Fundamentals of Dynamics (13)

Department of Mechanical and Control Engineering

Hiroshi Yamaura

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Multi-degree-of-freedom Vibration System (2)

Analytical Solution

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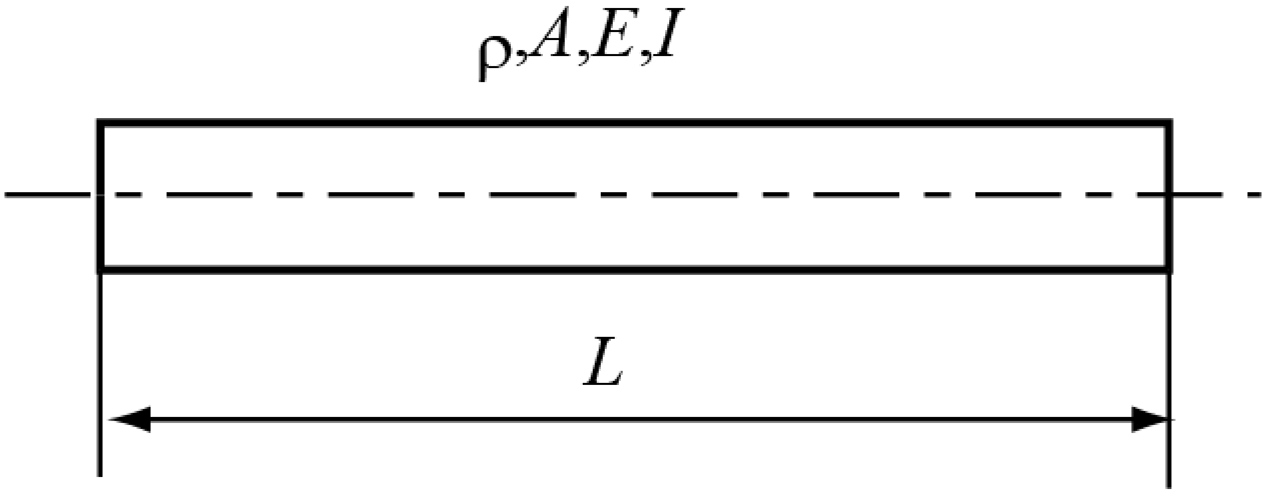
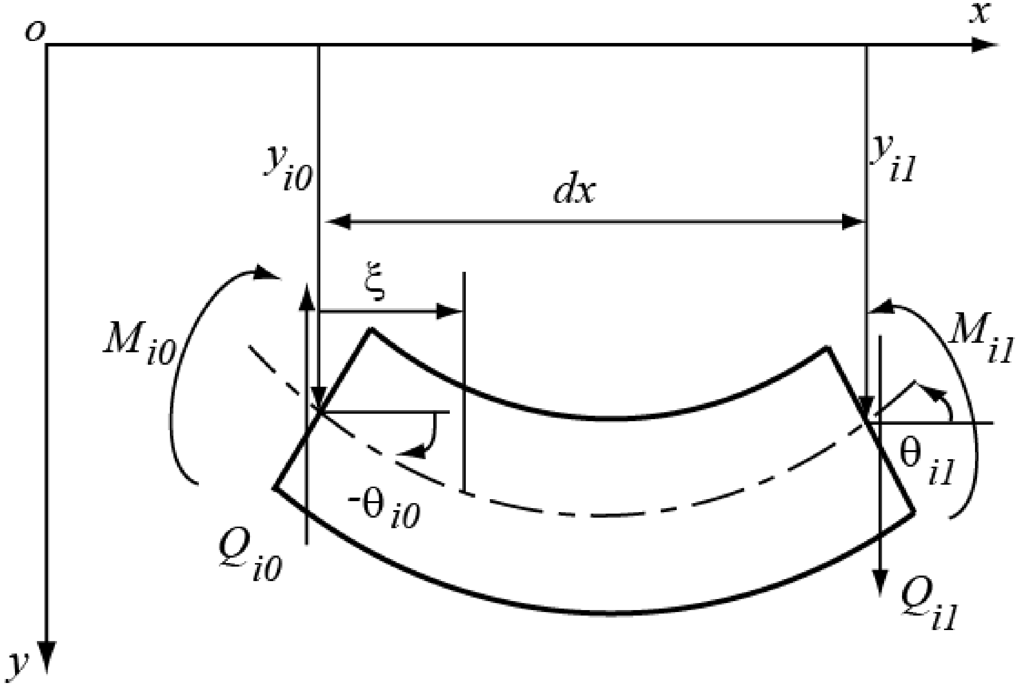
Bending Vibration of a Beam (3)



Uniform Beam

Bending Vibration of a Beam (4)



Equation of Motion

$$\rho A \frac{\partial^2 y}{\partial t^2} + EI \frac{\partial^4 y}{\partial x^4} = 0$$

Coordinates and acting forces on an infinitesimal element

Analytical Solution (1)

$$\rho A \frac{\partial^2 y}{\partial t^2} + EI \frac{\partial^4 y}{\partial x^4} = 0$$

$$\longrightarrow \quad \rho A Y \ddot{T} + EITY^{(4)} = 0$$

$$y(x,t) = Y(x)T(t)$$

$$\frac{\ddot{T}}{T} = \frac{-EIY^{(4)}}{\rho A Y} \equiv -\omega^2$$

Analytical Solution (2)

(a) $Y^{(4)} - a^4 Y = 0$ where $a^4 = \omega^2 \dfrac{\rho A}{EI}$

$$Y(x) = Y_1 e^{ax} + Y_2 e^{-ax} + Y_3 e^{iax} + Y_4 e^{-iax}$$

$Y(x)$: Eigen function

a is determined from boundary conditions of $Y(x)$

a_i : i-th solution of the characteristic equation

Analytical Solution (3)

$$Y_i(x) = Y_{i1}e^{a_i x} + Y_{i2}e^{-a_i x} + Y_{i3}e^{ia_i x} + Y_{i4}e^{-ia_i x}$$

$$\{Y_{i1}\ Y_{i2}\ Y_{i3}\ Y_{i4}\} = \{\alpha_i\ \alpha_i y_{2i}\ \alpha_i y_{3i}\ \alpha_i y_{4i}\}$$

Orthogonality of eigen functions

$$\int_0^L Y_i(x)Y_j(x)dx = \begin{cases} 0 & i \neq j \\ \beta & i = j \end{cases}$$

Either α_i or β can be given arbitrarily.

Analytical Solution (4)

(b) $\ddot{T}_i + \omega_i^2 T_i = 0$

$\rightarrow$

$$T_i(t) = T_i(0)\cos(\omega_i t) + \frac{\dot{T}_i(0)}{\omega_i}\sin(\omega_i t)$$

where $$\omega_i^2 = \frac{EI}{\rho A} a_i^4$$

Analytical Solution (5)

(c) General Solution

$$y(x,t) = \sum_{n=1}^{\infty} Y_i(x) T_i(t)$$

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Analytical Solution (6)

(d) Initial Value Response

$y(x,0)$ and $\dot{y}(x,0)$ are given.

The value of β is set as 1.

$$\rightarrow \quad T_i(0) = \int_0^L y(x,0) Y_i(x)\, dx$$

$$\dot{T}_i(0) = \int_0^L \dot{y}(x,0) Y_i(x)\, dx$$