Physics and Engineering of CMOS Devices

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Saturation Velocity (1)

The velocity-field relationship is simplified as

$$v = \frac{\mu E}{1 + E / E_c}$$

Using velocity equation above, the drain current is expressed as

$$I_{d} = WC_{g} \left(V_{g} - V_{th} - V(y) \right) \frac{\mu E}{1 + \mu E / E_{c}}$$
$$I_{d} \left(1 + \frac{\mu}{E_{c}} \frac{dV}{dy} \right) = \mu WC_{g} \left(V_{g} - V_{th} - V(y) \right) \frac{dV}{dy}$$

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Saturation Velocity (2)

$$I_d \left(dy + \frac{\mu}{E_c} dV \right) = \mu W C_g \left(V_g - V_{th} - V(y) \right) dV$$

$$I_d \left(L + \frac{\mu}{E_c} V_d \right) = \mu W C_g \left[\left(V_g - V_{th} \right) V_d - \frac{m}{2} {V_d}^2 \right]$$

$$I_d \left(1 + \frac{\mu}{E_c} \frac{V_d}{L} \right) = \mu \frac{W}{L} C_g \left[\left(V_g - V_{th} \right) V_d - \frac{m}{2} {V_d}^2 \right]$$

$$I_d = \frac{\mu \frac{W}{L} C_g \left[\left(V_g - V_{th} \right) V_d - \frac{m}{2} {V_d}^2 \right]}{1 + \frac{\mu}{E_c} \frac{V_d}{L}}$$

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Saturation Velocity (3)

The saturation voltage is obtained by solving $dI_d/dV_d = 0$.

$$V_{dsat} = \frac{2\left(V_g - V_{th}\right)/m}{1 + \sqrt{1 + \frac{2\mu\left(V_g - V_{th}\right)}{mv_{sat}L}}}$$
$$< \left(V_g - V_{th}\right)/m$$

$$I_{dsat} = C_g W v_{sat} \left(V_g - V_{th} \right) \frac{\sqrt{1 + 2\mu \left(V_g - V_{th} \right) / (m v_{sat} L)} - 1}{\sqrt{1 + 2\mu \left(V_g - V_{th} \right) / (m v_{sat} L)} + 1}$$

Saturation Velocity (4)

 $I_{dsat} \approx \mu C_g \frac{W}{L} \frac{\left(V_g - V_{th}\right)^2}{2m}$ $I_{dsat} \approx C_g W v_{sat} \left(V_g - V_{th}\right)$

$$\left(L \gg \frac{2\mu \left(V_g - V_{th}\right)}{m v_{sat} L}\right)$$
$$\left(L \ll \frac{2\mu \left(V_g - V_{th}\right)}{m v_{sat} L}\right)$$

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Saturation Velocity (5)



Mobility

The mobility is the magnitude of the drift velocity per unit electric field.



Mobility Universality

It is often found experimentally that the measured effective mobility of silicon MOSFETs follows a "universal" curve independent of substrate doping and gate oxide thickness when plotted against effective transverse electric field.



S. Takagi et al., IEEE Trans. Electron Devices, 41 (1994) 2357.

Effective Electric Field

$$\boldsymbol{E}_{eff} = \frac{\boldsymbol{q}}{\boldsymbol{\varepsilon}_{Si}} (\boldsymbol{N}_{dep} + \boldsymbol{\eta} \cdot \boldsymbol{N}_{s})$$

 ϵ_{si} : permittivity of Si N_{dep}: depletion charge density (N_{sub}xW_{dep})

Experimental:

η	(100)	(110)	(111)			
ELECTRON	1/2	1/3	1/3			
HOLE	1/3					

S. Takagi et al., IEEE Trans. Electron Devices, 41 (1994) 2363.

Theoretical:

$$\boldsymbol{E}_{eff} = \frac{\boldsymbol{q}}{\boldsymbol{\varepsilon}_{Si}} \left(\boldsymbol{N}_{dep} + \frac{11}{32} \boldsymbol{N}_{s} \right)$$

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F. Stern, Phys Rev. B 5 (1971) p4891.

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Scattering Rate

Fermi's Golden Rule The scattering rate from a $|\mathbf{k}\rangle$ state to a $|\mathbf{k}'\rangle$ state is given by $P(\mathbf{k}, \mathbf{k}') = \frac{2\pi}{\hbar} |\langle \mathbf{k}' | H_p(\mathbf{r}) | \mathbf{k} \rangle|^2 \delta(E_{\mathbf{k}'} - E_{\mathbf{k}} \mp \hbar \omega)$ Time-dependent perturbation theory. $H = H_0 + H_p \qquad H: \text{Hamiltonian of the system}$ $H_0: \text{ Unperturbed Hamiltonian}$ $H_0: \mathbf{k}\rangle = E_{\mathbf{k}} | \mathbf{k} \rangle \qquad H_p: \text{ Perturbation}$ Assuming that H_p is expanded by Fourier series, H_p is expressed as $H_p(\mathbf{r}) = \sum_{\mathbf{k}} H_p(\mathbf{q}) \exp(i\mathbf{q} \cdot \mathbf{r}) \qquad H_p(\mathbf{q}) = \frac{1}{V} \int d\mathbf{r} H_p(\mathbf{r}) \exp(-i\mathbf{q} \cdot \mathbf{r})$ $\langle \mathbf{k}', \mathbf{q}' | H_p(\mathbf{r}) | \mathbf{q}, \mathbf{k} \rangle = \langle \mathbf{k}', \mathbf{q}' | \sum_{\mathbf{q}'} H_p(\mathbf{q}'') \exp(i\mathbf{q} \cdot \mathbf{r}) | \mathbf{q}, \mathbf{k} \rangle$ $\approx \langle \mathbf{q}' | \sum_{\mathbf{q}'} H_p(\mathbf{q}'') \delta(\mathbf{k} - \mathbf{k}' + \mathbf{q}) | \mathbf{q} \rangle$ Momentum conservation 10 Physics and Engineering of CMOS Devices $\langle \mathbf{q}' | H_p(\mathbf{k}' - \mathbf{k}) \rangle | \mathbf{q} \rangle$

Mobility Limited by Various Scattering

Coulomb Scattering

$$\mu_{ ext{Coulomb}} = N_{sub}^{-1} \cdot N_{inv}^{+1}$$

Phonon Scattering

$$\mu_{\rm Phonon} = E_{eff}^{-1/3} \cdot T^{-1.25}$$

Roughness Scattering

$$\mu_{
m Roughness} = E_{eff}^{-m}$$

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2D Electron System in MOS Structures



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Scattering in 2D electron system

In 2D electron systems such as inversion-layer electrons in MOSFETs, 1) electrons are confined with potentials and 2) subbands are formed in each valley.

$$\Psi_{\mathbf{k}_{//},n} = \frac{1}{\sqrt{S}} \exp(i\mathbf{k}_{//} \cdot \mathbf{r}_{//}) \zeta_n(z)$$
$$= \zeta_n(z) |\mathbf{k}_{//}\rangle$$

$$\begin{aligned} \left\langle \mathbf{k}' \right| H_{p}(\mathbf{r}) \left| \mathbf{k} \right\rangle &= \left\langle \mathbf{k}' \right| \sum_{\mathbf{q}} H_{p}(\mathbf{q}) \exp(i\mathbf{q} \cdot \mathbf{r}) \left| \mathbf{k} \right\rangle \\ &= \left\langle \mathbf{k}'_{\prime\prime} \right| \zeta_{m}^{*}(z) \sum_{\mathbf{q}} H_{p}(\mathbf{q}) \exp(i\mathbf{q} \cdot \mathbf{r}) \zeta_{n}(z) \left| \mathbf{k}_{\prime\prime} \right\rangle \\ &= \sum_{\mathbf{q}} H_{p}(\mathbf{q}) \delta(\mathbf{k}_{\prime\prime} - \mathbf{k}'_{\prime\prime} + \mathbf{q}_{\prime\prime}) \int_{0}^{W} \zeta_{m}^{*}(z) \zeta_{n}(z) \exp(iq_{z}z) dz \end{aligned}$$

ct.
$$\langle \mathbf{k}' | H_p(\mathbf{r}) | \mathbf{k} \rangle = \sum_{\mathbf{q}} H_p(\mathbf{q}) \delta(\mathbf{k} - \mathbf{k}' + \mathbf{q})$$

P. J. Price, Annals of Physics, **133** (1981) p217. Physics and Engineering of CMOS Devices, Ken Uchida, Tokyo Tech, June 10, 2009

Form Factor

$$\delta(\mathbf{k} - \mathbf{k'} \pm \mathbf{q}) \rightarrow I_{mn}(q_z)\delta(\mathbf{k}_{\prime\prime} - \mathbf{k}_{\prime\prime}' \pm \mathbf{q})$$

where I_{mn} is the form factor. $I_{mn}(q_z) = \int_0^W \zeta_m(z)^* \zeta_n(z) \exp(iq_z z) dz$

$$\begin{split} \int_{-\infty}^{\infty} |I_{mn}(q_{z})|^{2} dq_{z} &= \int dq_{z} \int_{0}^{W} dz \int_{0}^{W} dz' \zeta_{m}(z)^{*} \zeta_{n}(z) \zeta_{m}(z') \zeta_{n}(z')^{*} \exp\left[iq_{z}(z-z')\right] \\ &= \int_{0}^{W} dz \int_{0}^{W} dz' \zeta_{m}(z)^{*} \zeta_{n}(z) \zeta_{m}(z') \zeta_{n}(z')^{*} \delta(z-z') \\ &= \int_{0}^{W} dz |\zeta_{m}(z)|^{2} |\zeta_{n}(z)|^{2} \\ &= \frac{\pi}{b_{mn}} \end{split}$$

 b_{mn} describes the effective extent of the interactions in the z-direction.

P. J. Price, Annals of Physics, 133 (1981) p217.

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Phonon Scattering in Bulk Si

- Lattice vibrations are an inevitable source of scattering
- Dominant scattering at high temperatures ~ RT

> Negligible contribution at low temperatures



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Phonon Dispersion Relationship



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Intra-Valley Phonon Scattering

Intra-valley acoustic-phonon scattering

transitions between states within a single valley via acoustic phonons phonons with low energies (elastic process)

Intra-valley optical-phonon scattering

transitions between states within a single valley via optical phonons phonons of low momentum and high energy

Selection rules	for	zone-center	phonons	in	intra-vallev	/	process
ocicection raies.	,	zone center	priorioris			·)	p100000

	Valley	Phonons	
	Г1	LA	
\triangleleft	X1	LA+TA	2
	L1	LA+TA+LO+TO	
	Γ 15	LA+TA+LO+TO	

B.K.Ridley, *Quantum Process in Semiconductors*, Oxford Univ. Press, 1999, p107.

Intra-valley optical-phonon scattering is forbidden in Silicon.

Intra-Valley Acoustic Phonon Scattering - Acoustic Deformation Potential-

The energy change, due to the deformation by acoustic phonon, is a function of the volume change of a crystal, $\delta V/V$.

$$H_{p} = D_{ac} \frac{\delta V}{V} = D_{ac} \operatorname{div} \mathbf{u}(\mathbf{r})$$
$$= D_{ac} \sum_{\mathbf{q}} \sqrt{\frac{\hbar}{2MN\omega_{q}}} \left(i\mathbf{e}_{q} \cdot \mathbf{q} \right) \left[a_{q} \mathrm{e}^{i\mathbf{q}\cdot\mathbf{r}} - a_{q}^{+} \mathrm{e}^{-i\mathbf{q}\cdot\mathbf{r}} \right]$$

$$\left|\left\langle \mathbf{k}',\mathbf{q}'\right|H_{p}\left(\mathbf{r}\right)\left|\mathbf{q},\mathbf{k}\right\rangle\right|^{2}=D_{ac}^{2}q^{2}\frac{\hbar}{2V\rho\omega_{q}}\left(n_{q}+\frac{1}{2}\mp\frac{1}{2}\right)$$

 $ho = {MN\over V}$ Physics and Engineering of CMOS Devices, Ken Uchida, Tokyo Tech, June 10, 2009

Intra-Valley Acoustic Phonon Scattering - Mobility Calculation -

$$P(\mathbf{k},\mathbf{k}') = \frac{2\pi}{\hbar} D_{ac}^{2} q^{2} \frac{\hbar}{2V\rho\omega_{q}} \left(n_{q} + \frac{1}{2} \mp \frac{1}{2} \right) \delta \left(E_{k} - E_{k'} \pm \hbar\omega_{q} \right)$$

$$\approx \frac{2\pi}{\hbar} \frac{D_{ac}^{2} k_{B} T}{2V\rho v_{s}^{2}} \delta \left(E_{k} - E_{k'} \pm \hbar\omega_{q} \right) \frac{k_{B} T}{\hbar v_{s} q}$$

$$\frac{1}{\tau(E)} = \frac{V}{8\pi^{3}} \int d^{3} \mathbf{k}' P(\mathbf{k},\mathbf{k}') \left\{ 1 - \frac{\mathbf{e}_{//} \cdot \mathbf{k}'}{\mathbf{e}_{//} \cdot \mathbf{k}} \right\} = \frac{(2m^{*})^{3/2} D_{ac}^{-2} k_{B} T}{2\pi \hbar^{4} \rho v_{s}^{2}} E^{1/2}$$

$$\langle \tau \rangle = \frac{2\pi \hbar^{4} \rho v_{s}^{2}}{(2m^{*})^{3/2} D_{ac}^{-2} k_{B} T} (k_{B} T)^{-1/2} \Gamma \left(\frac{5}{2} - \frac{1}{2} \right) / \Gamma \left(\frac{5}{2} \right) = \frac{2^{3/2} \sqrt{\pi} \hbar^{4} \rho v_{s}^{2}}{3m^{*3/2} D_{ac}^{-2} (k_{B} T)^{3/2}}$$

$$\mu_{ac} = \frac{2^{3/2} \sqrt{\pi} e \hbar^{4} \rho v_{s}^{2}}{3m^{*5/2} D_{ac}^{-2} (k_{B} T)^{3/2}} \propto T^{-1.5}$$

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Inter-Valley Optical Phonon Scattering - Optical Deformation Potential-

Selection rules for phonons in inter-valley scattering processes

Initial Valley	Final Valley		Phonons
Δ1	Δ1	g-scattering	LO
Δ1	Δ1	f-scattering	LA+TO

B.K.Ridley, Quantum Process in Semiconductors, Oxford Univ. Press, 1999, p110.

In a crystal with two or more atoms in a unit cell, optical-mode lattice vibration occurs due to the relative displacement between two atoms in the unit cell. The potential induced by the relative displacement results in scattering of carriers.

$$H_p = D_{op} \cdot \mathbf{u}$$

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Inter-Valley Optical Phonon Scattering

Randomizing Collision Approximation

$$1 - \frac{\mathbf{e}_{//} \cdot \mathbf{k}'}{\mathbf{e}_{//} \cdot \mathbf{k}} = 0$$

Under this approximation, the inverse of relaxation time is equivalent to the scattering rate. Then, the momentum relaxation time is evaluated to be

$$\tau_{op} = \frac{\left(2m^{*}\right)^{3/2}}{4\pi\hbar^{3}\rho} \frac{D_{op}^{2}}{\omega_{0}} \left[\left(n_{q}+1\right)\sqrt{E-\hbar\omega_{0}} + n_{q}\sqrt{E+\hbar\omega_{0}} \right]$$

$$\mu_{op} = \frac{4\sqrt{2\pi}e\hbar^{2}\rho\sqrt{\hbar\omega_{0}}}{3m_{d}^{*3/2}m_{c}^{*}D_{op}^{2}}f(x_{0})$$

$$f(x_0) = x_0^{5/2} \left(e^{x_0} - 1 \right) \int_0^\infty x e^{-x} \left[\left(1 + \frac{x_0}{x} \right)^{1/2} + e^{x_0} \left(1 - \frac{x_0}{x} \right)^{1/2} \right]^{-1} dx$$

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Summary

- Impact of Saturation Velocity on Drain Current
- Mobility in MOS Transistors
- 2D Electron System in MOS Transistors
- Phonon Scattering in Bulk Si

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