Benevolence

Norimasa Kobayashi

Department of Value and Decision Science (VALDES)
Tokyo Institute of Technology

The Golden Rule – True Agape Love?

Do unto others as you would have them do unto you.

– Jesus (c. 5 BC - AD 32) in the Gospels,
Matthew 7:12, Luke 6:31, Luke 10:27

None of you truly believes until he loves for his brother what he loves for himself.

– Muhammad (c. AD 571 - 632) in a Hadith.

(taken from Wikipedia English)

To implement the Golden Rule correctly may be much harder than you think.

Common Knowledge – Dig Deep into His Mind!

The difficulty with the golden rule is that your wise and loving partner has the golden rule in his mind as well as you do.

Example

Ann "does unto Bob as Ann would have Bob do unto Ann."
But,
Bob "does unto Ann as Bob would have Ann do unto Bob."

The example causes the infinite self-referring nested structure.
Q. Can Ann and Bob solve this problem and find out what they really want?
When a stable solution exists to this infinite chain of reasoning and is known to all the members, the solution is common knowledge (a jargon of logic!).
i.e.) Ann knows that Bob knows that Ann knows ...

Interdependent Utilities

We always start with the simplest example and train our brains!

Definition (Interdependent utilities)

For each consequence $s \in S$,

$$U_A(s) = F_A(s, U_B(s))$$

$$U_B(s) = F_B(s, U_A(s))$$

Interdependency may be:

Benevolence F_i is increasing in U_j.

Spite F_i is decreasing in U_j.

Interdependency and Individual Utilities

We deal with a special case in which the interdependent utility of each individual is defined with the sum of her individual utility with the opponent's utility.

Linear Model [Bergstrom(1999)]

$$U_A(s) = F_A(s, U_B(s)) = u_A(s) + \lambda_A U_B(s)$$
$$U_B(s) = F_B(s, U_A(s)) = u_B(s) + \lambda_B U_A(s)$$

λ_i is interdependency parameter:

Benevolence $\lambda_i > 0$

Spite $\lambda_i < 0$

Selfish $\lambda_i = 0$

Notice the self-referring structure in the linear model!

Feasibility of Interdependency

When $\lambda_A \lambda_B \geq 1$, the interdependency is unstably strong.
Both Ann and Bob do not have a clear preference on the social states.

Example

- $\lambda_A, \lambda_B > 0$ – Ann and Bob love each other badly. Both Ann and Bob say "I am ok with anything. I like to go with whatever you like!"
- $\lambda_A, \lambda_B < 0$ – Ann and Bob dislike each other badly.

- Perhaps, love relationship is more stable, if at least one of the two has something which she or he can enjoy alone, and love promotes that. Both purely loving each other with little self-interest can be quite destructive.
- In a stable marriage, "life" always coexists with love. (Actually, marriage (at least in terms of conventional language) is more of life contract than love relationship!)

Feasible Solution

Henceforth, we assume $\lambda_A \lambda_B < 1$.
Then, by solving the linear interdependence model with respect to $u_A(s), u_B(s)$, the following function is obtained.

$$U_A(s) = G_A(u_A(s), u_B(s)) = \frac{1}{1 - \lambda_A \lambda_B}(u_A(s) + \lambda_A u_B(s))$$

$$U_B(s) = G_B(u_A(s), u_B(s)) = \frac{1}{1 - \lambda_A \lambda_B}(u_B(s) + \lambda_B u_A(s))$$

Application 1 – Division of a Cake

Ann and Bob want to allocate $100s\%$ to Ann and $100(1-s)\%$ to Bob. What is the optimal allocation for whom?

Example

Assume $u_A(s) \equiv 0$ and $u_B(s) \geq 0$. Then, the optimal allocation for both Ann and Bob allocates 0 to Ann and 1 to Bob.
However, this result does not imply that Bob is selfish. He takes all the cake only because he knows that makes Ann happiest as well.

Application II – Prisoners' Dilemma Game

We introduce the famous game of prisoners' dilemma as an example.

Definition (Prisoners' Dilemma Game)

Ann \ Bob	Cooperate	Defect
Cooperate	c_A, c_B	b_A, a_B
Defect	a_A, b_B	d_A, d_B

$$a_i > c_i > d_i > b_i (i = Ann, Bob)$$

- (Defect, Defect) is the unique Nash (as well as dominant-strategy) equilibrium of the game and is Pareto-inefficient.
- (Cooperate, Cooperate) is Pareto-efficient but is not stable (not incentive compatible).

Q. What happens if Ann and Bob become benevolent?

Prisoners' Dilemma Game with Benevolence

With a linear transformation of utilities, we obtain

$1\backslash 2$	C	D
C	$c_1 + \lambda_1 c_2, c_2 + \lambda_2 c_1$	$b_1 + \lambda_1 a_2, a_2 + \lambda_2 b_1$
D	$a_1 + \lambda_1 b_2, b_2 + \lambda_2 a_1$	$d_1 + \lambda_1 d_2, d_2 + \lambda_2 d_1$

Case I – Mutual Cooperation

Both Ann and Bob are sufficiently benevolent (λ_A and λ_B are sufficiently large).
More precisely,

Mutual Cooperation

If $c_1 + \lambda_1 c_2 > a_1 + \lambda_1 b_2$, $c_2 + \lambda_2 c_1 > a_2 + \lambda_2 b_1$, then,
the unique dominant-strategy as well as Nash equilibrium is (C, C).

This is the most established case of social norm sort of argument in sociology and other fields, in which love can establish a Pareto-efficient outcome.
However, love is not necessarily this uniform!!! There may be many forms of love.

Case II – Free Ride

Ann is benevolent and Bob is selfish (λ_A is large and λ_B is small).
Consider the simplest case in which $\lambda_B = 0$.

Free Ride

If $b_A + \lambda_A a_B > d_A + \lambda_A d_B$, then,
the unique dominant-strategy as well as Nash equilibrium is (C, D).

Bob free rides on Ann's benevolence in this case. Ann is happy with that.
(Sometimes, women love selfish men, don't they?)

Case III – Concentration of Resources (Battle of Sexes)

It is not necessarily the case that if (C, D) is realized, then Bob is selfish. In some cases, it makes sense to concentrate resources on one side like division of labor.

Bob may agree to his becoming physically happy in order to make Ann happy as well. In fact, in some drastic cases, he may want less physical happiness as well, but compromises.

More precisely,

Benevolent Version of Battle of Sexes

(Detailed conditions omitted)

Both (C, D) and (D, C) can become Nash equilibrium.

Some Japanese women loved men such as poor artists pursuing dreams. Such men pursued dreams not only for themselves but to respond to the support of the loving partner. It was often difficult for the men to give up dreams because it may disappoint the partner.

Conclusion – How to implement Golden Rule Wisely

- There may be various forms of love. One side may take all the resources even when both love each other strongly.
- Benevolence is not about "giving" only.
  A true lover is also very good at "accepting".
  Imagine always that he may also want to give you, if you want to give him.
- Observe whether he is really happy or not carefully.
  Show him you are happy.
- Do not assume easily that others have the same taste as you. Guess his unique taste utilizing all your sensitivity instead.
  Utilize Pareto efficiency principle wisely.

T. C. Bergstrom.
Systems of benevolent utility functions.
Journal of Public Economic Theory, 1:71–100, 1999.