Fading Theory

- In many circumstances, it is too complicated to describe all reflection, diffraction, and scattering processes that determine the different Multi-path Components.

  Rather, it is often preferable to describe the probability that a channel parameter attains a certain value.

Deterministic vs. Stochastic

- Deterministic case : "x=y" means 2=2.
- Stochastic case : "x=y" means "p(x)=p(y)".
- For example, x = 1-x holds

when x is a uniform distributed random variable in the interval [0,1]

z: zero-mean Complex Gaussian Noise

$\therefore$ "$z=-z=z^*=-z^*$"

Z:zero-mean Complex Gaussian Vector

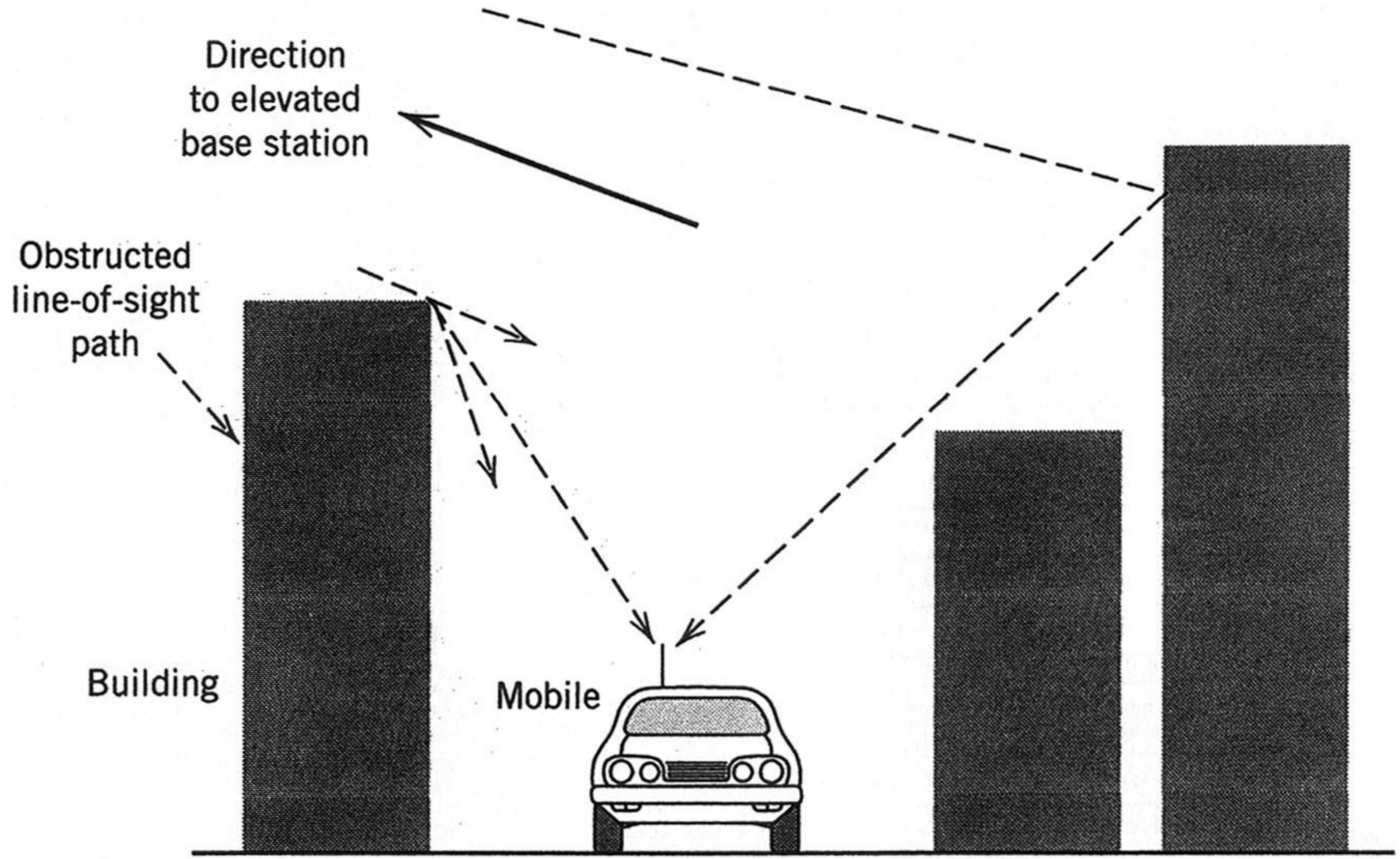
$\therefore$ "$\mathbf{Z}=U\mathbf{Z}$" where U: Unitary matrix

Contents

- Path Loss Formula
- Log-normal distribution
- Rayleigh/Rice distribution
- Envelope/Phase distribution
- Power Spectrum & Doppler effect
- Fading Coefficient
- MAP Estimation of Fading Channel in PHS

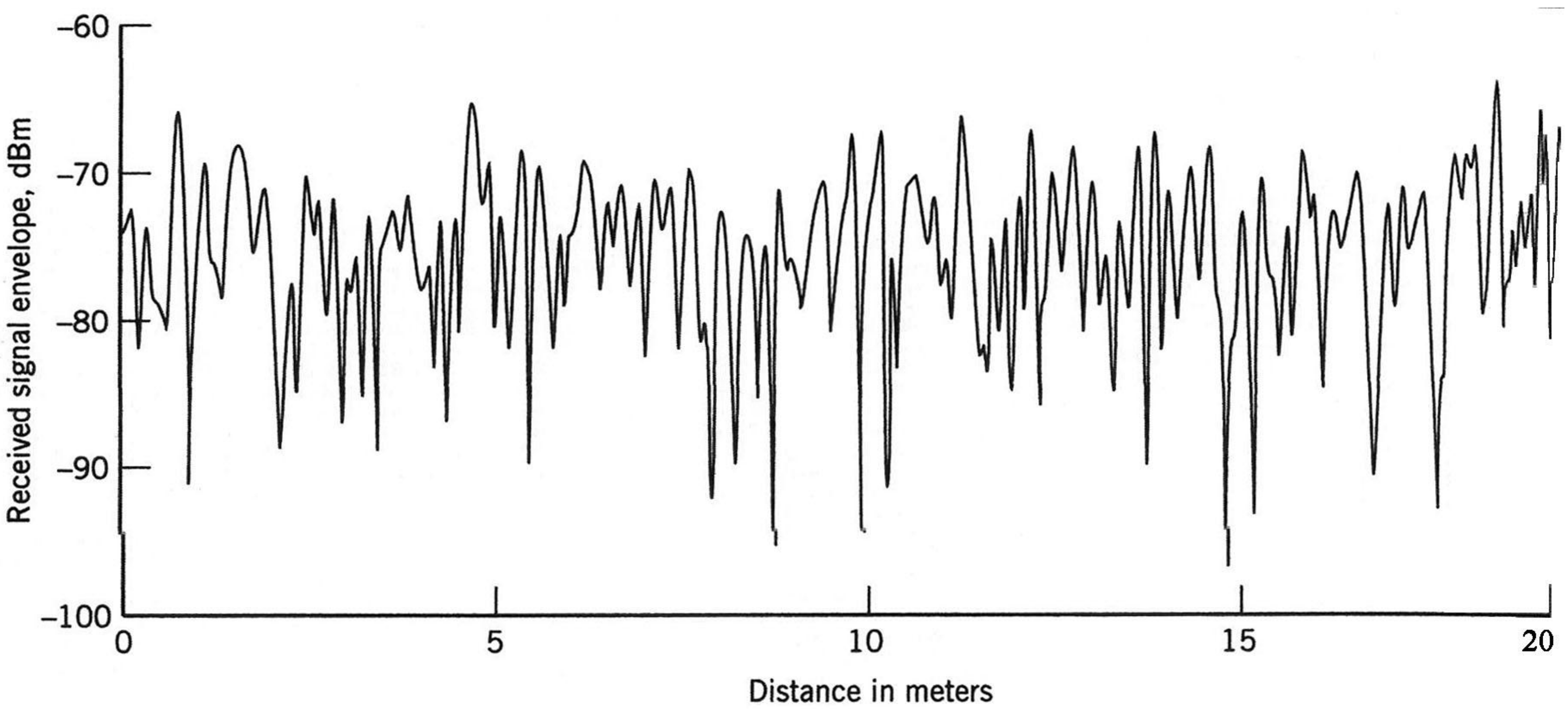
Mobile Communication Channel

In addition to Direct wave, there are many Reflection, Refraction and Diffraction waves.



- Received signal fluctuates dramatically

→ Fading (Long-range, Medium-range, Short-range)



Hierarchical stochastic structure

- Path loss : The large-scale mean itself depends on the "distance" between transmitter and receiver.
- Log-normal : Mean power, averaged over about 10 wavelengths, itself shows fluctuations due to "shadowing" by large objects.
- Rayleigh and Nakagami-Rice : On a very-short-distance scale, power fluctuates around a local mean value due to "interference" between different MPCs.

Path loss and Power Control

- For 3G Wireless Communication System, i.e. W-CDMA (Wideband Code Division Multiple Access) Power Control is used in order to alleviate "Near-Far Problem".

  Dynamic Range for Power Control is required more than 74dB.

Path Loss Formula

- **Land mobile electromagnetic wave propagation**
  Propagation characteristics are important in designing a cell size, a transmitter and a receiver.
  - **Long** distance variation (Okumura curve): The CCIR adopted the basic formula for the median path loss, based on Okumura's measurements.

$$L = 69.55 + 26.16\log(f) - 13.82\log(H_b) + [44.9 - 6.55\log(H_b)]\log(d) + a_x(H_m)$$

f : frequency in MHz

H_b : Base station antenna height in meter

d : Range in Km

H_m : Mobile station antenna height in meter

$a_x(H_m)$: Correction factor

– **Middle** distance variation (**Log-normal** distribution: Shadowing) Median over several ten or hundred wavelengths obeys a log-normal distribution.

$$E_r = T_1 \times T_2 \times T_3 \times \cdots \times E_s$$

E_r : Signal Strength at the receiver

E_s : Signal Strength at the transmitter

T_i : Transmission coefficent at the i-th obstacle

$$\therefore \quad \log E_r = \log T_1 + \log T_2 + \cdots + \log E_s$$

Central Limit Theorem

– The sum of statistically independent and identically distributed random variables with finite mean and variance approaches to a **Gaussian distribution** as the number of variables increases.
– **Gaussian distribution** is characterized only by mean and variance (2 parameters).

Shadowing effect

- Typical shadowing range is around 4-10dB
- 3GPP Channel model:

  Suburban Macro 8dB

  Urban Macro 8dB

  Urban Micro 10dB(NLOS) 4dB(LOS)

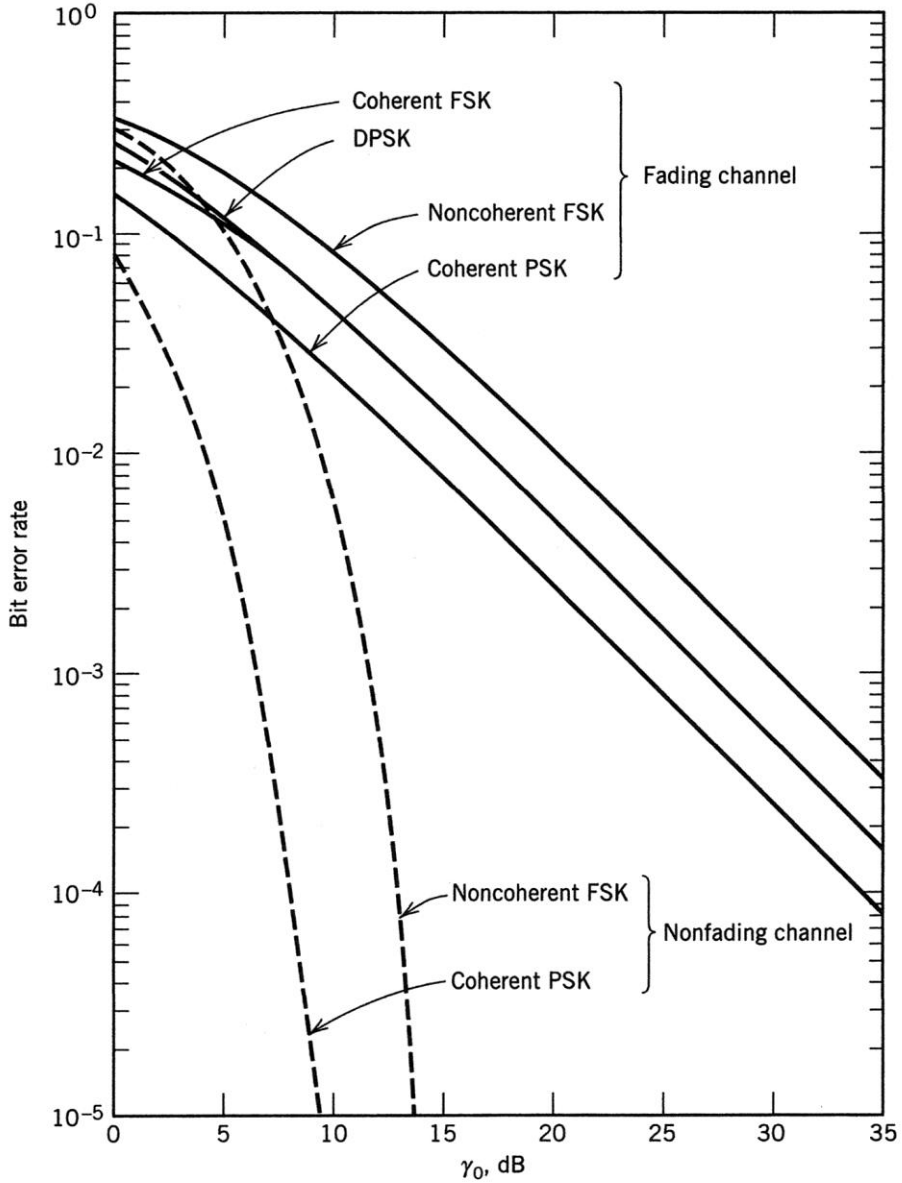
Rayleigh Fading

– **Short** distance variation (Rayleigh Fading) There are so many reflection and diffraction waves to generate a complicate standing wave pattern. The mobile station moves through there.

BER Performance in Rayleigh Fading Channel

- BER (Bit Error Rate) is proportional to an exponential function of SNR in non-fading channel (AWGN channel).
- BER is proportional to an inverse of SNR in fading channel.
- Because SNR in fading channel is a random variable of which PDF (probability density function) is an exponential function.

- Fading significantly deteriorates QoS (i.e. bit error rate).



BER in Rayleigh channel

- Instantaneous BER:

$$Pe \cong \exp(-\gamma)/2$$

- Averaged BER:

$$\overline{Pe} = \int Pe \times P(\gamma)d\gamma = 1/\{2(\Gamma+1)\}$$

- Pdf of SNR: $$P(\gamma) = \exp(-\gamma/\Gamma)/\Gamma$$

  where Γ: average SNR

Interference between Multi-path Components

- **Rayleigh Fading Model**

The n-th elementary arriving wave $e_n(t)$ at an angle of ϕ_n

$$e_n(t) = \mathrm{Re}\left[z_n(t)\exp\left(j2\pi\left[f_c + f_D\cos(\phi_n)\right]t\right)\right]$$

$\mathrm{Re}[\quad]$: Real part complex number

$z_n(t)$: Complex envelope

f_c : Carrier frequency

f_D : Maximum Doppler frequency shift $(= v/\lambda)$

v : Velocity of mobile station

λ : Wavelength $(= c/f_c)$

– Envelope and phase distribution

Received signal $e(t)$ iscomposed of N elementary waves.

$$e(t) = \sum_{n=1}^{N} e_n(t)$$

$$= \mathrm{Re}\left[\sum_{n=1}^{N} z_n(t) \exp\left(j2\pi f_c t\right)\right]$$

$$z(t) = \sum_{n=1}^{N} z_n(t)$$

$$= x(t) + jy(y)$$

$$x(t): \text{In-phase component} = R(t)\cos(\theta(t))$$
$$y(t): \text{Quadrature component} = R(t)\sin(\theta(t))$$

In the limit $(N \to \infty)$, $x(t)$ and $y(t)$ become an independent Gaussian random variable with zero mean.

Thus, a joint pdf (probability density function) of x and y

$$p(x, y) = \exp\left(-\frac{x^2 + y^2}{2b_0}\right) \Big/ 2\pi b_0$$

where $2b_0$: average received power $= E\left[x^2 + y^2\right] = E\left[R^2\right]$

A joint pdf of R and θ is

$$p(R, \theta) = \frac{R}{2\pi b_0} \exp\left(-\frac{R^2}{2b_0}\right) = p(R)\, p(\theta)$$

where R : envelope

θ : phase

Rayleigh Distribution

A pdf of envelope R is a **Rayleigh** distribution

$$p(R)=\frac{R}{b_0}\exp\left(-\frac{R^2}{2b_0}\right)$$

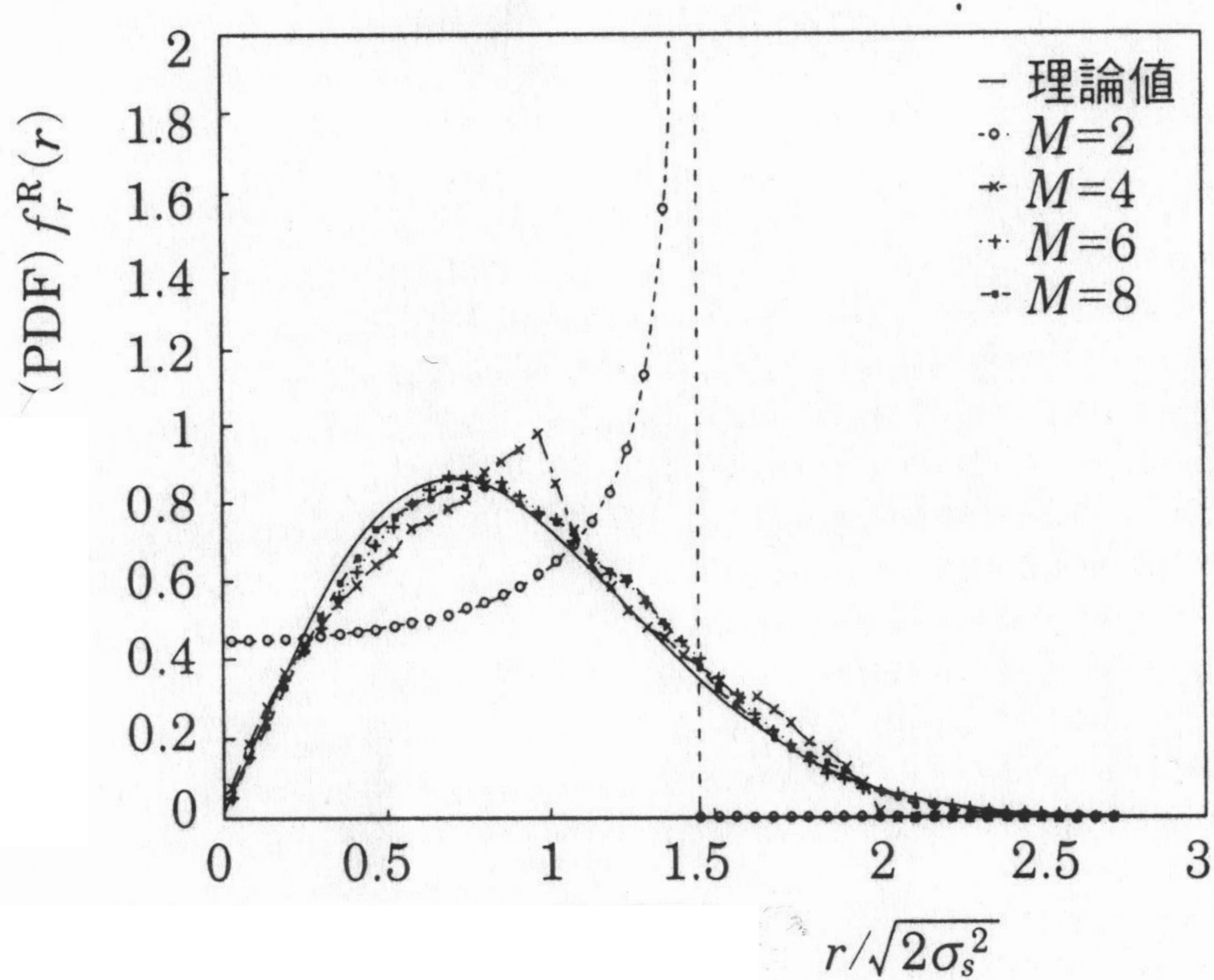
A pdf of phase θ is a **uniform** distribution

$$p(\theta)=1/2\pi$$

CNR (Carrier-to-noise radio), $\gamma = R^2/p_n$ is **exponential** distribution with noise power of p_n

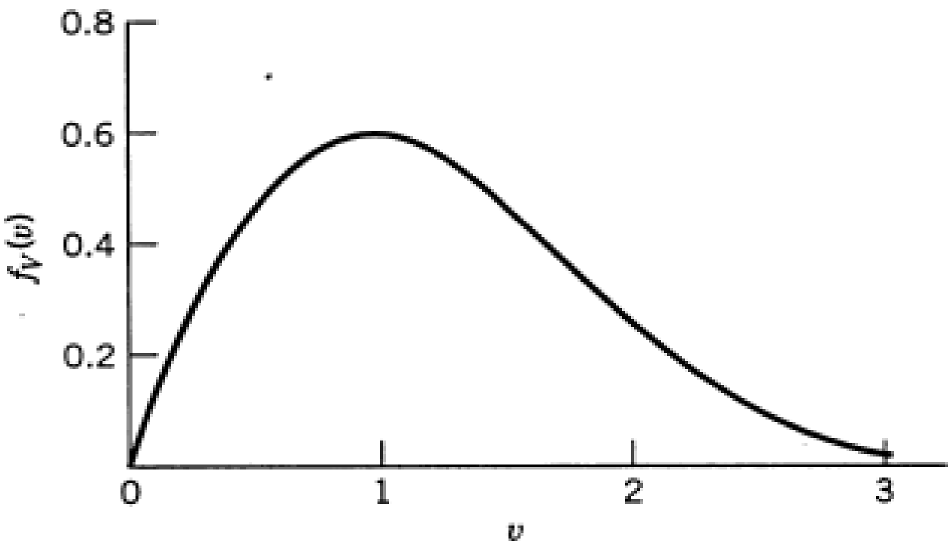
$$p(\gamma)=\frac{1}{\Gamma}\exp\left(-\frac{\gamma}{\Gamma}\right)$$

where Γ : Average CNR

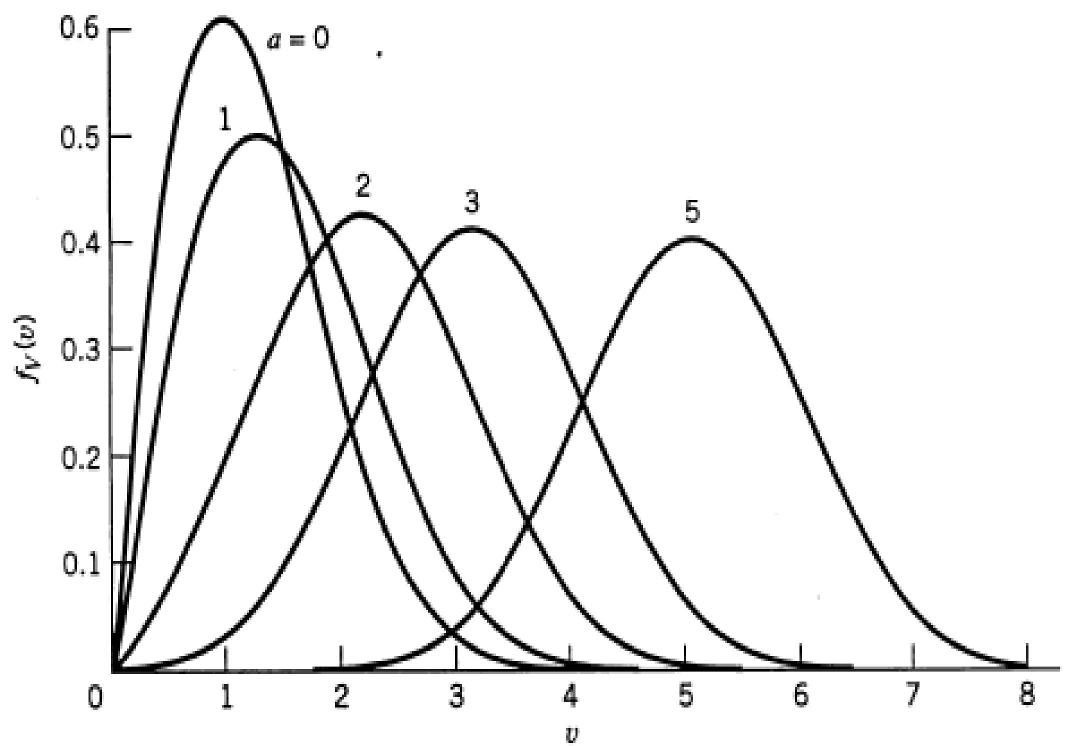


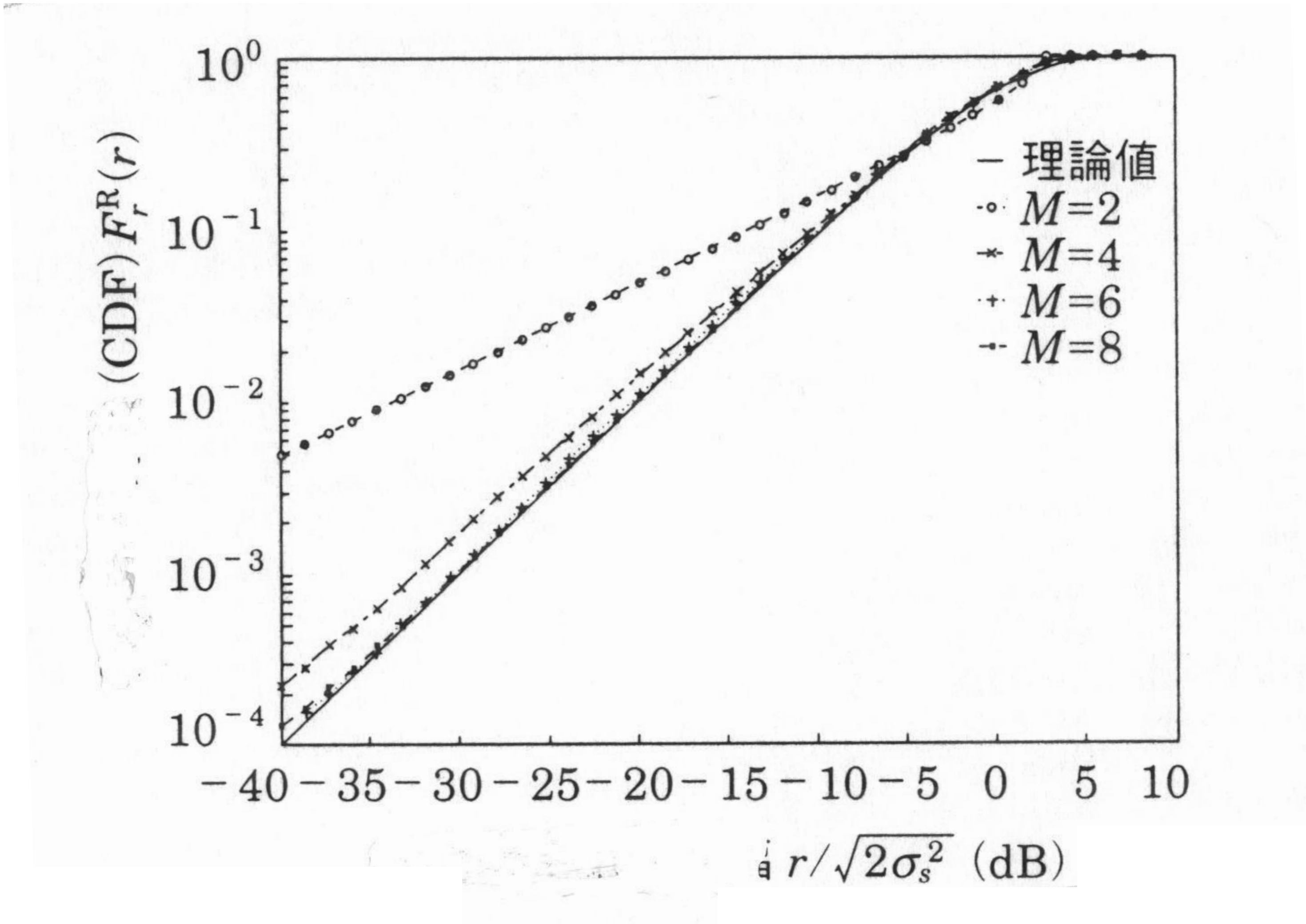
(PDF) $f_r^{\mathrm{R}}(r)$
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3
$r/\sqrt{2\sigma_s^2}$
理論値
M=2
M=4
M=6
M=8

Normalized Rayleigh Distribution



Normalized Nakagami/Rice Distribution





(CDF) $F_r^R(r)$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
− 理論値
M=2
M=4
M=6
M=8
−40 −35 −30 −25 −20 −15 −10 −5 0 5 10
$r/\sqrt{2\sigma_s^2}$ (dB)

- **Power spectrum & Doppler effect**

Elementary wave of arrival angle ϕ has a different frequency from f_c due to the Doppler effect.

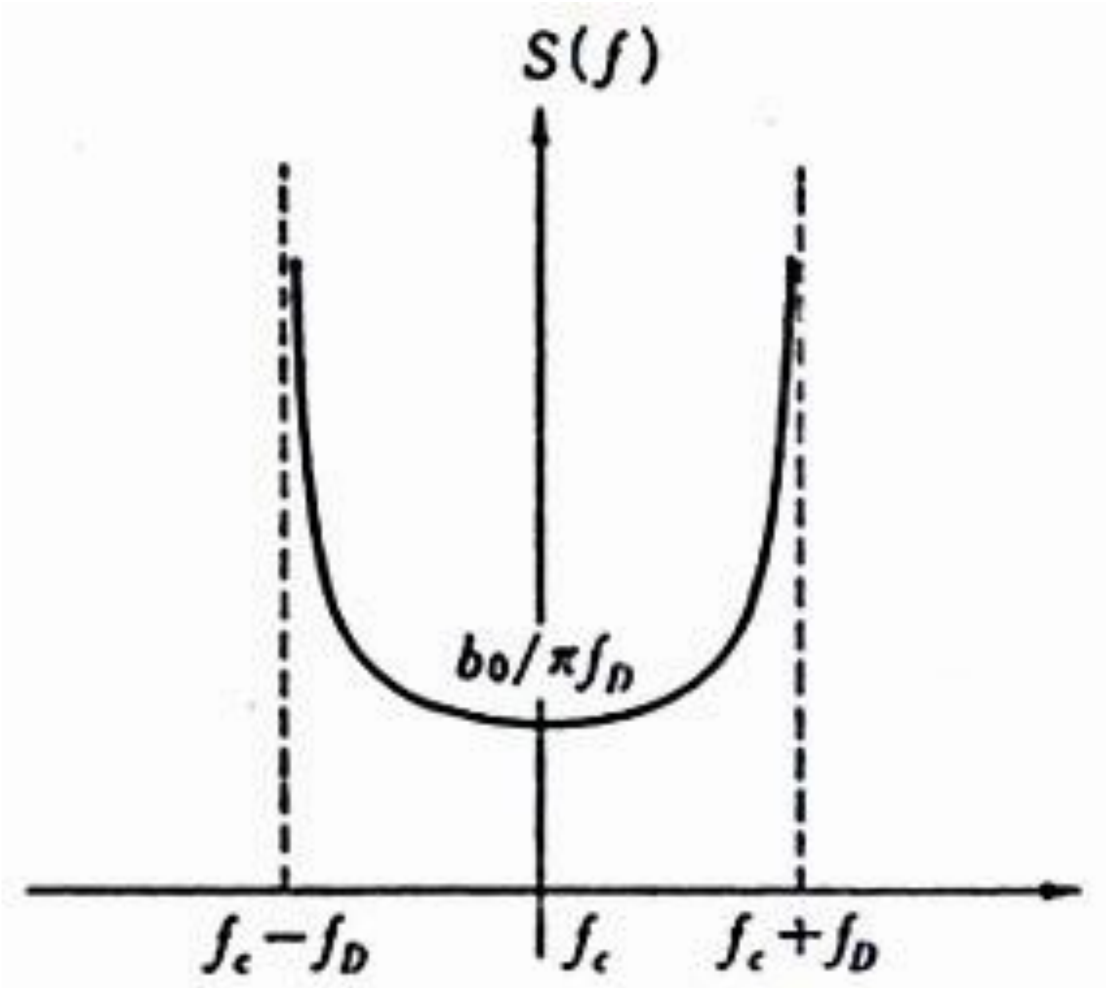
$$f = f_c + f_D \cos\phi$$

Arriving angle is uniformly distributed so that received power $S(f)\,df$ in the range $[f,\ f + df]$ is

$$S(f)df = 2 \times \frac{b_0}{2\pi} df$$

$$= \frac{b_0}{\pi f_D \sqrt{1 - [(f - f_c)/f_D]^2}} df$$

(cf. $f_c = 1.5\text{GHz}$, $v = 50\text{km/h}$, $f_D = 135\text{Hz}$)



Power Spectrum

Time derivative of random variables

$$dx(t)/dt = dR(t)/dt \times \cos(\theta(t)) - R(t) \times \sin(\theta(t)) \times d\theta(t)/dt$$

$$dy(t)/dt = dR(t)/dt \times \sin(\theta(t)) + R(t) \times \cos(\theta(t)) \times d\theta(t)/dt$$

$$pdf(x, y, dx/dt, dy/dt) \rightarrow pdf(R, \theta, dR/dt, d\theta/dt)$$

- **Level crossing number & Fade duration**

  They are important parameters for mobile communication quality.

  - **Level crossing number**

$\dot{R}$: time derivative of envelope R

A joint pdf of R and $\dot{R}$, $p\left(R, \dot{R}\right)$ is

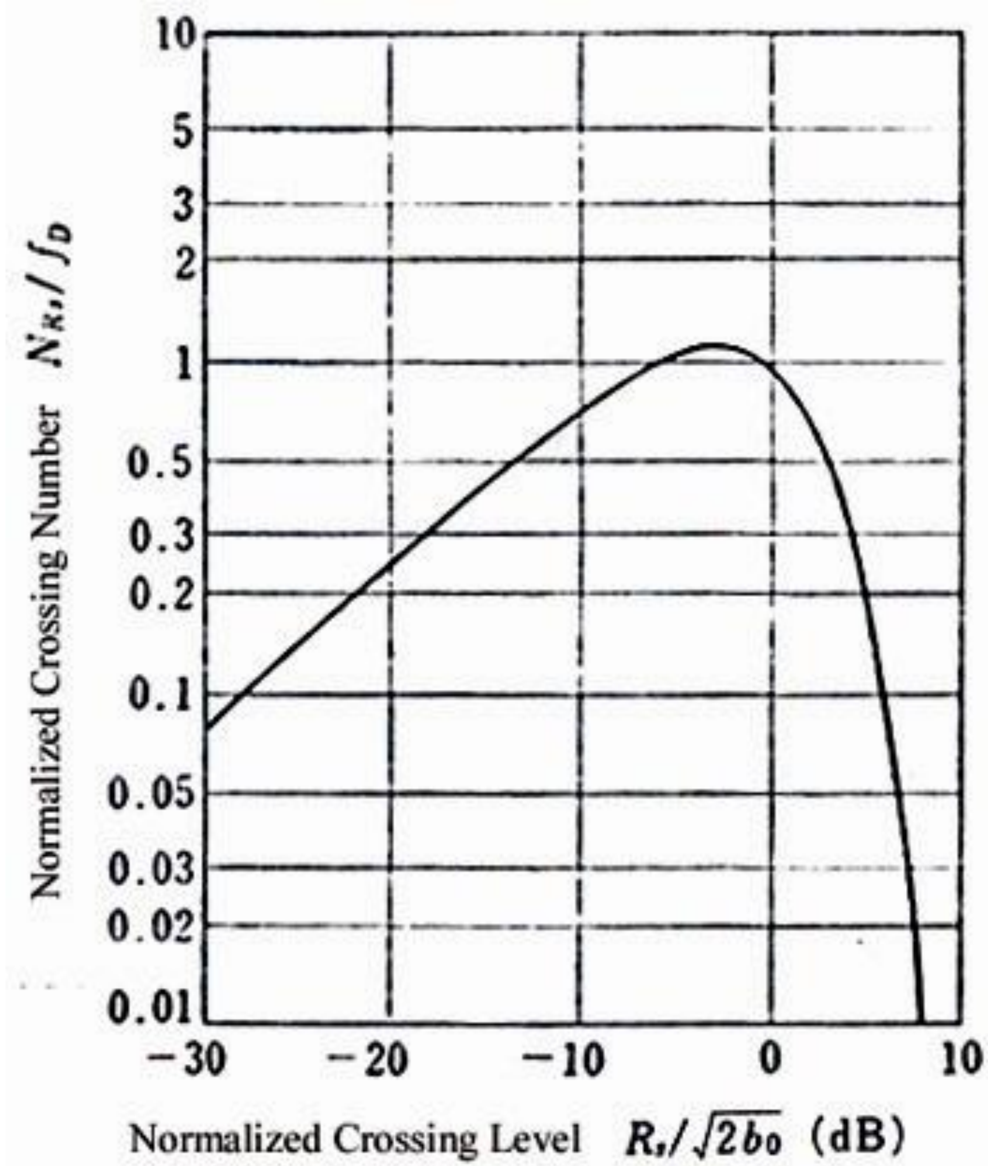
$$p\left(R, \dot{R}\right) = \frac{R}{b_0} \exp\left[-\frac{R^2}{2b_0}\right] \frac{1}{\sqrt{2\pi b_2}} \exp\left[-\frac{\dot{R}^2}{2b_2}\right]$$

Level crossing number of envelope per unit time $N\left(R_s\right)$ at the level R_s

$$N\left(R_s\right) = \int_0^{\infty} \dot{R} \cdot P\left(R_s, \dot{R}\right) d\dot{R}$$

where $b_2 = E\left[\dot{R}^2\right]$

Therefore



10
5
3
2
1
0.5
0.3
0.2
0.1
0.05
0.03
0.02
0.01
−30
−20
−10
0
10
Normalized Crossing Number N_{R_s}/f_D
Normalized Crossing Level $R_s/\sqrt{2b_0}$ (dB)

$$N\left(R_s\right) = \sqrt{\frac{b_2}{\pi b_0}}\,\frac{R_s}{\sqrt{2b_0}}\exp\left[-\frac{R_s^{\ 2}}{2b_0}\right]$$

$$= \sqrt{2\pi}\, f_D\, \frac{R_s}{\sqrt{2b_0}}\exp\left[-\frac{R_s^{\ 2}}{2b_0}\right]$$

$$N\left(\sqrt{b_0}\right)_{\max} = f_D\sqrt{\pi/e}$$

– Average fade duration time at the level R_s, $\bar{\tau}$

$$\bar{\tau} = \frac{\Pr\left[R(t) \le R_s\right]}{N(R_s)}$$

$$= \frac{\sqrt{2b_0}}{\sqrt{2\pi} f_D R_s}\left[\exp\left(\frac{{R_s}^2}{2b_0}\right) - 1\right]$$

(cf. When $R_s \big/ \sqrt{2b_0} = 0.1$ (20dB down), $f_c = 1.5$GHz, $v = 50$km/h, $\bar{\tau} = 2$ms)

- Random FM noise

$\theta(t)$ fluctuates randomly $\rightarrow$ FM noise

A pdf of $\dot{\theta}$, $p(\dot{\theta})$ is

$$p(\dot{\theta}) = \frac{1}{2}\sqrt{\frac{b_0}{b_2}}\left[1 + \frac{b_0}{b_2}\dot{\theta}^2\right]^{-3/2}$$

Random FM noise is independent on average received power.

This determines a lower bound of bit error rate.

- **Fading correlation**
  The correlation characteristics are necessary for the design of diversity system.
  - Time correlation

$$\rho(\tau) = \frac{E\left[z^*(t)z(t+\tau)\right]}{E\left[z(t)^* z(t)\right]} = J_0\left(2\pi f_D \tau\right)$$

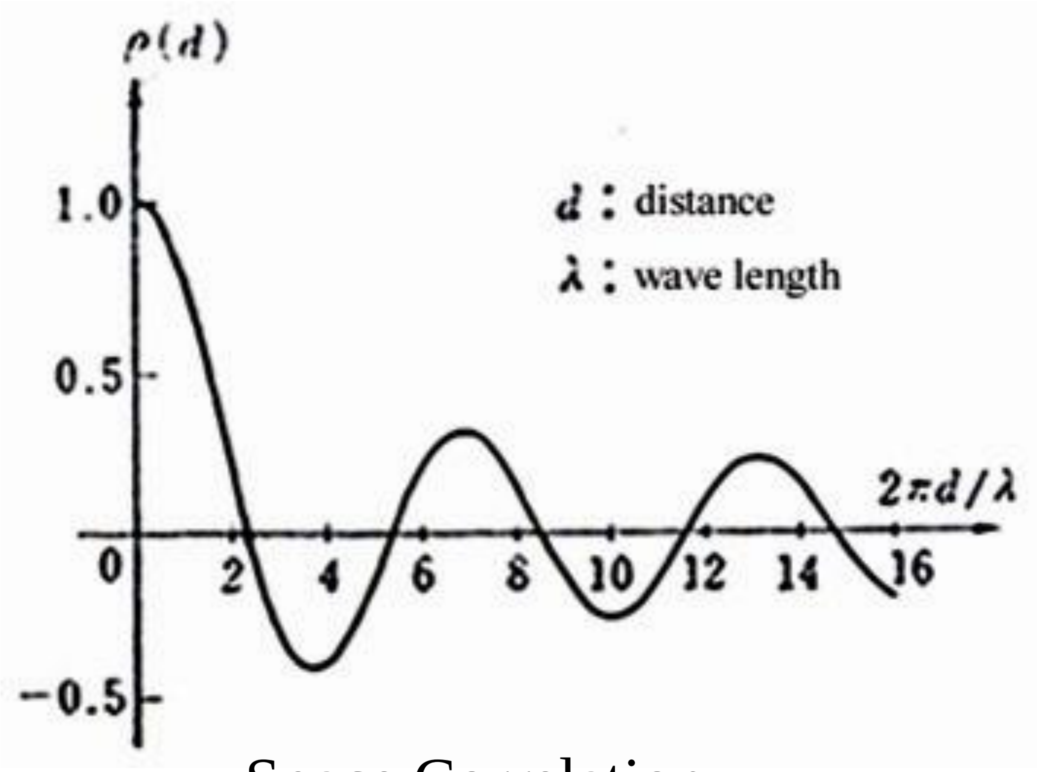
$J_0(\quad)$: 0-th order Bessel function of the first kind

– Space correlation

Space distance $d = v\tau$

$$\rho(d) = J_0(2\pi d/\lambda)$$

Around half wavelength spacing $(d \sim \lambda/2) \rightarrow$ no correlation



Space Correlation

MIMO Transmission and Antenna correlation

- Antenna correlation decreases MIMO channel capacity if average SNR at RX antenna is equal to each other.

– **Frequency correlation**
This is important parameter for Wide-band transmission.

$$\rho(\Omega) = \frac{1}{1 + j2\pi\Omega(\delta\ell/c)} \exp\left(j2\pi\ell_0/c\right)$$

ℓ_0 : minimum path length

$\delta\ell$: deviation in path length

(cf. For $\delta\ell = 200\text{m}$, coherent bandwidth is 400kHz)

A MAP Estimation of Rayleigh Fading Channel

-- A Filter Theory of Complex Gaussian Process –
and Its Application to PHS SDMA

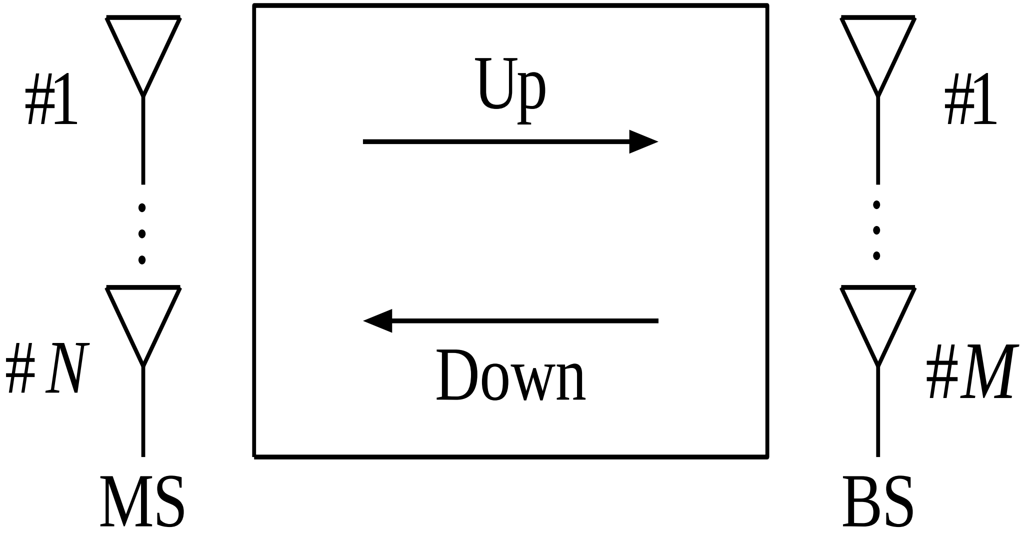
Contents

- Background & Motivation
- Complex Gaussian Stochastic Process
- Noisy Rayleigh Fading Channel
- MAP Estimation of Channel Transfer Coefficient
- Numerical Results
- Conclusion
- Future Work

Background & Motivation

- Recursive Simulation Method for Rayleigh Fading Channel.
  - How to write a computer program ?
- Fading Channel Coefficients should be estimated in SDMA PHS Systems

- Mobile Communication Channel with MIMO Systems
  - Time Variant Linear Reciprocal System



For $(N+M)$-port Circuit, a $(N+M)\,(N+M)$ scattering matrix S is defined;

$$S(f,t)=\begin{array}{c} \leftarrow N \rightarrow \quad \leftarrow M \rightarrow \\ \begin{bmatrix} S_{MM} & S_{BM} \\ S_{MB} & S_{BB} \end{bmatrix} \end{array} \begin{array}{c} \uparrow N \downarrow \\ \uparrow M \downarrow \end{array}$$

where

$S_{BM}: M \times N$ Transfer Matrix of Up-Link from MS to BS

$S_{MB}: N \times M$ Transfer Matrix of Down-Link from BS to MS

By the reciprocity,

$$S = S^{t}$$

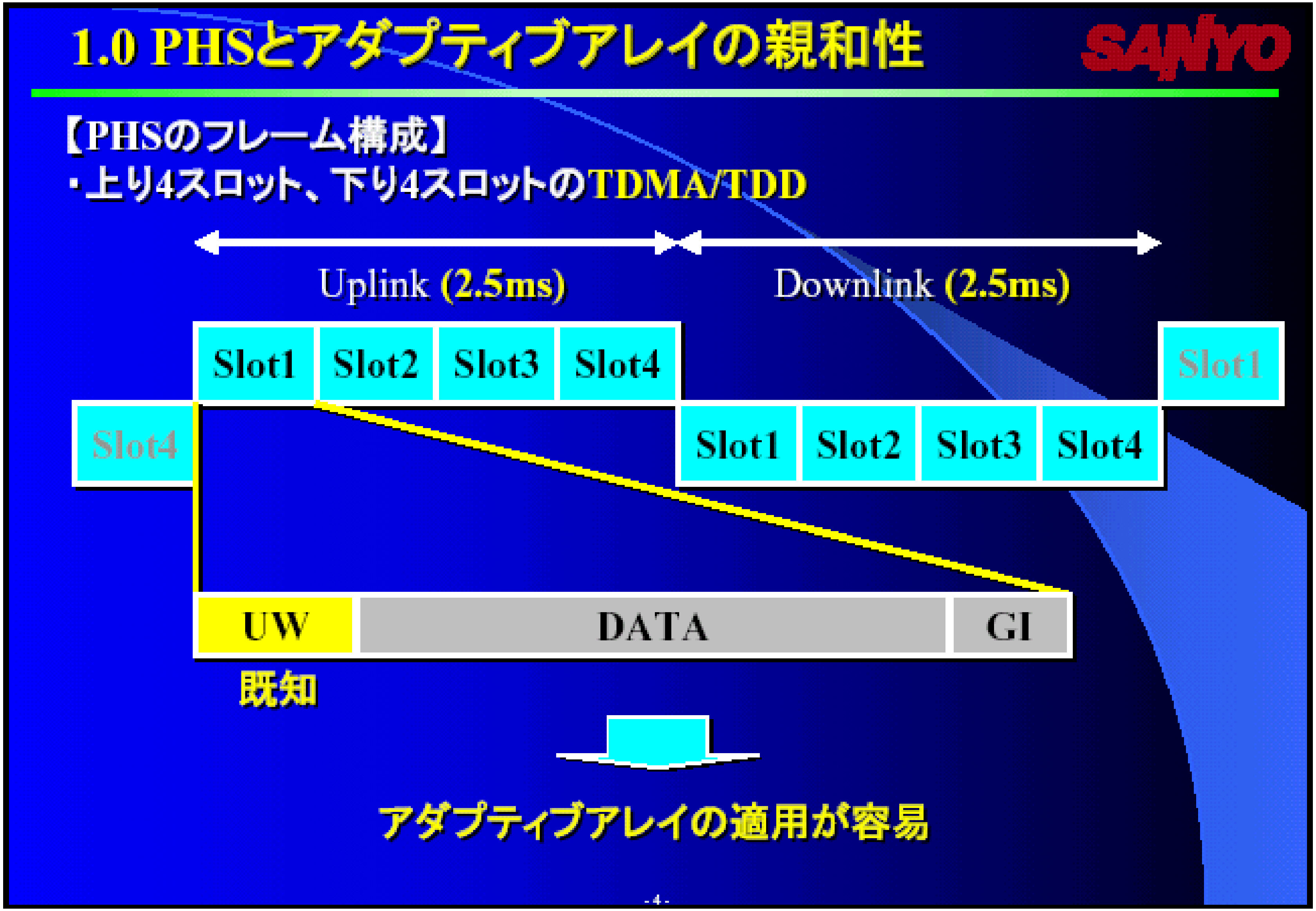
$$\therefore \quad S_{MB}(f,t) = S_{BM}(f,t)^{t}$$

Thus, the Down-Link Transfer Characteristics can be determined by the Up-Link one.

The above equality, however, holds only for the same frequency and time.

PHS system

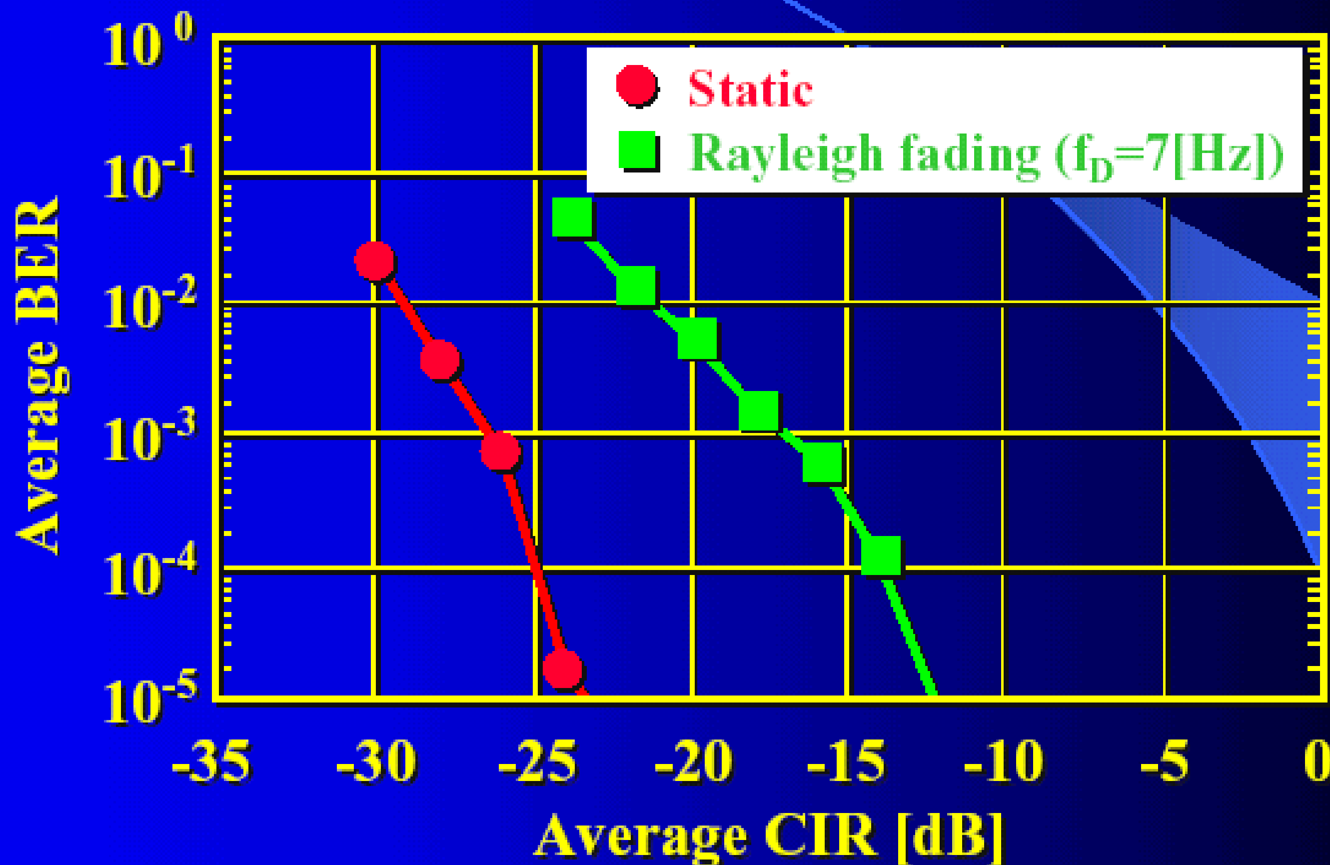
- TDD (Time Domain Duplex)
- TDMA (Time Domain Multiple Access)
- 4 Time Slot Segmentation
- Introduction of SDMA increases a channel capacity by 3 times or more.
- At the PHS base station, 4 antennas are installed.
- At most 4 data streams can be transmitted simultaneously by pre-coding at BS for down link.
- The idea is used in “i-Burst” system (IEEE802.20)


1.0 PHSとアダプティブアレイの親和性
SANYO
【PHSのフレーム構成】
・上り4スロット、下り4スロットのTDMA/TDD
Uplink (2.5ms)
Downlink (2.5ms)
Slot4
Slot1
Slot2
Slot3
Slot4
Slot1
Slot2
Slot3
Slot4
Slot1
UW
DATA
GI
既知
アダプティブアレイの適用が容易
-4-

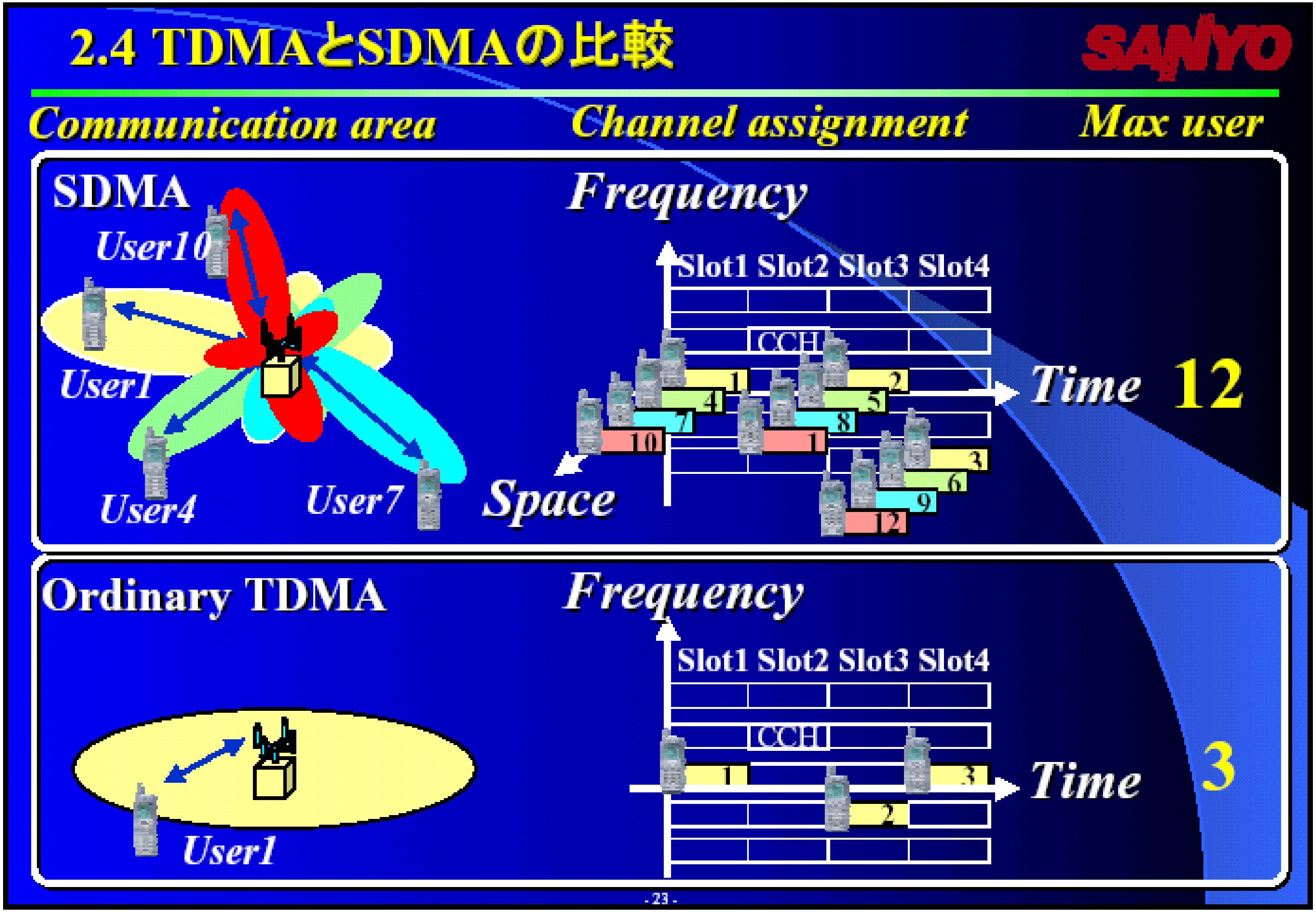
1.6.2 アダプティブアレイ基地局の特性

SANYO

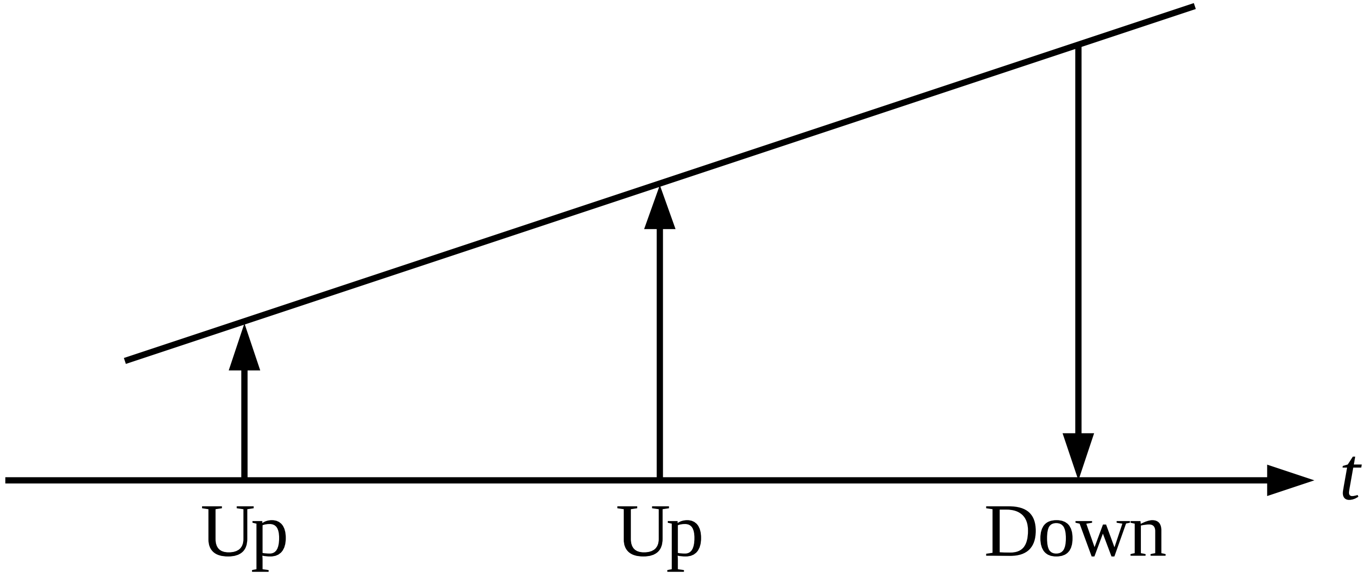
CIR 対 BER特性



- 14 -


2.4 TDMAとSDMAの比較
SANYO
Communication area
Channel assignment
Max user
SDMA
User10
User1
User4
User7
Frequency
Slot1 Slot2 Slot3 Slot4
CCH
Time
12
Space
Ordinary TDMA
User1
Frequency
Slot1 Slot2 Slot3 Slot4
CCH
Time
3
- 23 -

- Conventionally
  - Linear Extrapolation for Channel coefficient is used.



  - Noise Filtering is not taken into account.

Complex Gaussian Stochastic Process

1) Rayleigh (or Rice) Fading Coefficient : $X(t)$
2) Random White Gaussian Noise : $Y(t)$

$$\downarrow$$

3) Rayleigh Fading Coefficient contaminated with Noise:

$$Z(t) = X(t) + Y(t)$$

Stationary Gaussian Process can be characterized only by Autocorrelation Function

$$R_{ZZ}(\tau) = \overline{Z(t)Z(t+\tau)}$$
$$= R_{XX}(\tau) + R_{YY}(\tau)$$

where

$$R_{XX}(\tau) = A\,\mathrm{J}_0\left(2\pi f_D \tau\right)$$

$A = \overline{|X(t)|^2}$: Average Fading Level

J_0 : 0th Order Bessel Function of First Kind

f_D : Maximum Doppler Frequency $\left(= f\,\frac{v}{c}\right)$

f : Carrier Frequency

v : velocity of MS

c : velocity of Light

$$R_{YY}(\tau) = \begin{cases} N & (\tau = 0) \\ 0 & (\tau \neq 0) \end{cases}$$

$N = \overline{|Y(t)|^2}$: Average Noise Level

For MAP Estimation, Cross-correlation Function is also needed

$$R_{ZX}(\tau) = \overline{Z(t)X(t+\tau)} = \overline{(X(t)+Y(t))X(t+\tau)}$$

$$= \overline{X(t)X(t+\tau)} = R_{XX}(\tau)$$

$\because$ $X(t)$ and $Y(t)$ are independent.

- MAP (LS) Estimation and Optimal Noise Reduction
  - Wiener-Hopf Equation

Optimal Linear Combination Estimator Vector : $\mathbf{b}$

$$\begin{bmatrix} 1+\frac{N}{A} & J_0\left(2\pi f_D\left(t_1-t_0\right)\right) & \cdots & J_0\left(2\pi f_D\left(t_{n-1}-t_0\right)\right) \\ & \ddots & & \\ & & \ddots & \\ & & & 1+\frac{N}{A} \end{bmatrix} [\mathbf{b}] = \begin{bmatrix} J_0\left(2\pi f_D\left(t_n-t_0\right)\right) \\ \vdots \\ \vdots \\ J_0\left(2\pi f_D\left(t_n-t_{n-1}\right)\right) \end{bmatrix}$$

MAP Estimator for $X(t_n)$

$$X(t_n)_{\mathrm{MAP}} = \mathbf{b}^{\dagger}\,\mathbf{Z}$$

where

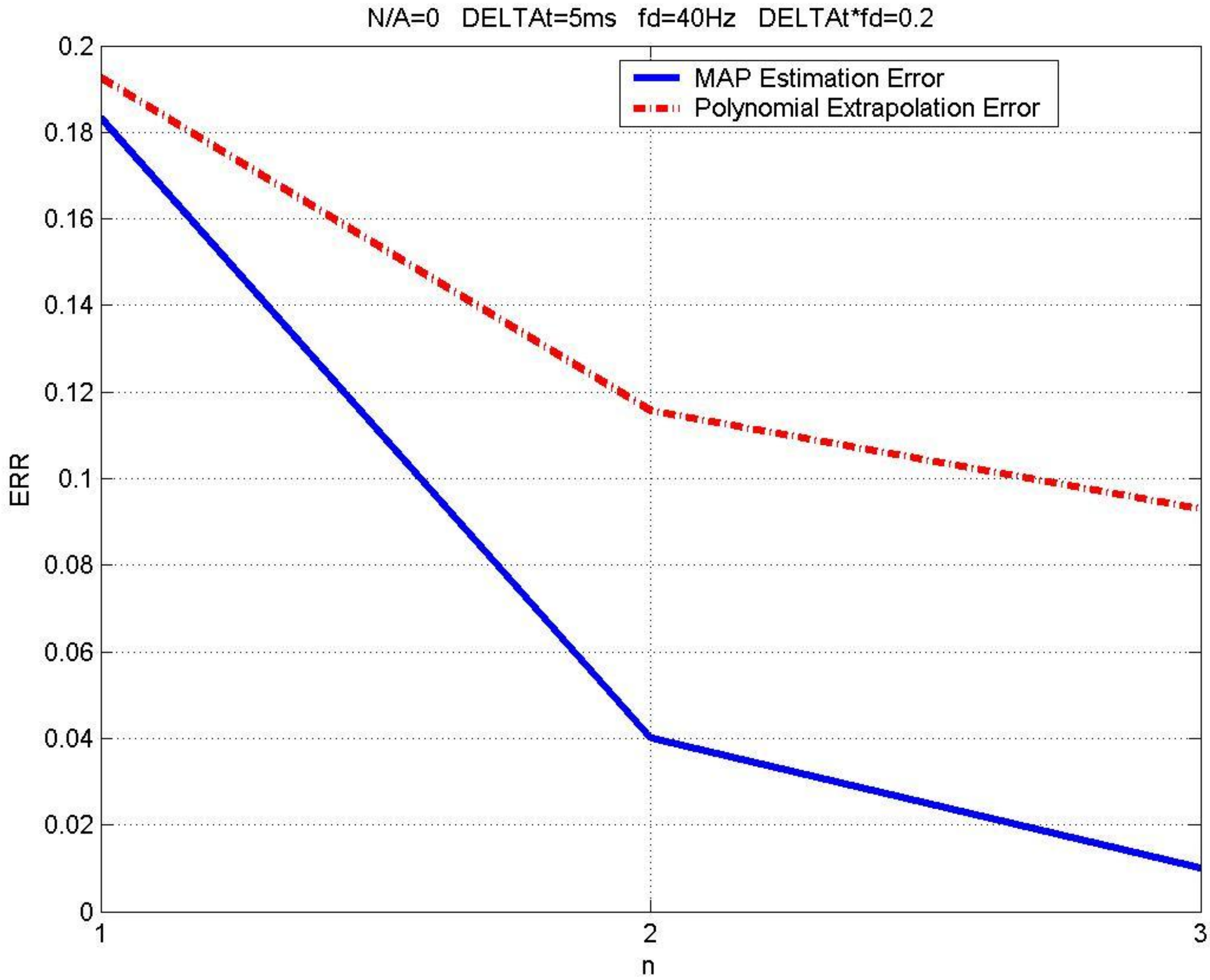
$$\mathbf{Z} = \left(Z(t_0), \ldots, Z(t_{n-1})\right)^{t} : \text{Observed Noisy Data}$$

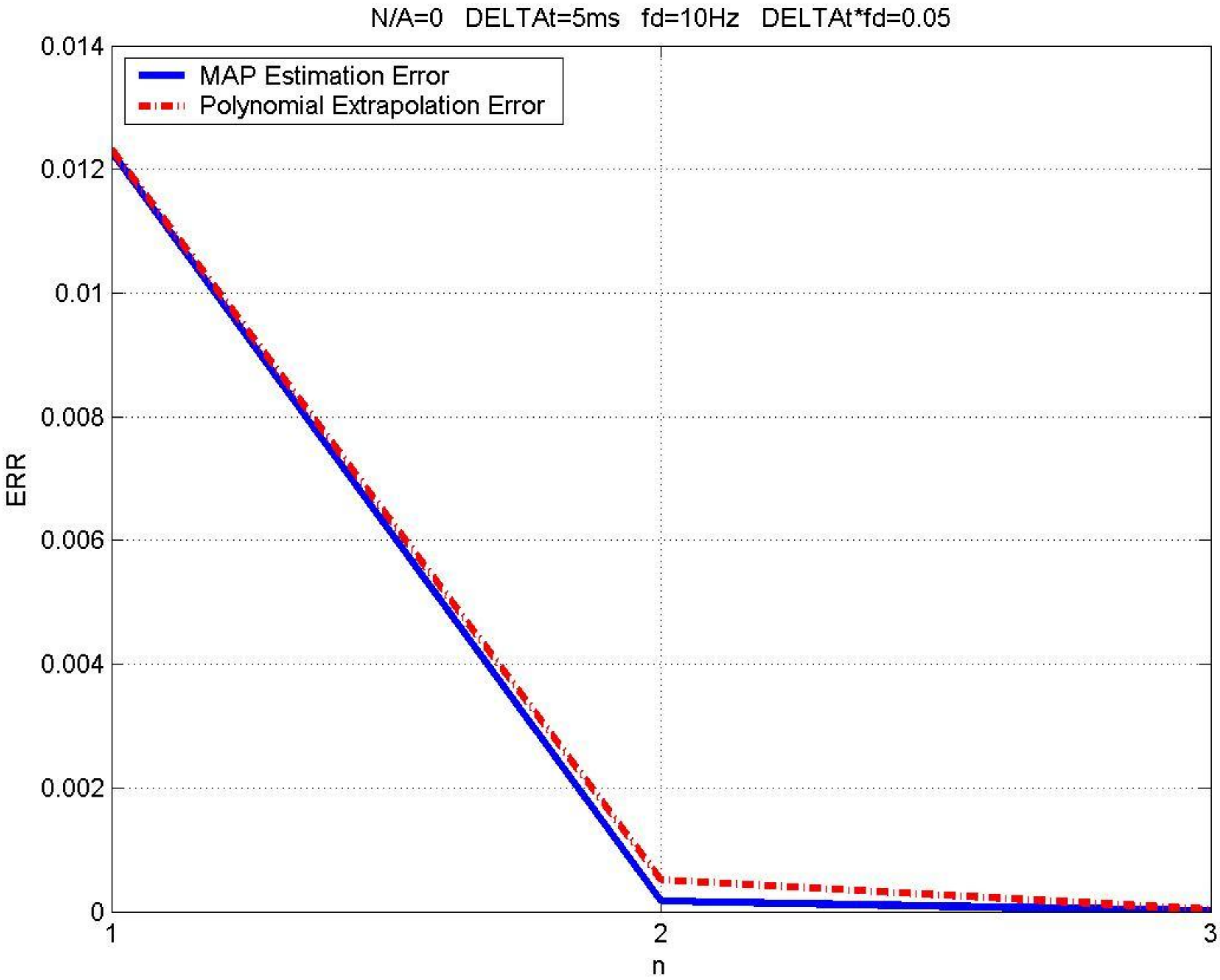
Numerical Results

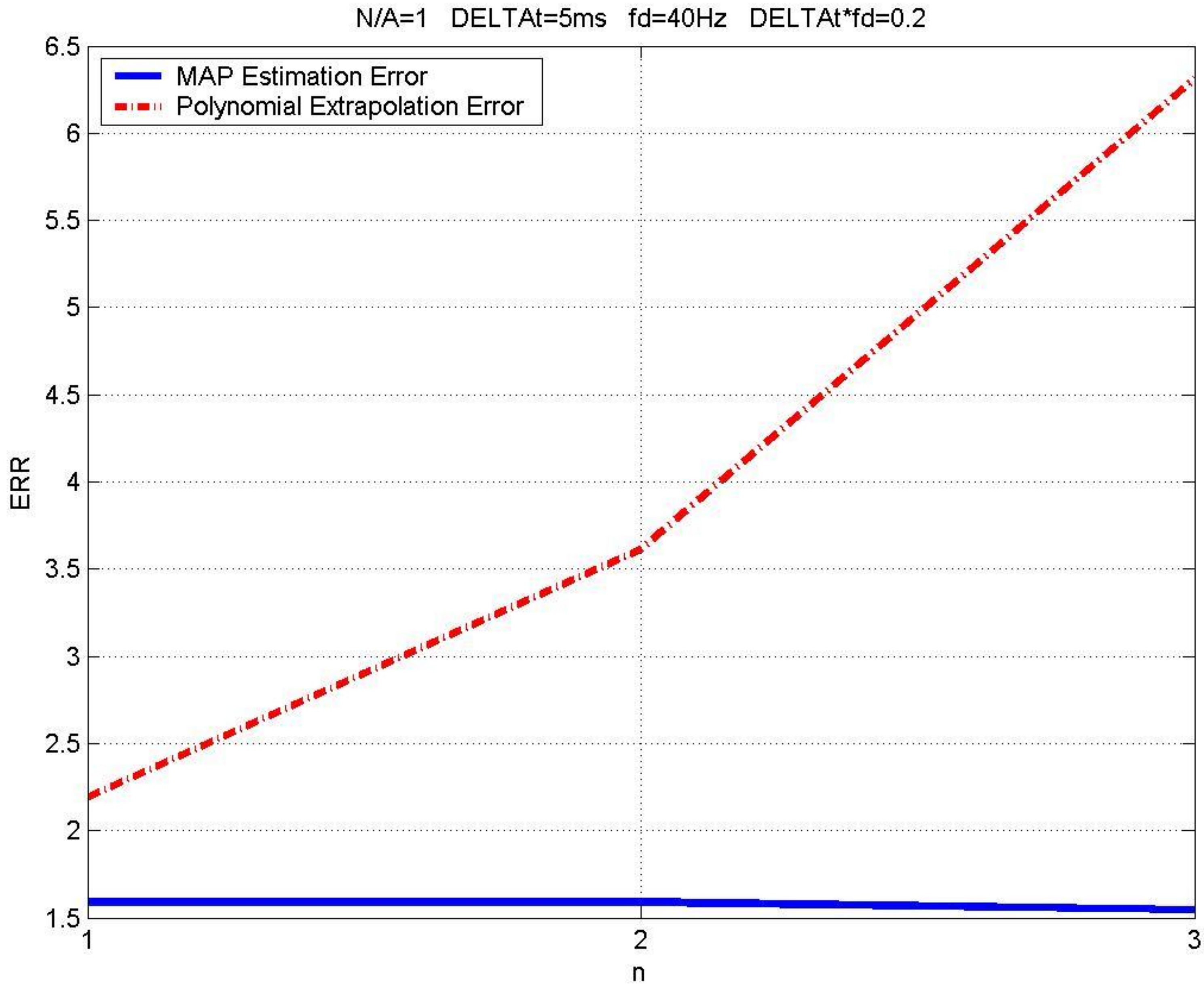
(1) Noise Level : $N/A = 0,\ 0.1,\ 1$

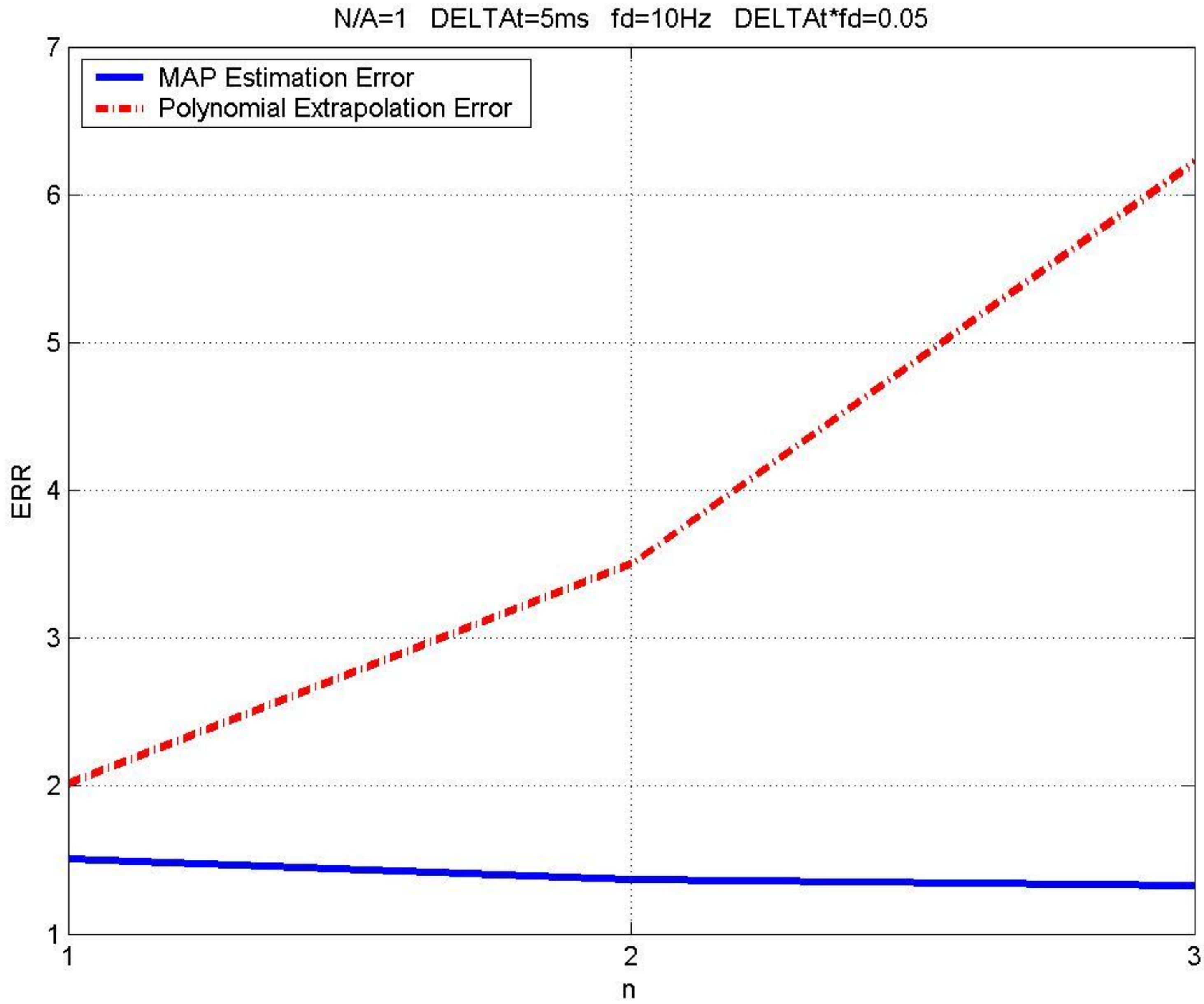
(2) Doppler Frequency : $f_D = 10,\ \ 40[\mathrm{Hz}]$

(3) No. of Data : $n = 1,\ 2,\ 3$


N/A=0 DELTAt=5ms fd=40Hz DELTAt*fd=0.2
MAP Estimation Error
Polynomial Extrapolation Error
ERR
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
1
2
3
n


N/A=0 DELTAt=5ms fd=10Hz DELTAt*fd=0.05
MAP Estimation Error
Polynomial Extrapolation Error
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
ERR
1
2
3
n


N/A=1 DELTAt=5ms fd=40Hz DELTAt*fd=0.2
MAP Estimation Error
Polynomial Extrapolation Error
ERR
n
6.5
6
5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
2
3


N/A=1 DELTAt=5ms fd=10Hz DELTAt*fd=0.05
MAP Estimation Error
Polynomial Extrapolation Error
ERR
n
7
6
5
4
3
2
1
1
2
3

Conclusion

- Estimation of Fading Coefficient is useful for TDMA/ TDD.
- Conventional Estimation is not satisfactory.
- Estimation Error can be greatly reduced by MAP Estimation.