Fundamentals of Dynamics (8)

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Two-degree-of-freedom Positioning Mechanism (5)

Optimal Control (1)

Basic Concept based on the variation principal

Criteria (Functional) $$J = \int V(\mathbf{z}(t), \dot{\mathbf{z}}(\mathbf{t}))\, dt$$

Boundary conditions (Fixed Ends) $$\mathbf{z}(0) = \mathbf{z}_0,\ \mathbf{z}(T) = \mathbf{z}_1$$

$$\delta J = 0 \iff \frac{dV}{d\mathbf{z}} - \frac{d}{dt}\left(\frac{dV}{d\dot{\mathbf{z}}}\right) = 0$$

Euler's Equation

Optimal Control (2)

Special Case for Access Control

Constraint $\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}$ or $\boxed{\ddot{\mathbf{y}} + \Lambda \mathbf{y} = \mathbf{f}_m}$

New Criteria $J' = \int V'(\mathbf{z}(t), \dot{\mathbf{z}}(t), \ddot{\mathbf{z}}(t))\, dt$

$$V' = V + \mathbf{p}^T (\ddot{\mathbf{y}} + \Lambda \mathbf{y} - \mathbf{f}_m)$$

$$\mathbf{z}(t) = \begin{Bmatrix} \mathbf{y}(t) \\ f_1(t) \end{Bmatrix}$$

$\mathbf{p}$: Lagrange Multiplier

Optimal Control (3)

$$\delta J' = 0$$

$$\Leftrightarrow \frac{dV'}{d\mathbf{z}} - \frac{d}{dt}\left(\frac{dV'}{d\dot{\mathbf{z}}}\right) + \frac{d}{dt^2}\left(\frac{dV'}{d\ddot{\mathbf{z}}}\right) = 0$$

Optimal Control (4)

Example (1)

$$J = \int \frac{1}{2} f_1^{\,2}\, dt \qquad V = \frac{1}{2} f_1^{\,2}$$

Example (2)

$$J = \int \frac{1}{2}\left(\frac{df_1}{dt}\right)^2 dt \qquad V = \frac{1}{2}\left(\frac{df_1}{dt}\right)^2$$