Fundamentals of Dynamics (5)

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Two-degree-of-freedom Positioning Mechanism (2)

Modal Analysis (5)

Meaning of the eigen vectors

$$\mathbf{v}_1 = \begin{Bmatrix} \alpha \\ \alpha \end{Bmatrix}$$

Displacement of two mass have the same sign and the same value.

→ Rigid body mode

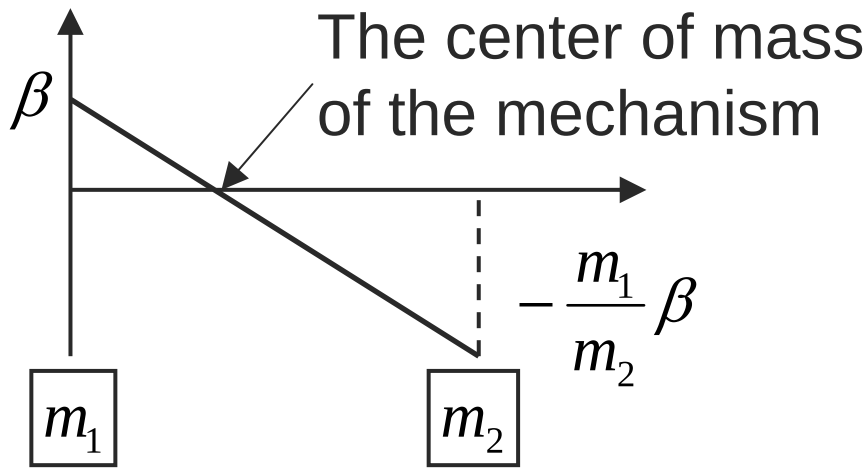
$$\mathbf{v}_2 = \begin{Bmatrix} \beta \\ -\dfrac{m_1}{m_2}\beta \end{Bmatrix}$$

Displacement of two mass have the different sign and the different value.

→ Vibration mode

Modal Analysis (6)

$$\mathbf{v}_2 = \begin{Bmatrix} \beta \\ -\dfrac{m_1}{m_2}\beta \end{Bmatrix}$$



The center of mass of the mechanism doesn't move with the vibration mode.

Modal Analysis (7)

Equation of motion with no excitation force

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{0}$$

Coordinate transformation

$$\mathbf{x} = \mathbf{V}\mathbf{y}$$

$$\mathbf{V} = \begin{bmatrix} \dfrac{1}{\sqrt{m_1 + m_2}} & \sqrt{\dfrac{m_2}{m_1(m_1 + m_2)}} \\ \dfrac{1}{\sqrt{m_1 + m_2}} & -\sqrt{\dfrac{m_1}{m_2(m_1 + m_2)}} \end{bmatrix}$$

$\mathbf{x} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$ Physical coordinate

$\mathbf{y} = \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix}$ Modal coordinate

y_i : Displacement of the i-th mode

Modal Analysis (8)

Equation of motion with no excitation force

$$\mathbf{V}^T\mathbf{M}\mathbf{V}\ddot{\mathbf{y}} + \mathbf{V}^T\mathbf{K}\mathbf{V}\mathbf{y} = \mathbf{0}$$

$$\longrightarrow \quad \ddot{\mathbf{y}} + \Lambda\mathbf{y} = \mathbf{0}$$

where

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

Asymmetric elements are 0.

Responses are divided into two independent modes.

Initial Value Response (1)

$$\ddot{y}_1 = 0 \longrightarrow y_1(t) = y_1(0) + \dot{y}_1(0)t$$

$$\ddot{y}_2 + \lambda_2 y_2 = 0$$

$$\longrightarrow y_2(t) = y_2(0)\cos(\omega_2 t)$$

$$+ \frac{\dot{y}_2(0)}{\omega_2}\sin(\omega_2 t)$$

where

$$\omega_2 = \sqrt{\lambda_2} = \sqrt{\frac{k(m_1 + m_2)}{m_1 m_2}}$$

Initial Value Response (2)

Initial conditions

$$\begin{Bmatrix} y_1(0) \\ y_2(0) \end{Bmatrix} = \mathbf{V}^{-1} \begin{Bmatrix} x_1(0) \\ x_2(0) \end{Bmatrix}, \qquad \begin{Bmatrix} \dot{y}_1(0) \\ \dot{y}_2(0) \end{Bmatrix} = \mathbf{V}^{-1} \begin{Bmatrix} \dot{x}_1(0) \\ \dot{x}_2(0) \end{Bmatrix}$$

$$\mathbf{V}^{-1} = -\sqrt{m_1 m_2} \begin{bmatrix} -\dfrac{m_1}{m_2}\beta & -\beta \\ -\alpha & \alpha \end{bmatrix} = \begin{bmatrix} \dfrac{m_1}{\sqrt{m_1+m_2}} & \dfrac{m_2}{\sqrt{m_1+m_2}} \\ \dfrac{\sqrt{m_1 m_2}}{\sqrt{m_1+m_2}} & \dfrac{-\sqrt{m_1 m_2}}{\sqrt{m_1+m_2}} \end{bmatrix} \quad \text{in case (3)}$$

Initial Value Response (3)

Reconstruction in Physical coordinates

$$\mathbf{x}(t) = \mathbf{V}\mathbf{y}(t)$$