(u_{gA} - u_{gB})}{1 + \gamma} \right\}$$
(1.71)

$$F_B = k_B \left\{ u + \frac{\gamma(u_{gA} - u_{gB})}{1 + \gamma} \right\}$$
(1.72)

On the other hand, the shear force developed at support A and B when the mass is subjected to the same ground motion is obtained as

$$F_A = k_A u \tag{1.73}$$

$$F_B = k_B u \tag{1.74}$$

Therefore, by taking the peak value of Eqs. (1.71)-(1.74), one can define the maximum shear force as; F_A^{AB} and F_B^{AB} : shear force developed at supports A and B for multiple excitation; F_A^A and F_B^A : shear force developed at supports A and B for excitation by \ddot{u}_{gA} ; F_A^B and F_B^B : shear force developed at supports A and B for excitation by \ddot{u}_{gB} . Effect of multiple excitations on the shear force is then defined as

$$\xi_{A}^{A} = F_{A}^{AB} / F_{A}^{A}, \quad \xi_{B}^{A} = F_{B}^{AB} / F_{B}^{A}$$

$$\xi_{A}^{B} = F_{A}^{AB} / F_{A}^{B}, \quad \xi_{B}^{B} = F_{B}^{AB} / F_{B}^{B}$$
(1.75)

For evaluating the multiple response spectra, a dense array data at PWRI field was analyzed. Two ground accelerations as shown in Fig. 24 which were measured at 100 m apart at 50 m below the ground surface were used. They were induced by an earthquake which occurred on September 24, 1980 at 42 km from the measuring point. Magnitude of the earthquake was 6.0. Peak ground acceleration and displacement of the data were shown in Table 1.7. The soil around the measuring point is of diluvium clay and sand covered by top soil with 3 m deep. Spatial variation of soil properties at the measuring point is small.

Components	NS-component		EW-component	
Supports	Support A	Support B	Support A	Support B
Peak Acceleration (cm/s2)	14.8	28.1	24.6	20.3
Peak Displacement (cm)	0.17	0.16	0.20	0.16

Table 1.7 Ground M	Motion Used for	Multiple Res	ponse Spectra
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Fig. 1.45 shows S^{AB} , S^{A} and S^{B} computed for the records assuming *h*=0.05 and γ =1.0. Fig. 1.46 shows S^{AB}/S^{A} and S^{AB}/S^{B} computed from Eqs. (1.69) and (1.70). It is noted that S^{AB}/S^{A} and S^{AB}/S^{B} are generally smaller than 1.0. It means that the acceleration response developed at the mass is smaller in multiple excitation than rigid excitation.

On the other hand, Fig. 1.47 shows the shear force ratio. It is important to note that the shear force is large at short natural period and approaches to 1.0 as the natural period increases. At natural period of about 0.5s, the shear ratio is almost less than 2. This implies that the multiple excitation effect on shear force (or bending moment in actual bridges) becomes significant as the system becomes rigid. This may be understood from Eq. (28) that u_s becomes more predominant than u.



Fig. 1.45 Absolute Acceleration Response Spectra



Fig. 1.46 Absolute Acceleration Response Spectra Ratio



Fig. 1.47 Shear Force Ratio

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