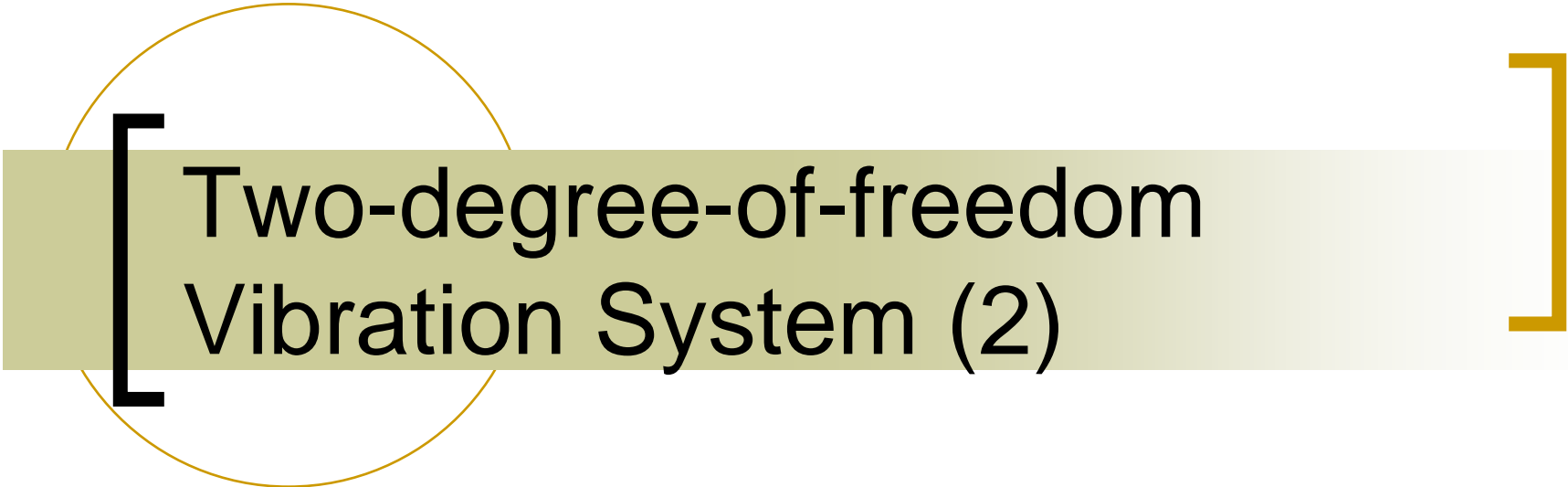




# Fundamentals of Dynamics (10)

Department of Mechanical and  
Control Engineering

Hiroshi Yamaura



# Two-degree-of-freedom Vibration System (2)

# [Modal Analysis (3)]

Equation of motion with no excitation force

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{0}$$

Coordinate transformation

$$\mathbf{x} = \mathbf{V}\mathbf{y}$$

$$\mathbf{x} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$$

Physical coordinate

$$\mathbf{y} = \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix}$$

Modal coordinate

$y_i$  : Displacement of  
the i-th mode

# [Modal Analysis (4)]

Equation of motion with no excitation force

$$\mathbf{V}^T \mathbf{M} \mathbf{V} \ddot{\mathbf{y}} + \mathbf{V}^T \mathbf{K} \mathbf{V} \mathbf{y} = \mathbf{0}$$

$$\longrightarrow \ddot{\mathbf{y}} + \Lambda \mathbf{y} = \mathbf{0}$$

where

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

Asymmetric elements are 0.

Responses are divided into two independent modes.

# [ Initial Value Response (1) ]

$$\ddot{y}_1 + \lambda_1 y_1 = 0$$

$$\longrightarrow y_1(t) = y_1(0) \cos(\omega_1 t) + \frac{\dot{y}_1(0)}{\omega_1} \sin(\omega_1 t)$$

$$\ddot{y}_2 + \lambda_2 y_2 = 0$$

$$\longrightarrow y_2(t) = y_2(0) \cos(\omega_2 t) + \frac{\dot{y}_2(0)}{\omega_2} \sin(\omega_2 t)$$

where

$$\omega_i = \sqrt{\lambda_i} \quad (i = 1, 2)$$

# [ Initial Value Response (2) ]

Initial conditions

$$\begin{Bmatrix} y_1(0) \\ y_2(0) \end{Bmatrix} = \mathbf{V}^{-1} \begin{Bmatrix} x_1(0) \\ x_2(0) \end{Bmatrix}, \quad \begin{Bmatrix} \dot{y}_1(0) \\ \dot{y}_2(0) \end{Bmatrix} = \mathbf{V}^{-1} \begin{Bmatrix} \dot{x}_1(0) \\ \dot{x}_2(0) \end{Bmatrix}$$

Reconstruction in Physical coordinates

$$\mathbf{x}(t) = \mathbf{V}\mathbf{y}(t)$$

# [ Initial Value Response (3) ]

Beat

