# Physics and Engineering of CMOS Devices

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#### Review of the Last Lecture (1) MOS Capacitor: φ<sub>s</sub> versus Q<sub>depl</sub>



### **Maximum Depletion-Layer Width**

The depletion layer width reaches its maximum value at the onset of the strong inversion, namely  $\phi_s = 2\phi_F$ . The maximum depletion-layer width,  $X_{dm}$ , is given as

$$X_{dm} = \sqrt{\frac{4\kappa_s \varepsilon_0 \phi_F}{qN_A}}$$
(14)

The depletion-layer capacitance under the strong inversion is called the maximum depletion-layer capacitance,  $C_{dm}$ , and is expressed as

$$C_{dm} = \sqrt{\frac{qN_A\kappa_s\varepsilon_0}{4\phi_F}}$$

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In gated PN junctions, we have  $E_{Fn}$  for electrons and  $E_{Fp}$  for holes when the junction is biased.

The point is that in the depletion region  $E_{Fn}$  should be used to calculate the electron densities.

(15)

#### **Review of the Last Lecture (3)**



$$V_{th} = V_{FB} + 2\phi_F + V_R + \frac{\sqrt{2qN_A\kappa_s\varepsilon_0\left(2\phi_F + V_R\right)}}{C_{ox}}$$
(13)

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# V<sub>th</sub> for Gated PN Junction

It should be noted that the threshold voltage may be defined as the substrate-to-gate voltage or drain(diffusion layer)-to-gate voltage at the onset of the strong inversion.

Threshold for substrate-to-gate voltage

$$V_{gb,th} = V_{FB} + 2\phi_F + V_R + \frac{\sqrt{2qN_A\kappa_s\varepsilon_0(2\phi_F + V_R)}}{C_{ox}}$$

Threshold for drain-to-gate voltage

$$V_{gd,th} = V_{FB} + 2\phi_F + \frac{\sqrt{2qN_A\kappa_s\varepsilon_0\left(2\phi_F + V_R\right)}}{C_{ox}}$$

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# **Charge-Sheet Model**

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# **Charge-Sheet Approximation**

- All the inversion-charges are located at the interface of Si and SiO<sub>2</sub>.
- There is no potential drop and no band bending across the inversion layer.

**Depletion Approximation & Charge-Sheet Approximation** 



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#### **Charge-Sheet Model of MOSFETs**



The channel is equivalent with the gated PN junction. At the position y, the quasi-Fermi energy level for electrons,  $E_{Fn}(y)$ , is given as

 $E_{Fn}(y) = V(y) + \phi_F$ 

The threshold voltage at the position y is expressed as

$$V_{th}(y) = V_{FB} + 2\phi_F + V(y) + \frac{\sqrt{2qN_A\kappa_s\varepsilon_0(2\phi_F + V(y))}}{C_{ox}}$$

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#### Charge-Sheet Model –cont'd

Inversion-charge density  $Q_i(y)$  at the position y is given by

$$Q_{i}(y) = C_{g} \left( V_{g} - V_{th}(y) \right)$$
$$= C_{g} \left( V_{g} - V_{FB} - 2\phi_{F} - V(y) - \sqrt{2qN_{A}\kappa_{s}\varepsilon_{0} \left(2\phi_{F} + V(y)\right)} \right)$$

The drain current is expressed as

$$I_{d} = WQ_{i}(y)\mu \frac{dV(y)}{dy}$$
$$= \mu C_{g}W \left( V_{g} - V_{FB} - 2\phi_{F} - V(y) - \sqrt{2qN_{A}\kappa_{s}\varepsilon_{0}\left(2\phi_{F} + V(y)\right)} \right) \frac{dV(y)}{dy}$$

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# Charge-Sheet Model –cont'd

By integrating the previous equation, the drain current  $I_d$  is obtained as follows.

$$I_{d} = \mu C_{g} \frac{W}{L} \left[ \left( V_{g} - V_{FB} - 2\phi_{F} \right) V_{d} - \frac{1}{2} V_{d}^{2} - \frac{2\sqrt{2\kappa_{s}\varepsilon_{0}}qN_{A}}{3C_{g}} \left( \left( 2\phi_{F} + V_{d} \right)^{3/2} - \left( 2\phi_{F} \right)^{3/2} \right) \right]$$
(15)

 $I_d$  is expanded in the series of  $V_d$ .

$$I_{d} \approx \mu C_{g} \frac{W}{L} \left[ \left( V_{g} - V_{FB} - 2\phi_{F} - \frac{\sqrt{4q\kappa_{s}\varepsilon_{0}N_{A}\phi_{F}}}{C_{g}} \right) V_{d} - \frac{1}{2} \left( 1 + \sqrt{\frac{q\kappa_{s}\varepsilon_{0}N_{A}}{4\phi_{F}}} \right) V_{d}^{2} \right]$$

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## Charge-Sheet Model –cont'd

$$I_d \approx \mu C_g \frac{W}{L} \left[ \left( V_g - V_{th0} \right) V_d - \frac{m}{2} {V_d}^2 \right]$$
(16)

H

Here,  

$$V_{th0} = V_{FB} + 2\phi_F + \frac{\sqrt{4qN_A\kappa_s\varepsilon_0\phi_F}}{C_{ox}}$$

$$m = 1 + \frac{C_{dm}}{C_{ox}}$$
(17)

*m* is the body-effect coefficient.

Eq (16) reaches its maximum at the saturation drain voltage  $V_{d,sat}$ given below.

$$V_{d,sat} = \frac{V_g - V_{th0}}{m}$$
(18)
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# Charge-Sheet Model –cont'd

The saturation drain current,  $I_{d,sat}$ , is expressed as

$$I_{d,sat} = \frac{1}{2} \mu C_{ox} \frac{W}{L} \frac{\left(V_g - V_{th0}\right)^2}{m}$$
(18)

In charge-sheet model, the effect of substrate impurities (acceptor ions in the case of nMOSFETs) is included.

Because of the depletion charges, the saturation voltage and the saturation current are lower than those in the simple analytical MOSFET model discussed in the first lecture.

When the threshold voltage of MOSFETs is discussed, the source-to-gate voltage  $V_{gs}$  is usually considered. In other words,  $V_{th}(0)$  in page 8 of this foil with V(y) = 0 is considered.

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# Substrate-Bias Effect

#### Substrate-Bias Effect

The threshold voltage of nMOSFETs is increased if a negative substrate bias  $(-V_b)$  is applied.

Let us consider the threshold voltage as a function of  $V_b$ .

$$V_{th}(0) = V_{FB} + 2\phi_F + \frac{\sqrt{4qN_A\kappa_s\varepsilon_0\phi_F}}{C_{ox}} = V_{FB} + 2\phi_F + \gamma\sqrt{2\phi_F}$$
$$V_{th}(V_b) = V_{FB} + 2\phi_F + \frac{\sqrt{2qN_A\kappa_s\varepsilon_0(\phi_F + V_b)}}{C_{ox}} = V_{FB} + 2\phi_F + \gamma\sqrt{2\phi_F} + V_b$$

Here, the body factor  $\gamma$  is defined as.

$$\gamma = \frac{\sqrt{2\kappa_s \varepsilon_0 q N_A}}{C_{ox}}$$

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#### Substrate-Bias Effect -cont'd

$$\Delta V_{th} = V_{th}(V_b) - V_{th}(0)$$
  
=  $\gamma \left( \sqrt{2\phi_F + V_b} - \sqrt{2\phi_F} \right)$  (19)

$$\frac{\partial V_{th}}{\partial V_b} = \frac{\gamma}{2} \frac{1}{\sqrt{2\phi_F + V_b}}$$

$$\lim_{V_b \to 0} \frac{\partial V_{th}}{\partial V_b} = \frac{\gamma}{2} \frac{1}{\sqrt{2\phi_F}} = \frac{C_{dm}}{C_{ox}} = m - 1$$
(20)

# Summary

- Charge-sheet approximation is introduced.
- Charge-sheet model, which is one of the most widely used MOSFET model, is derived.
- The substrate-bias effect is discussed. The threshold voltage shift induced by the substrate bias is obtained.

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