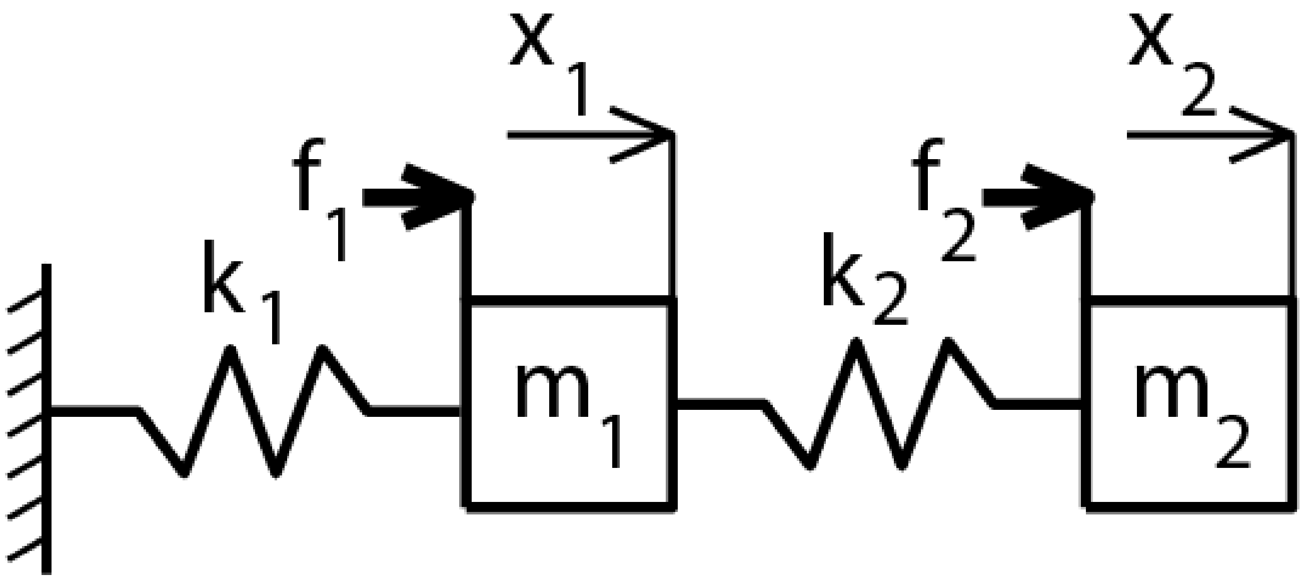
Fundamentals of Dynamics (9)

Department of Mechanical and Control Engineering

Hiroshi Yamaura

Two-degree-of-freedom Vibration System (1)

Analytical Model



Analytical model of a two-degree-of-freedom vibration system

Equation of Motion

$$m_1\ddot{x}_1 + k_1x_1 + k_2(x_1 - x_2) = f_1$$

$$m_2\ddot{x}_2 + k(x_2 - x_1) = f_2$$

These equations can be written as $\boxed{\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}}$

where $\mathbf{M} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$, $\mathbf{x} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$

$$\mathbf{K} = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}, \quad \mathbf{f} = \begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix}$$

Modal Analysis (1)

No exciting force = Free vibration = Initial value response

$$\boxed{\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{0}}$$

General eigen value problem

$$\boxed{(\mathbf{K} - \lambda\mathbf{M})\mathbf{v} = \mathbf{0}}$$

Solutions $\lambda = \lambda_1,\ \lambda_2$

$$\mathbf{v} = \mathbf{v}_1,\ \mathbf{v}_2$$

Modal Analysis (2)

Orthoganatity of eigen vectors

$$\mathbf{v}_i^T \mathbf{M} \mathbf{v}_j = \begin{cases} 0 & i \neq j \\ \overline{m}_i & i = j \end{cases}, \quad \mathbf{v}_i^T \mathbf{K} \mathbf{v}_j = \begin{cases} 0 & i \neq j \\ \overline{k}_i & i = j \end{cases}$$

$\overline{m}_i$: Modal mass, $\overline{k}_i$: Modal stiffness $\longrightarrow$ $\boxed{\lambda_i = \dfrac{\overline{k}_i}{\overline{m}_i}}$

If we define a modal matrix $\mathbf{V} \equiv [\mathbf{v}_1 \ \mathbf{v}_2]$,

then

$$\mathbf{V}^T \mathbf{M} \mathbf{V} = \begin{bmatrix} \overline{m}_1 & 0 \\ 0 & \overline{m}_2 \end{bmatrix}, \quad \mathbf{V}^T \mathbf{K} \mathbf{V} = \begin{bmatrix} \overline{k}_1 & 0 \\ 0 & \overline{k}_2 \end{bmatrix}.$$