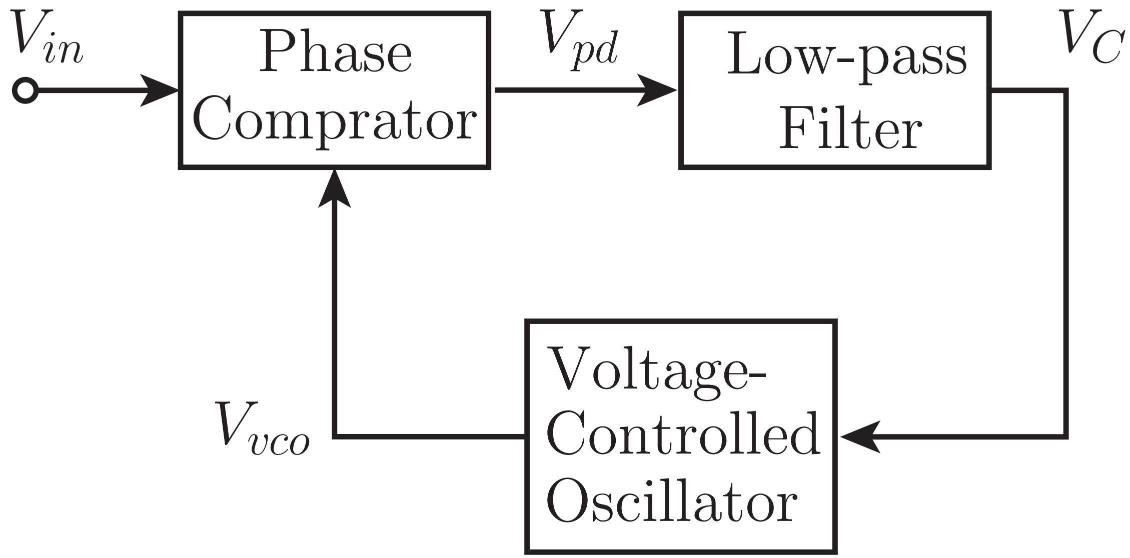
PLLの構成

(Phase Locked Loop)

(位相同期ループ)

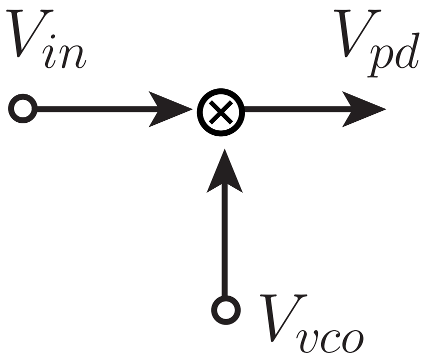
PLLの動作の概要



$$V_{in}(t) = A_{in}\sin(2\pi f_{in}t + \theta_{in})$$

$$V_{vco}(t) = A_{vco}\sin(2\pi f_{vco}t + \theta_{vco} + \frac{\pi}{2})$$

位相比較回路→アナログ乗算回路



$$V_{pd}(t) = K_{mul}V_{in}(t)V_{vco}(t)$$

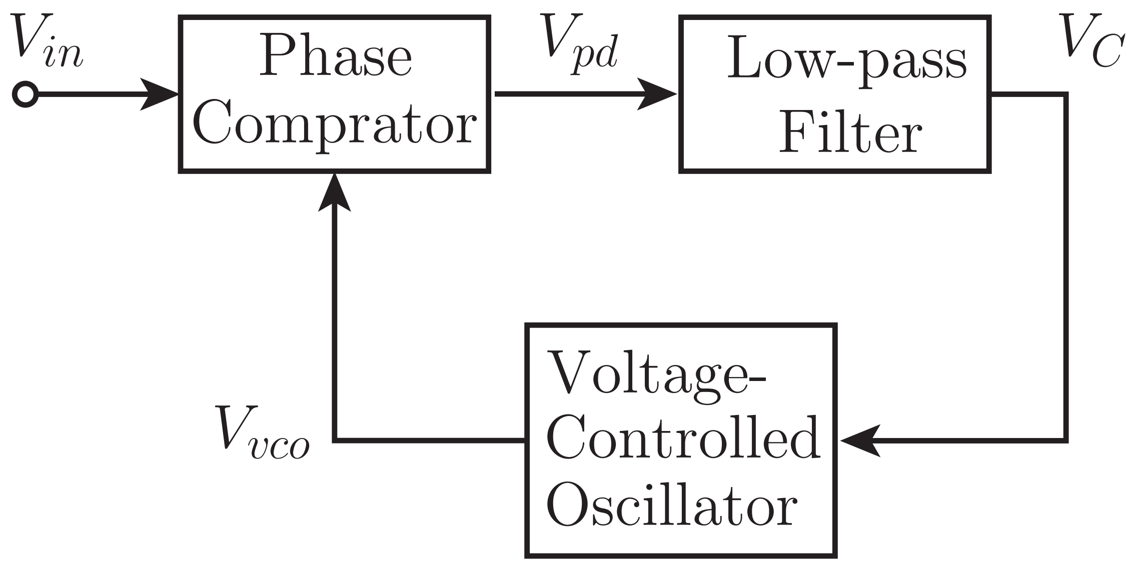
$$= \frac{K_{mul}A_{in}A_{vco}}{2}\left[\cos\left\{2\pi(f_{in} - f_{vco})t + (\theta_{in} - \theta_{vco}) - \frac{\pi}{2}\right\}\right.$$

$$\left. - \cos\left\{2\pi(f_{in} + f_{vco})t + (\theta_{in} + \theta_{vco}) + \frac{\pi}{2}\right\}\right]$$

高周波成分

$$= \frac{K_{mul}A_{in}A_{vco}}{2}\left[\sin(\theta_{in} - \theta_{vco}) + \sin\{4\pi f_{in}t + (\theta_{in} + \theta_{vco})\}\right]$$

$\boxed{f_{in} = f_{vco}\text{と仮定}}$



$$V_C = \frac{K_{lpf} K_{mul} A_{in} A_{vco}}{2} \sin(\theta_{in} - \theta_{vco})$$

$$\theta_{in} - \theta_{vco} = \sin^{-1}\left(\frac{2V_C}{K_{lpf} K_{mul} A_{in} A_{vco}}\right)$$

$$K_{lpf} K_{mul} A_{in} A_{vco} \to \infty$$

$$\boxed{\theta_{in} - \theta_{vco} \to 0}$$

$\boxed{f_{in} \neq f_{vco}}$と仮定

$$f_{vco} = K_{vco} V_C(t) + f_{free}$$

$$V_C(t) = \frac{K_{lpf} K_{mul} A_{in} A_{vco}}{2} \sin\{2\pi(f_{in} - f_{vco})t + (\theta_{in} - \theta_{vco})\}$$

$\boxed{f_{in} > f_{vco}}$

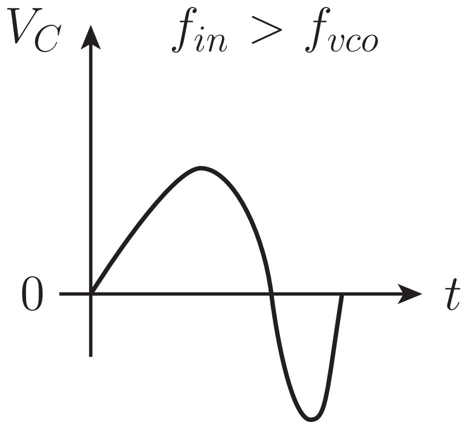
$V_C(t) > 0$ $f_{vco} \to$ **大** $|f_{in} - f_{vco}| \to$ **小** **長い**

$V_C(t) < 0$ $f_{vco} \to$ **小** $|f_{in} - f_{vco}| \to$ **大** **短い**

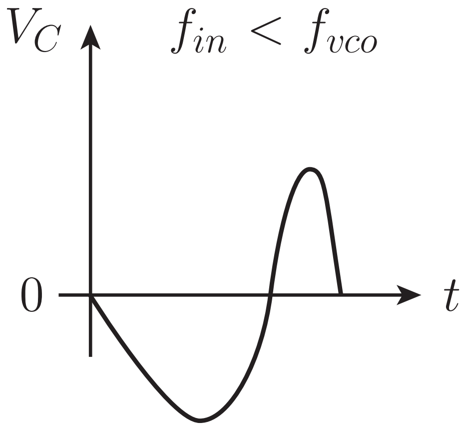
$\boxed{f_{in} < f_{vco}}$

$V_C(t) > 0$ $f_{vco} \to$ **大** $|f_{in} - f_{vco}| \to$ **大** **短い**

$V_C(t) < 0$ $f_{vco} \to$ **小** $|f_{in} - f_{vco}| \to$ **小** **長い**



(a)



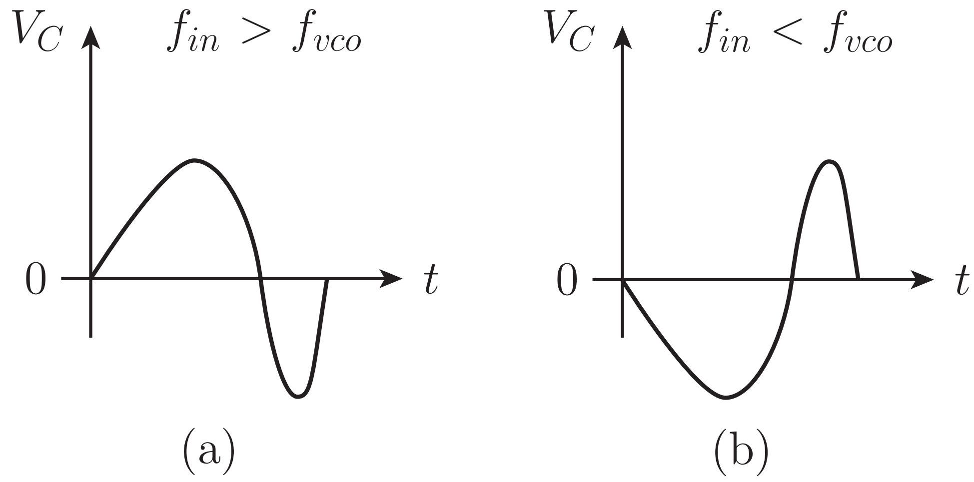
(b)

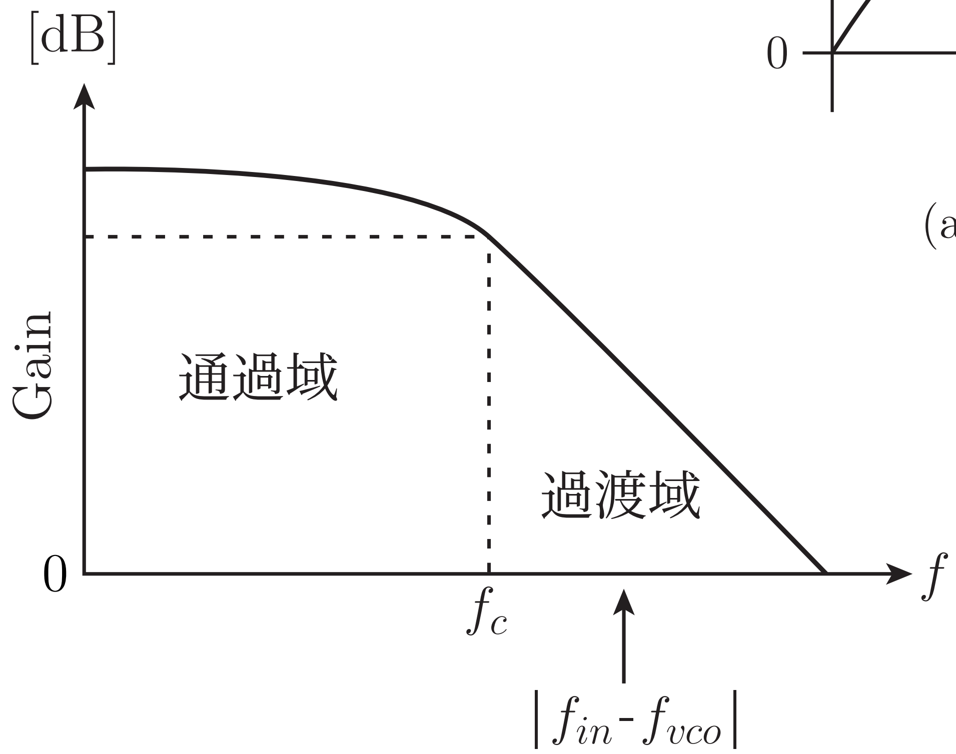
平均値 $V_C(t) > 0$ $V_C(t) < 0$

$f_{vco} \to$ **大** $f_{vco} \to$ **小**

$$\boxed{f_{vco} \to f_{in}}$$

低域通過フィルタ特性





$|f_{in} - f_{vco}| \to f_c$ (または0)　　利得増加 $\to V_C$: 大

$|f_{in} - f_{vco}| \to \infty$　　利得減少 $\to V_C$: 小

PLLの線形解析手法

$$K_{lpf} K_{mul} A_{in} A_{vco} \to \infty\ ?$$

$$\boxed{\theta_{in} - \theta_{vco} \neq 0}$$

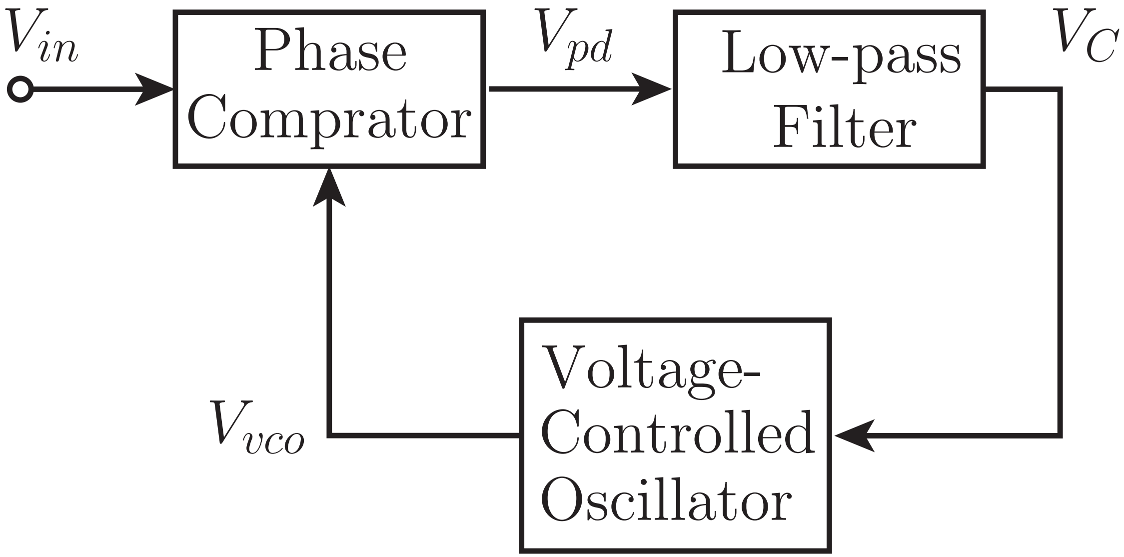
「同期」の定義

$$\frac{d\theta_{in}}{dt} = \frac{d\theta_{vco}}{dt}$$

「瞬時周波数」の定義

$$f_{inst} = \frac{1}{2\pi} \cdot \frac{d\theta}{dt}$$

「同期」とは $\boxed{f_{in} = f_{vco}}$



$$V_{pd}'(t) = \frac{K_{mul} A_{in} A_{vco}}{2} \sin(\theta_{in} - \theta_{vco})$$
$$\approx \frac{K_{mul} A_{in} A_{vco}}{2} (\theta_{in} - \theta_{vco})$$

ラプラス変換

$$\boxed{V_{pd}(s) = K_{pd}\{\Theta_{in}(s) - \Theta_{vco}(s)\}}$$

$$\boxed{V_C(s) = T_{lpf}(s) V_{pd}(s)}$$

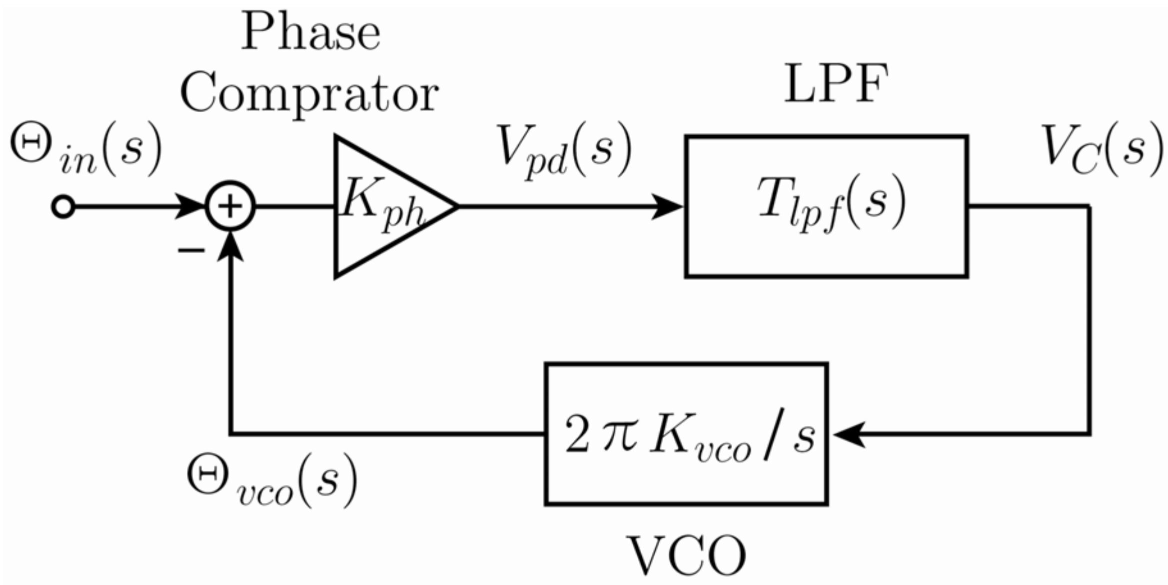
$$\boxed{f_{vco} = K_{vco} V_C(t) + f_{free}}$$

$$\boxed{f_{inst} = \frac{1}{2\pi} \cdot \frac{d\theta}{dt}}$$

$$\theta_{vco} = 2\pi \int (f_{vco} - f_{free}) dt + Const.$$

$$= 2\pi \int K_{vco} V_C(t) dt + Const.$$

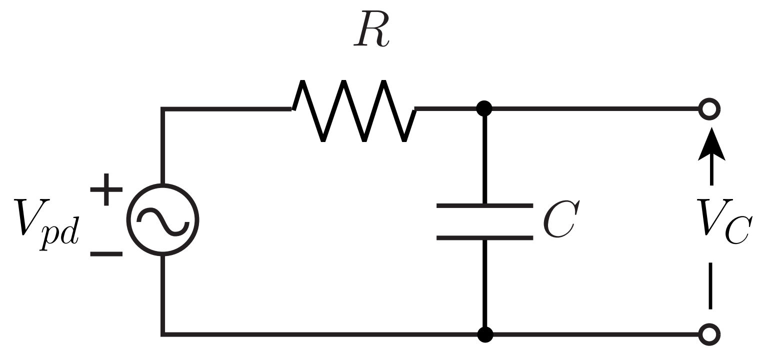
$$\boxed{\Theta_{vco}(s) = \frac{2\pi K_{vco} V_C(s)}{s}}$$



$$\boxed{\Theta_{vco}(s) = \frac{2\pi K_{vco} K_{pd} T_{lpf}(s)}{s + 2\pi K_{vco} K_{pd} T_{lpf}(s)} \Theta_{in}(s)}$$

$$\boxed{V_C(s) = \frac{s K_{pd} T_{lpf}(s)}{s + 2\pi K_{vco} K_{pd} T_{lpf}(s)} \Theta_{in}(s)}$$

$$\boxed{T_{lpf}(s) = \frac{K_{lpf} a_0}{a_0 + a_1 s}}$$



$$\boxed{\Theta_{vco}(s) = \frac{2\pi K_{vco} K_{pd} K_{lpf} a_0}{a_1 s^2 + a_0 s + 2\pi K_{vco} K_{pd} K_{lpf} a_0} \Theta_{in}(s)}$$

直流利得が1倍 $\longrightarrow$ θ_{in} **の初期値からの偏差**

$= \theta_{vco}$ **の初期値からの偏差**

$$\boxed{V_C(s) = \frac{s K_{pd} K_{lpf} a_0}{a_1 s^2 + a_0 s + 2\pi K_{vco} K_{pd} K_{lpf} a_0} \Theta_{in}(s)}$$

直流利得が0倍 $\longrightarrow$ θ_{in} **の初期値からの偏差は**

V_C **へ影響しない**

$$\boxed{\Theta_{vco}(s) = \frac{2\pi K_{vco} K_{pd} K_{lpf} a_0}{a_1 s^2 + a_0 s + 2\pi K_{vco} K_{pd} K_{lpf} a_0} \Theta_{in}(s)}$$

$$\Theta_{vco}(s) = \frac{\omega_0{}^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0{}^2} \Theta_{in}(s)$$

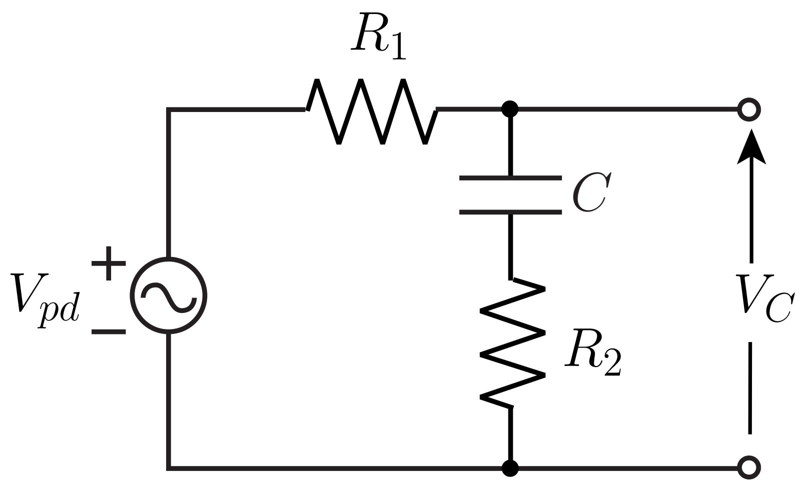
$$\omega_0 = \sqrt{\frac{2\pi K_{loop} a_0}{a_1}}$$

$$Q = \sqrt{\frac{2\pi K_{loop} a_1}{a_0}}$$

$$K_{loop} = K_{vco} K_{pd} K_{lpf} >> 1 \qquad a_1 >> a_0$$

$\boxed{Q\text{ は大}}$

$$\boxed{T_{lpf}(s) = \frac{K_{lpf}(a_0 + a_2 s)}{a_0 + a_1 s}}$$



$$\boxed{\Theta_{vco}(s) = \frac{2\pi K_{loop}(a_0 + a_2 s)}{a_1 s^2 + (a_0 + 2\pi K_{loop} a_2)s + 2\pi K_{loop} a_0}\Theta_{in}(s)}$$

$$\boxed{V_C(s) = \frac{sK_{pd}K_{lpf}(a_0 + a_2 s)}{a_1 s^2 + (a_0 + 2\pi K_{loop} a_2)s + 2\pi K_{loop} a_0}\Theta_{in}(s)}$$

$$\omega_0 = \sqrt{\frac{2\pi K_{loop} a_0}{a_1}} \qquad Q = \sqrt{\frac{2\pi K_{loop} a_1 / a_0}{1 + K_{loop} a_2 / a_0}}$$

ロックレンジ

発振周波数の範囲＝制御電圧の可変範囲

$$V_C(t) = \frac{K_{lpf} K_{mul} A_{in} A_{vco}}{2} \sin\left\{2\pi(f_{in} - f_{vco})t + (\theta_{in} - \theta_{vco})\right\}$$

$$\boxed{V_{Cmax} = \frac{K_{lpf} K_{mul} A_{in} A_{vco}}{2}}$$

$$f_{vco} = K_{vco} V_C(t) + f_{free}$$

$$\boxed{f_{lock} = K_{vco} V_{Cmax} \le \frac{K_{vco} K_{lpf} K_{mul} A_{in} A_{vco}}{2} = K_{loop}}$$

キャプチャレンジ

近似式

$$f_{capture} = \sqrt{\frac{2a_2}{a_1} - \left(\frac{a_2}{a_1}\right)^2}\, f_{lock} \quad (a_2 < a_1)$$

引き込み時間

近似式

$$\boxed{t_{pull-in} = \frac{Q\{2\pi(f_{in} - f_{vco})\}^2}{\omega_0{}^3}}$$

ω_0が大

θ_{in}の高い周波数成分を含む揺らぎがθ_{vco}へ伝達

(位相雑音)

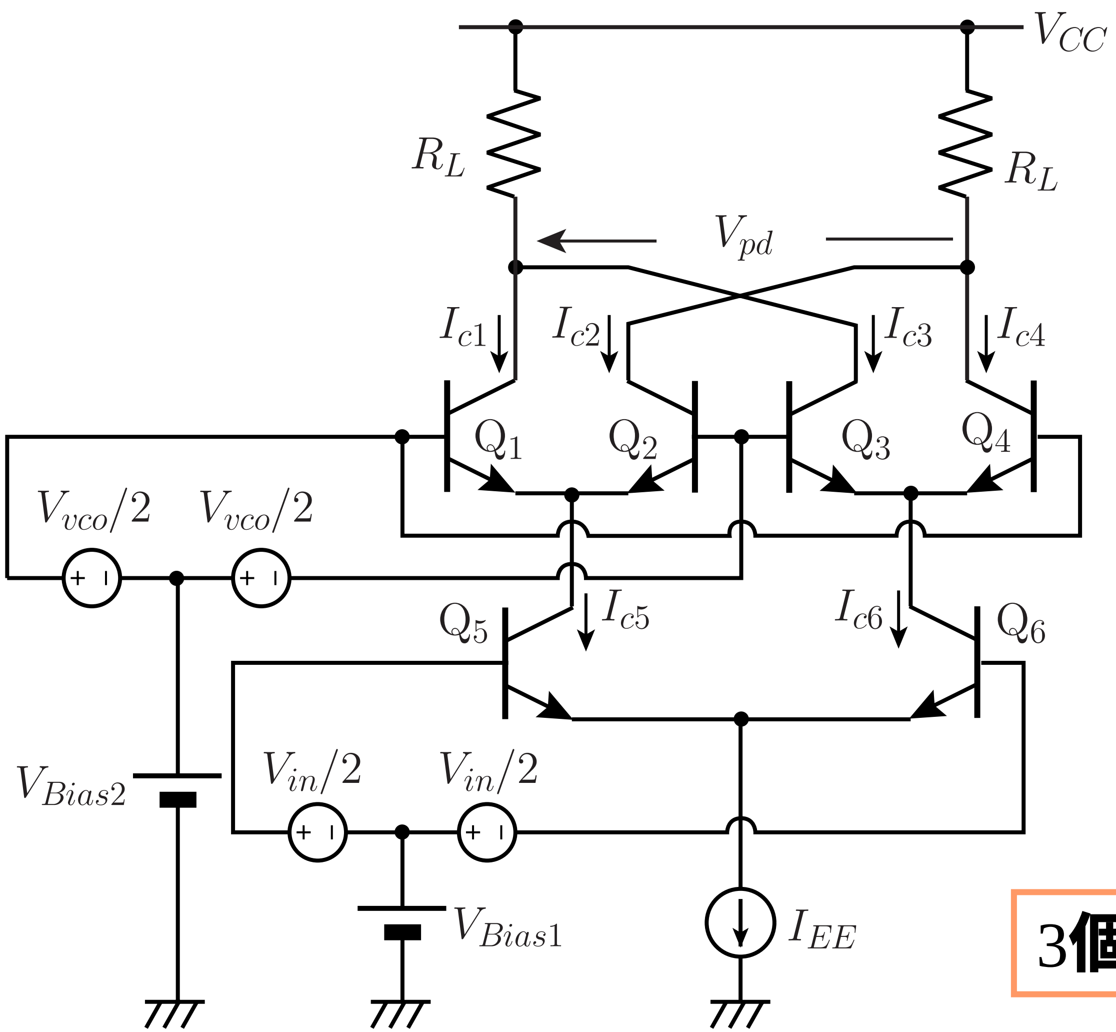
ω_0が小

引き込み時間が増加

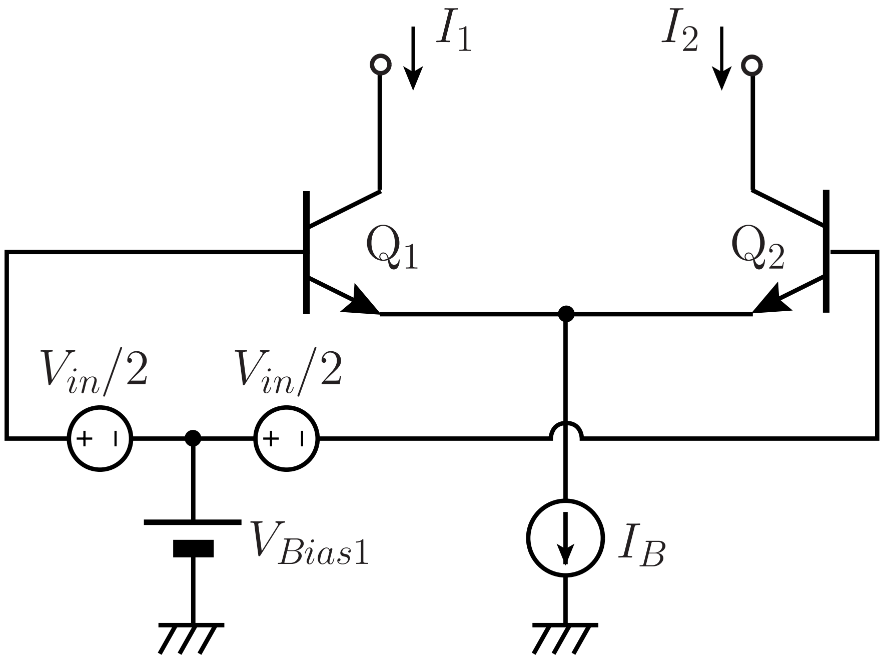
PLLの構成要素

位相比較回路

アナログ乗算回路



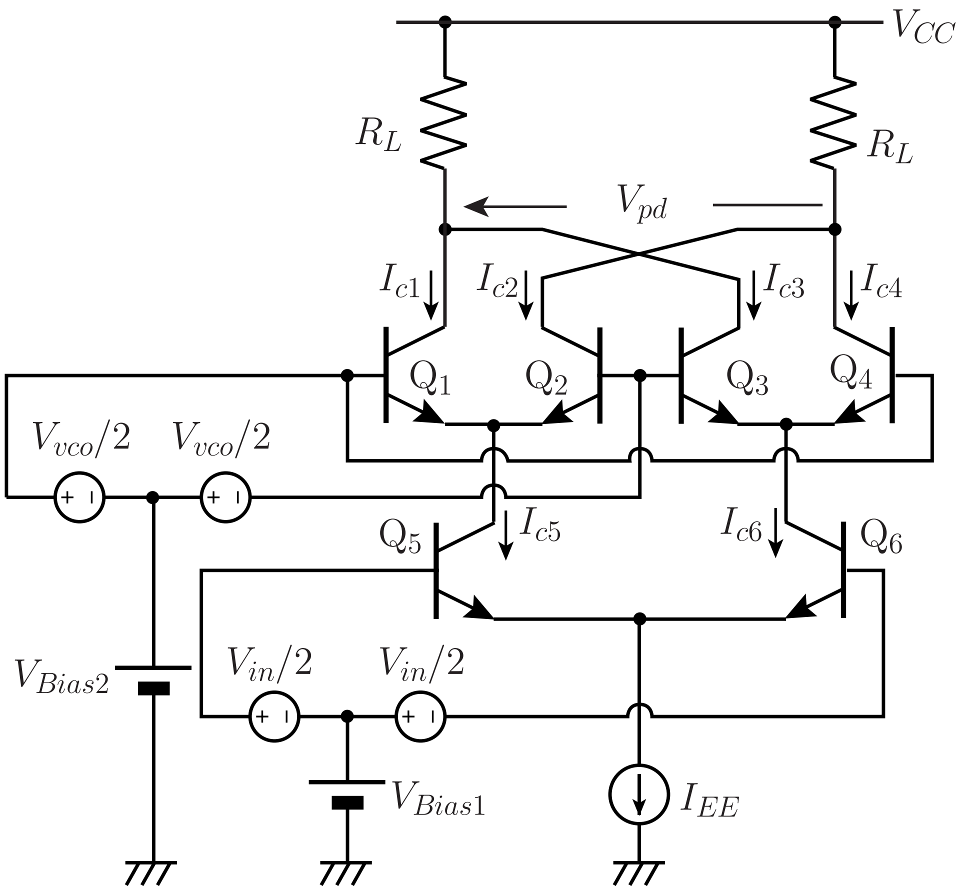
3個の差動増幅回路



$$\boxed{I_1 = \frac{I_B}{2} + \frac{1}{2r_e}V_{in}}$$

$$\boxed{I_2 = \frac{I_B}{2} - \frac{1}{2r_e}V_{in}}$$

$$r_e = \frac{kT}{q(I_B/2)} = 2\frac{kT}{qI_B}$$



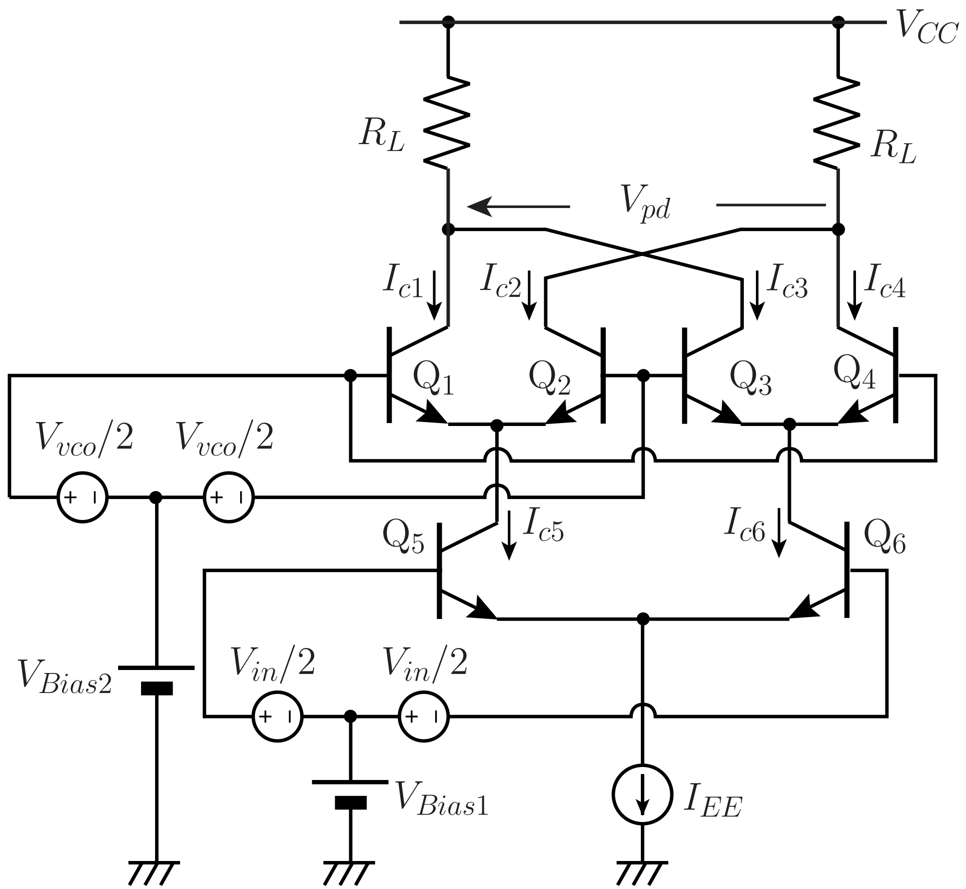
$$I_{c5} = \frac{I_{EE}}{2} + \frac{1}{2r_{e1}} V_{in}$$

$$I_{c6} = \frac{I_{EE}}{2} - \frac{1}{2r_{e1}} V_{in}$$

$$r_{e1} = 2\frac{kT}{qI_{EE}}$$

$$I_{c1} = \frac{I_{c5}}{2} + \frac{1}{2r_{e2}} V_{vco} \qquad I_{c2} = \frac{I_{c5}}{2} - \frac{1}{2r_{e2}} V_{vco} \qquad r_{e2} = 2\frac{kT}{qI_{c5}}$$

$$I_{c3} = \frac{I_{c6}}{2} - \frac{1}{2r_{e3}} V_{vco} \qquad I_{c4} = \frac{I_{c6}}{2} + \frac{1}{2r_{e3}} V_{vco} \qquad r_{e3} = 2\frac{kT}{qI_{c6}}$$



$$\boxed{V_{pd} = -R_L(I_{c1} - I_{c2} + I_{c3} - I_{c4})}$$

$$I_{c1} - I_{c2} = \frac{1}{r_{e2}} V_{vco}$$

$$I_{c3} - I_{c4} = -\frac{1}{r_{e3}} V_{vco}$$

$$I_{c5} - I_{c6} = \frac{1}{r_{e1}} V_{in}$$

$$V_{pd} = -R_L\left(\frac{V_{vco}}{r_{e2}} - \frac{V_{vco}}{r_{e3}}\right) = -R_L V_{vco}\left(\frac{qI_{c5}}{2kT} - \frac{qI_{c6}}{2kT}\right)$$

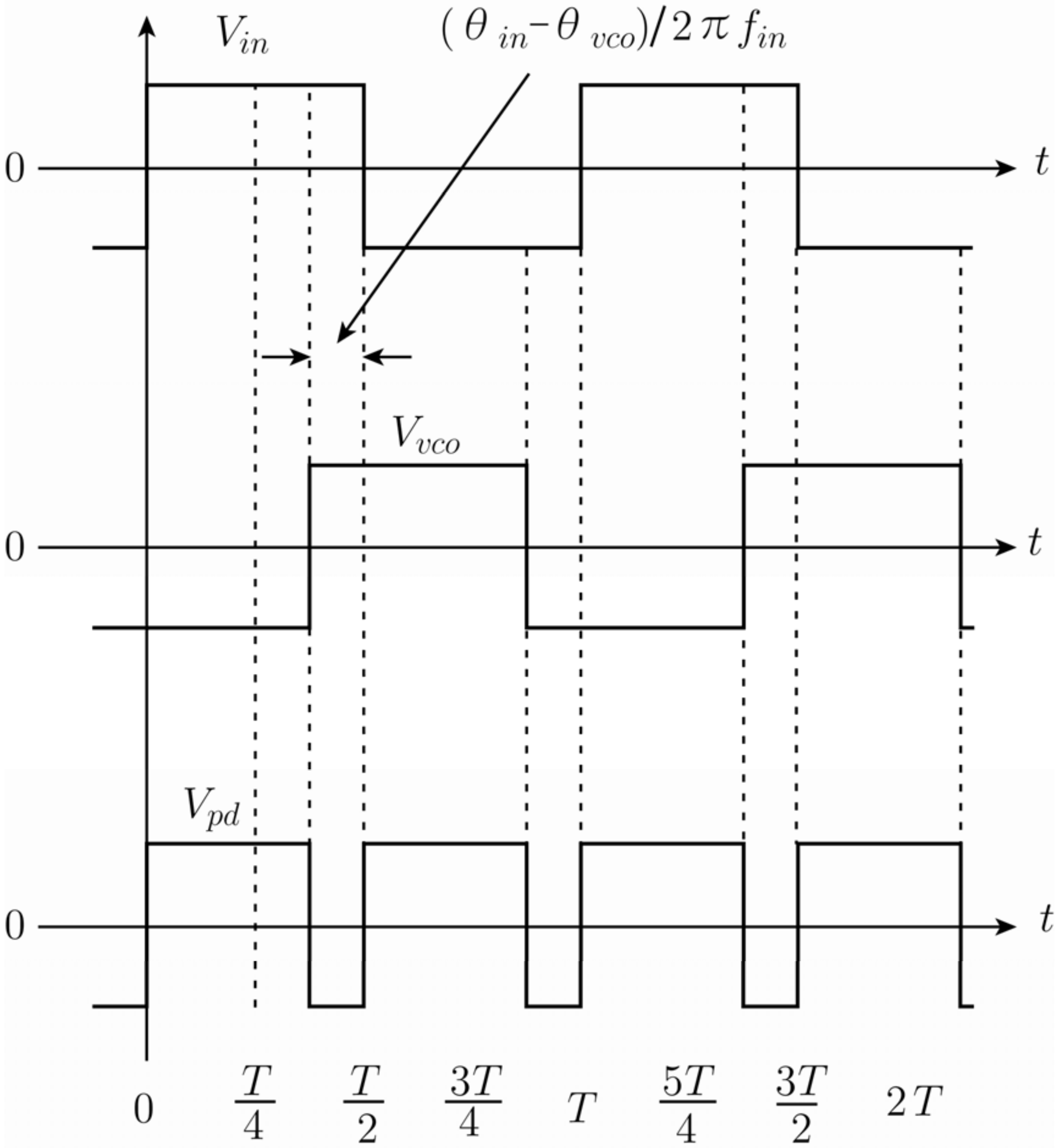
$$= -R_L V_{vco} \frac{qV_{in}}{2kTr_{e1}} = -R_L \frac{qI_{EE}}{(2kT)^2} V_{vco} V_{in}$$

Exclusive OR回路

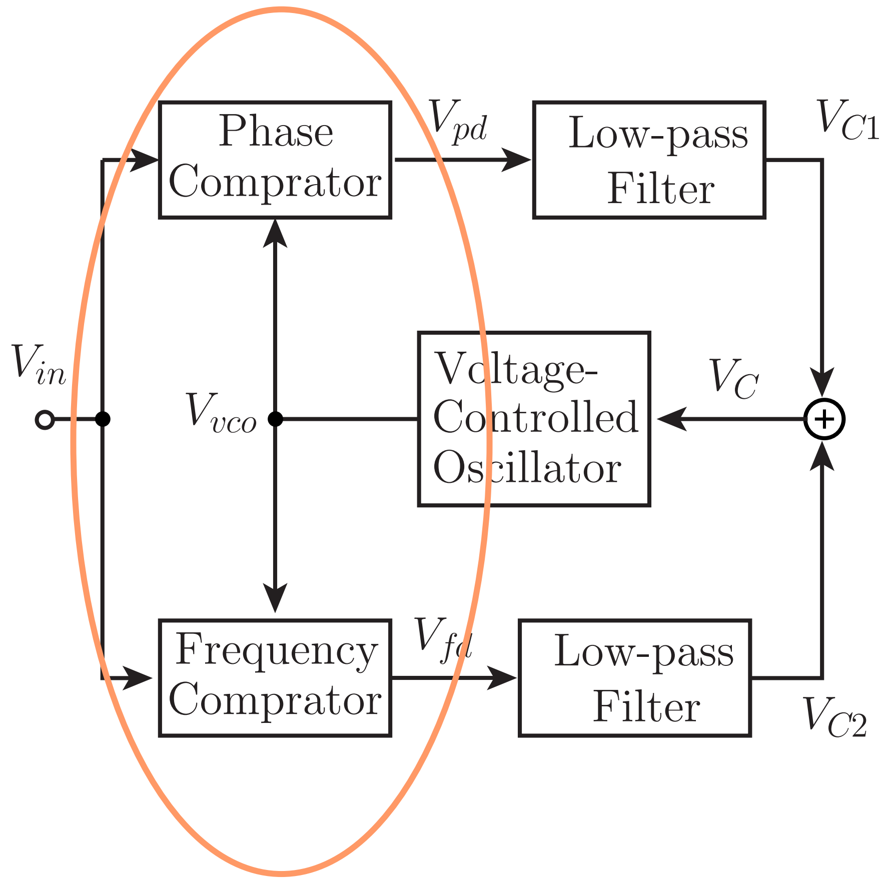
$$x \oplus y \rightarrow \begin{array}{l} 0 \quad 0 \rightarrow 0 \\ 0 \quad 1 \rightarrow 1 \\ 1 \quad 0 \rightarrow 1 \\ 1 \quad 1 \rightarrow 0 \end{array}$$

$0 \rightarrow +$(プラス)

$1 \rightarrow -$(マイナス)



チャージポンプ型位相比較回路



高速な同期を実現

周波数・位相差比較回路

方形波V_{in}とV_{vco}の立ち上がりに反応

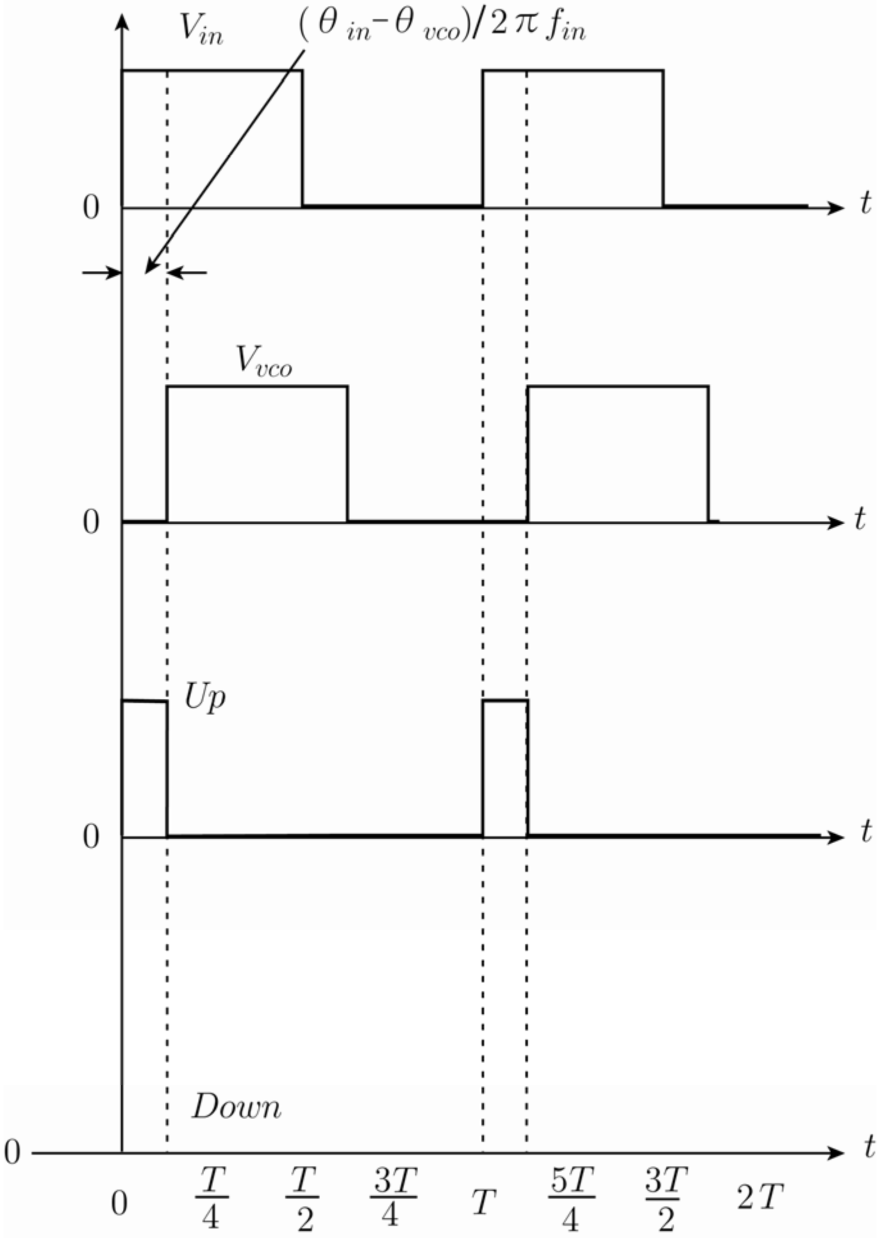
$V_{in}:H$レベル$\rightarrow Up=1$ ⟶ $V_{vco}:H$レベル$\rightarrow Up=0$

$Down=0$

$V_{vco}:H$レベル$\rightarrow Down=1$ ⟶ $V_{in}:H$レベル$\rightarrow Down=0$

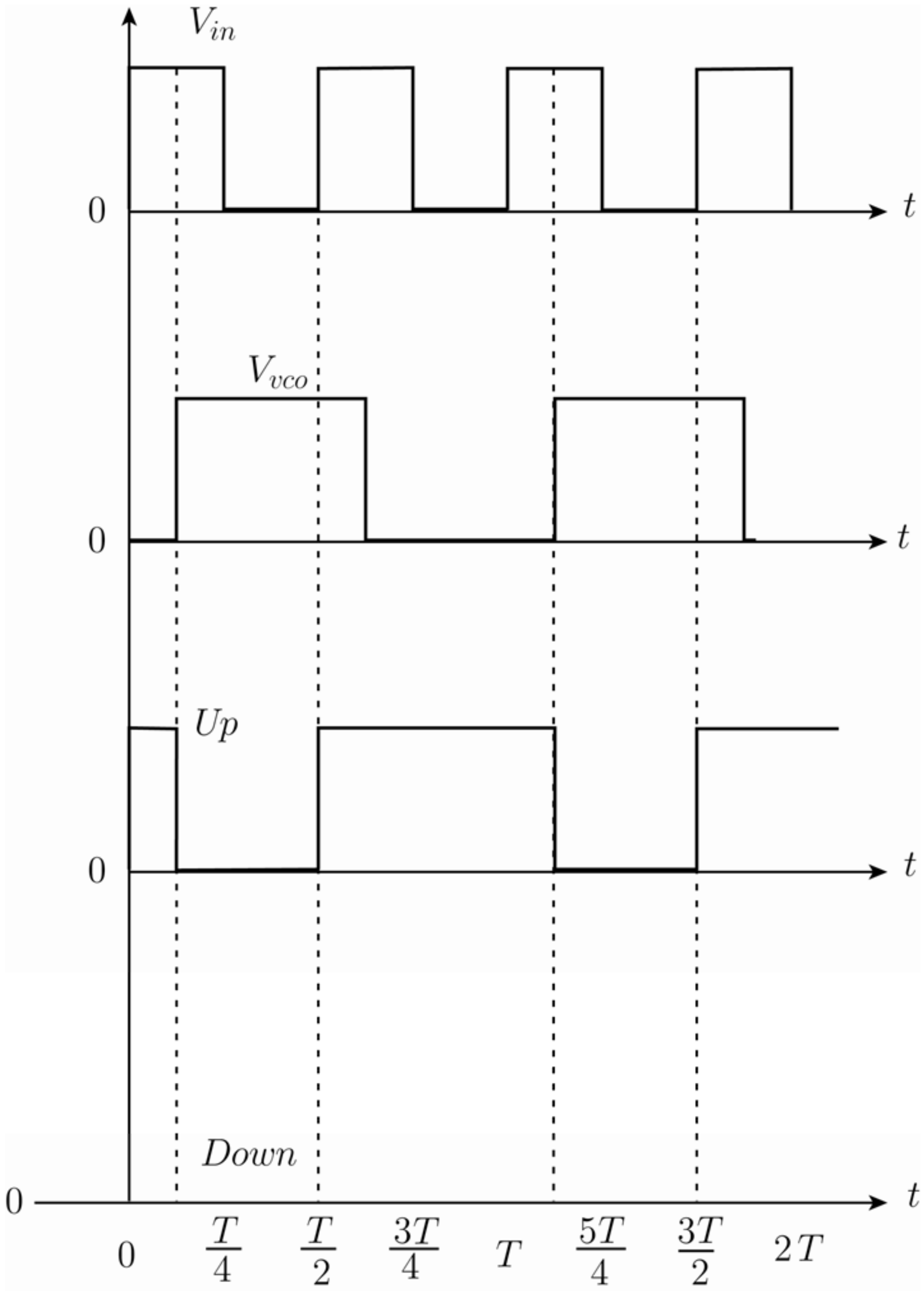
$Up=0$

周波数が等しく，V_{in}がV_{vco}よりも進んでいる場合



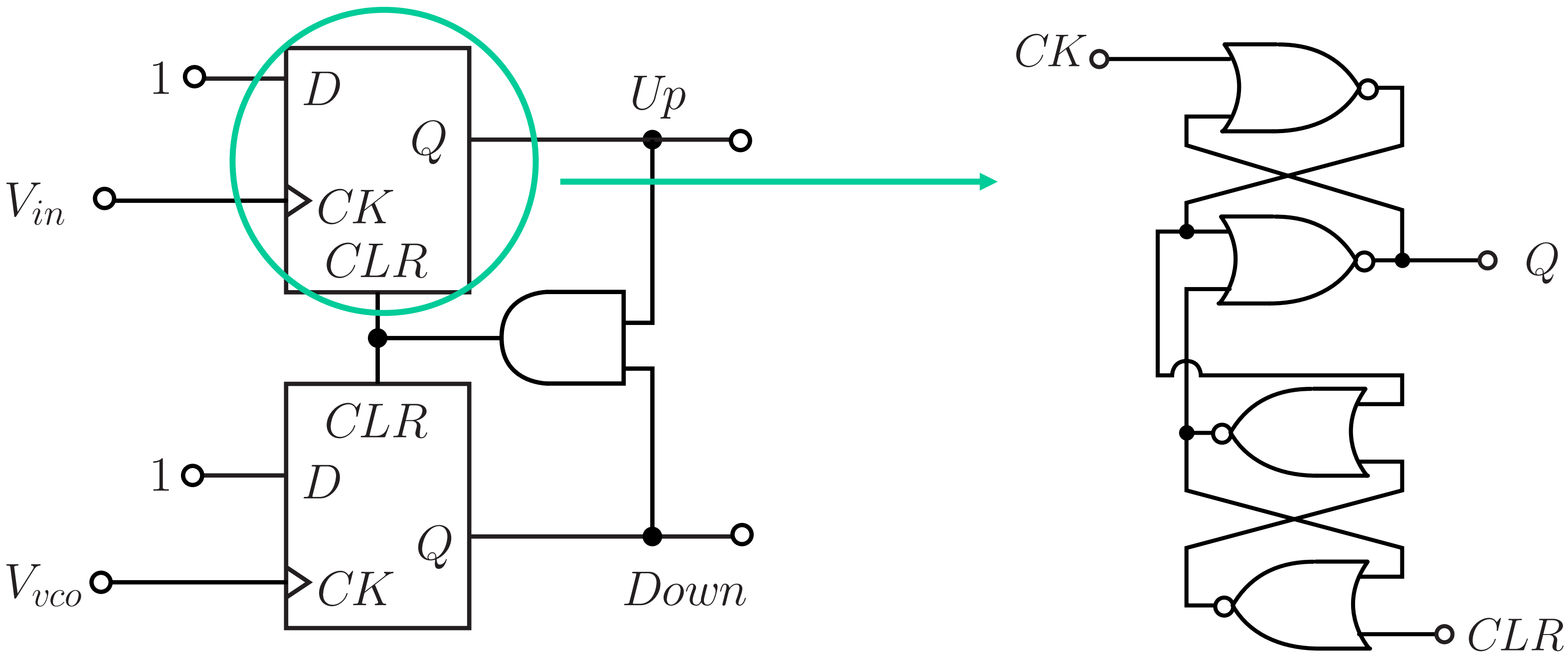
位相差の検出

周波数が異なる場合



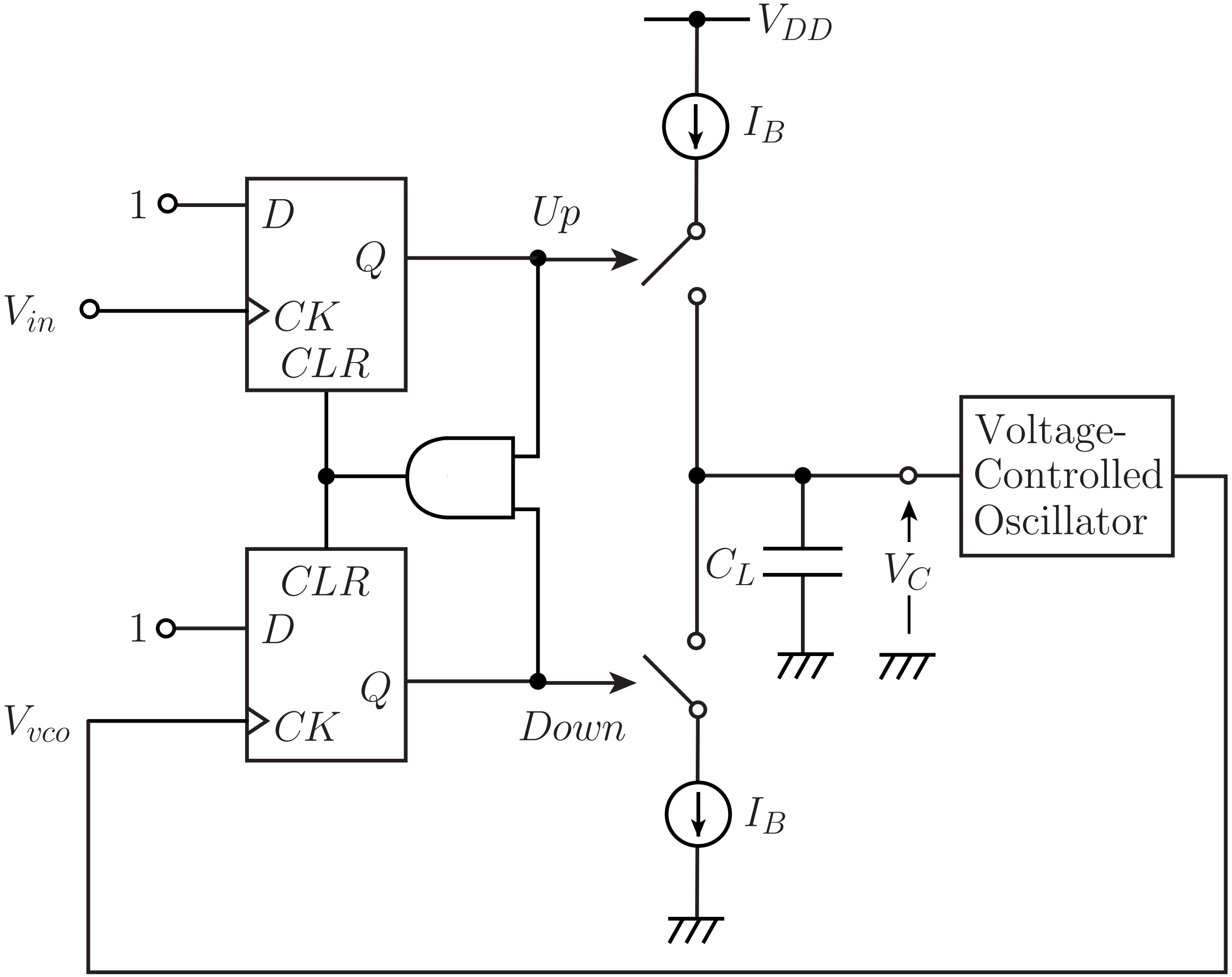
周波数差の検出

周波数・位相差比較回路の構成



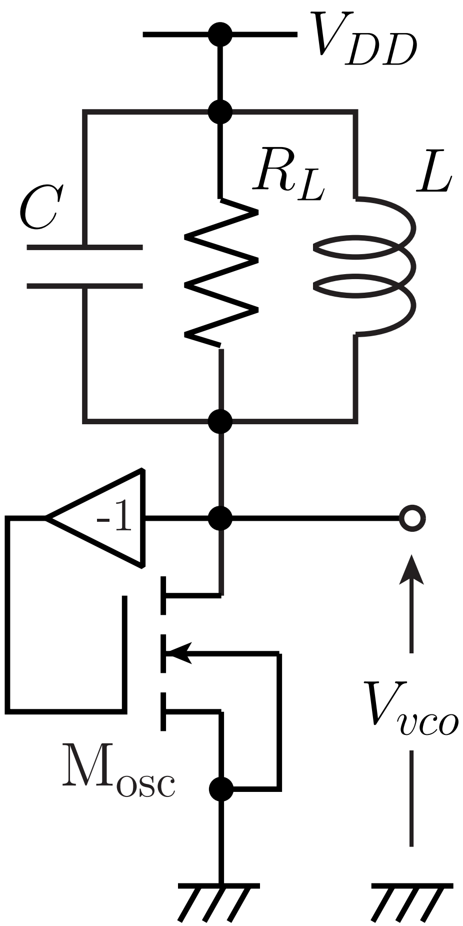
$Up = 1,\ Down = 1$という状態が存在

チャージポンプ型PLLの構成



$Up = 1, Down = 1$という状態には不感

電圧制御発振回路



$$f_{res} = \frac{1}{2\pi\sqrt{LC}}$$ **のとき**

負荷インピーダンス $Z_{LC} = R_L$

$\boxed{\mathrm{M}_{osc} + R_L \rightarrow \text{Inverter}}$

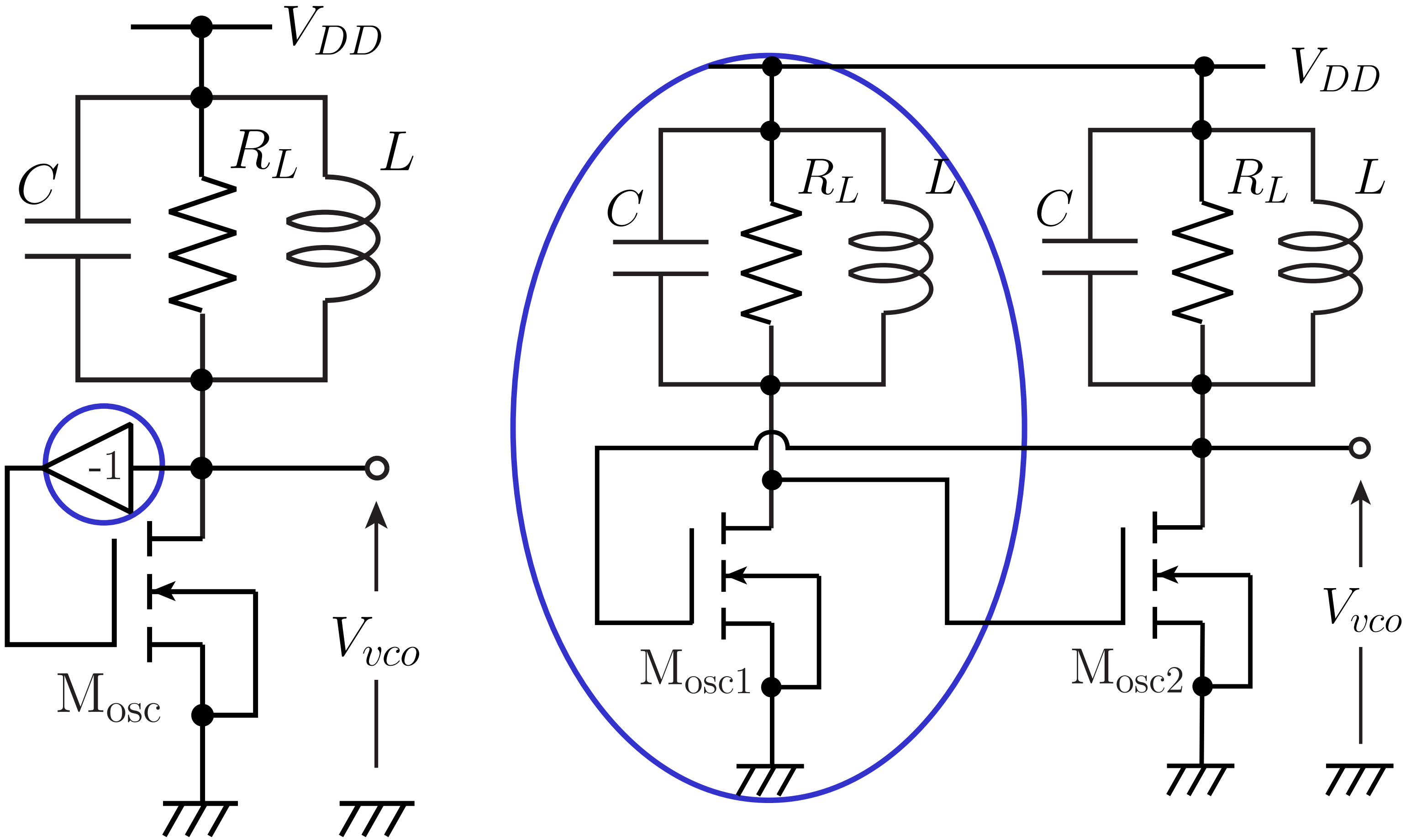
+

-1**倍の**Inverter

||

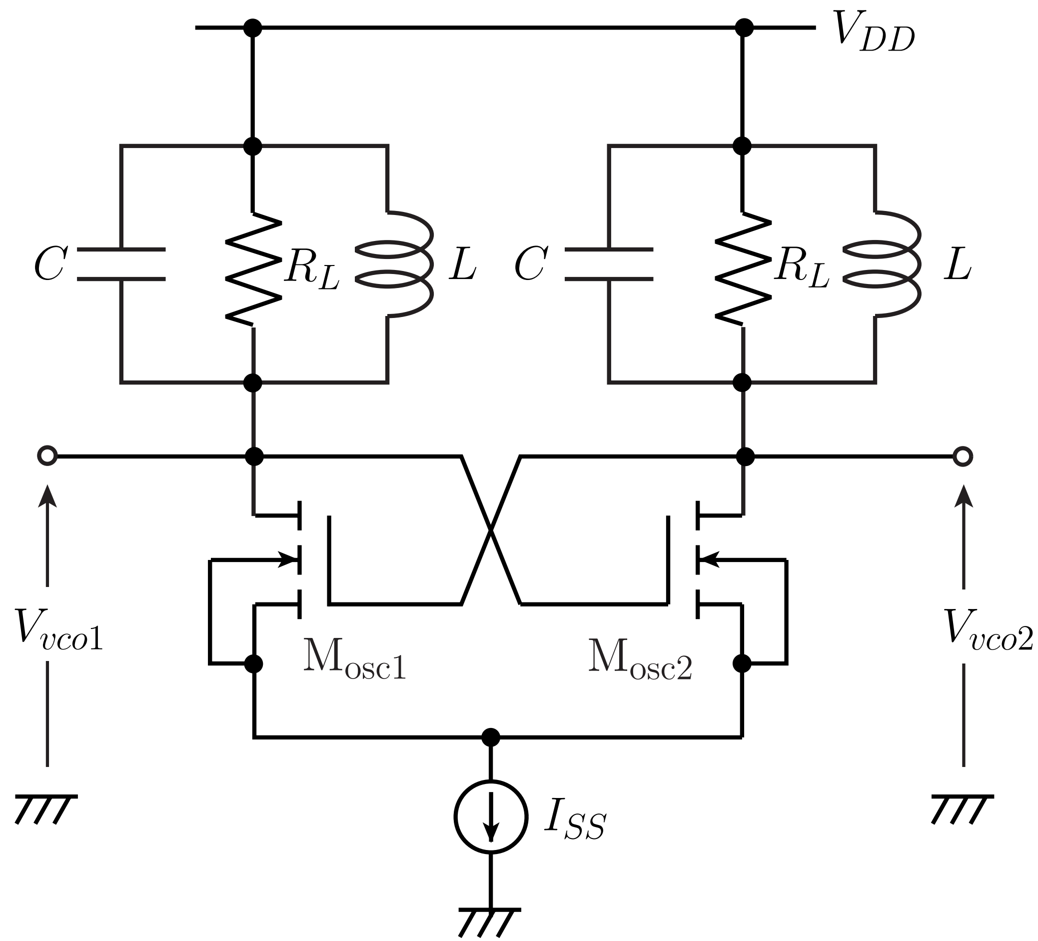
正帰還（発振）

1GHz**以上→インダクタの使用が可能**



消費電力を定めるのが困難

非飽和領域動作の可能性

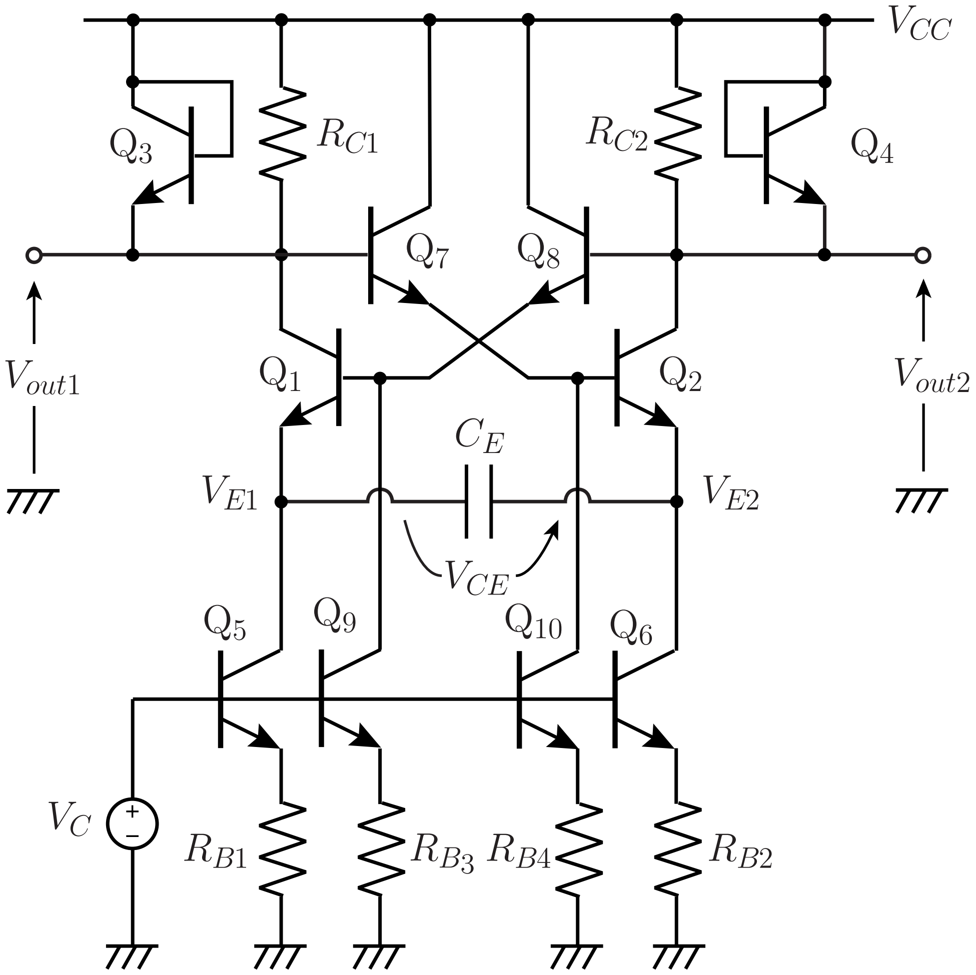


周波数条件： $f_{res} = \dfrac{1}{2\pi\sqrt{LC}}$

接合容量（可変容量）の使用

電力条件： $(g_m R_L)^2 \geq 1$

電圧制御マルチバイブレータ



Q_1：オン，　Q_2：オフ

$$V_{E1} = V_{CC} - 2V_{BE}$$

$$V_{CE} = V_{BE} \rightarrow -V_{BE}$$

$$V_{E2} = V_{CC} - V_{BE} \rightarrow V_{CC} - 3V_{BE}$$

$$\downarrow$$

Q_2：オン → Q_1：オフ

$$V_{E2} = V_{CC} - 3V_{BE} \rightarrow V_{CC} - 2V_{BE}$$

$$V_{CE} = -V_{BE}$$

$$V_{E1} = V_{CC} - 2V_{BE} \rightarrow V_{CC} - V_{BE}$$

1周期： $V_{CE} = V_{BE} \rightarrow -V_{BE} \rightarrow V_{BE}$

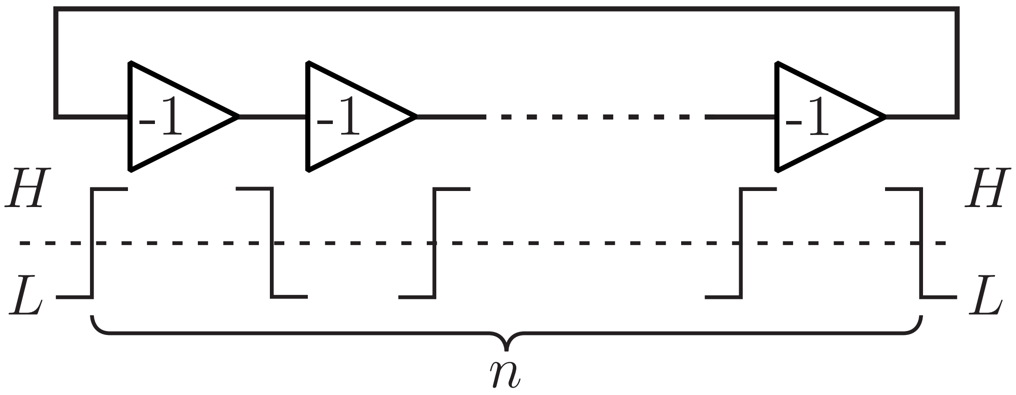
充電電流： $I_E = \dfrac{V_C - V_{BE}}{R_B}$

$\boxed{\text{周期}T:}$ $4V_{BE} = \dfrac{I_E}{C_E} T$

$$\boxed{T = \frac{4V_{BE}C_E}{I_E} = \frac{4C_E R_B V_{BE}}{V_C - V_{BE}}}$$

リングオシレータの原理

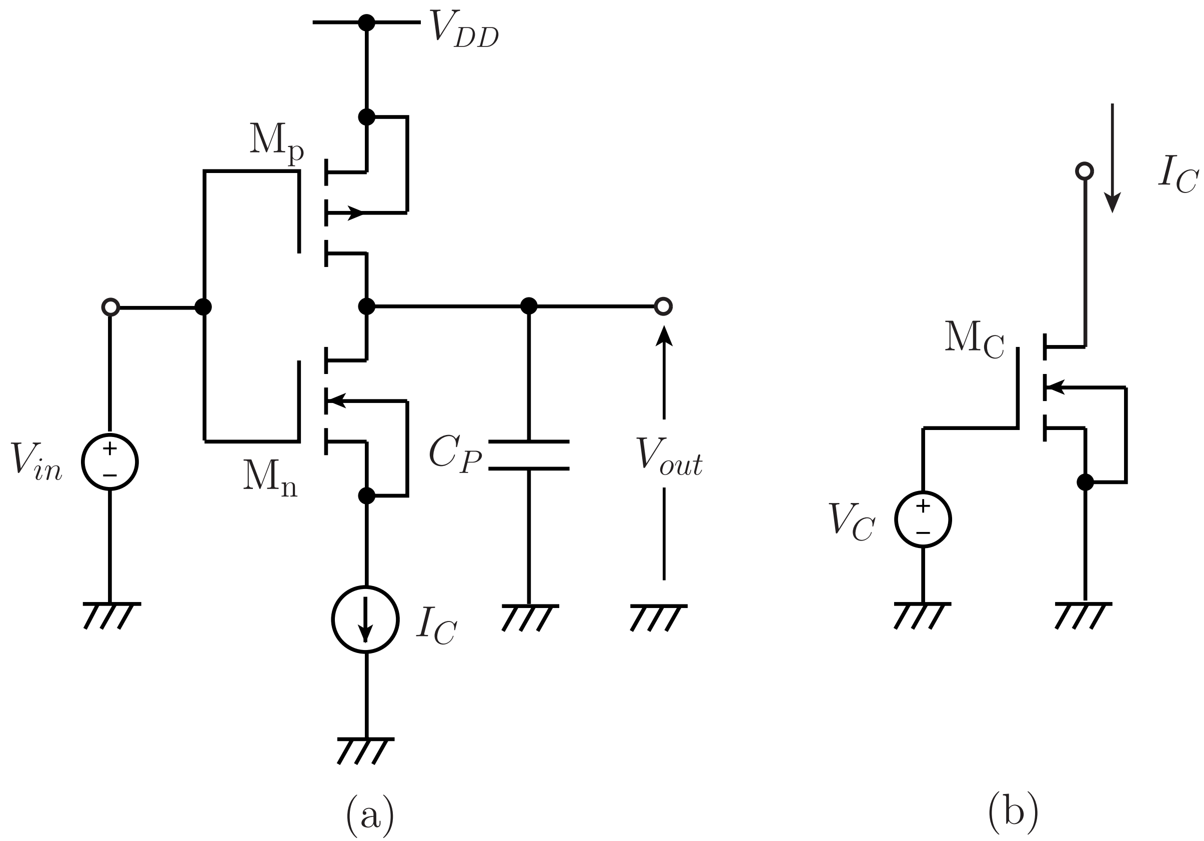
nは奇数



$$t_{delay} = nt_{inv}$$

周期$T = 2t_{delay}$

インバータの構成



I_CによりC_Pへの充電時間を調整

PLLの応用

- 復調回路
- 周波数シンセサイザ
- クロック信号の再生
- その他

振幅変調信号とその復調

搬送波：$V_{carrier}(t) = A_c \cos(2\pi f_{carrier} t + \theta_{carrier})$

振幅変調 $A_c \to K_{AM} S_{org}(t)$

$S_{org}(t)$: 変調波（信号）

$$\boxed{S_{org}(t) = V_{signal} \cos(2\pi f_{signal} t)}$$

被変調波：$V_{AM}(t) = K_{AM} V_{signal} \cos(2\pi f_{signal} t) \cos(2\pi f_{carrier} t + \theta_{carrier})$

$$V_{AM}(t) = K_{AM} V_{signal} \cos(2\pi f_{signal} t) \cos(2\pi f_{carrier} t + \theta_{carrier})$$

$$= \frac{1}{2} K_{AM} V_{signal} [\cos\{2\pi(f_{carrier} - f_{signal})t - \theta_{carrier}\}$$

$$- \cos\{2\pi(f_{carrier} + f_{signal})t + \theta_{carrier}\}]$$

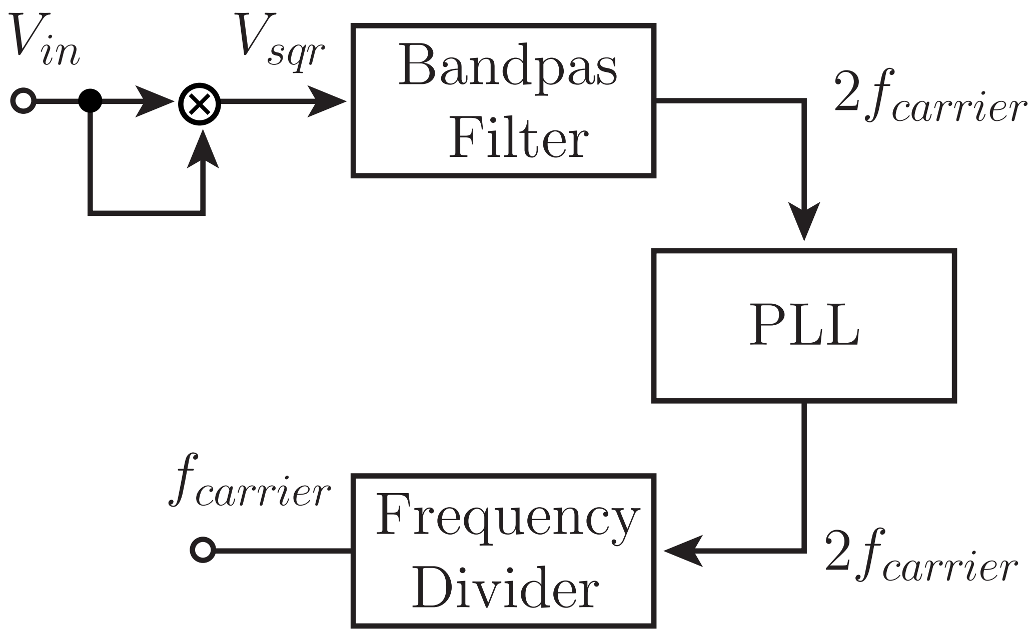
復調

$$f_{carrier} - f_{signal}, \quad f_{carrier} + f_{signal} \boxed{\times f_{carrier}}$$

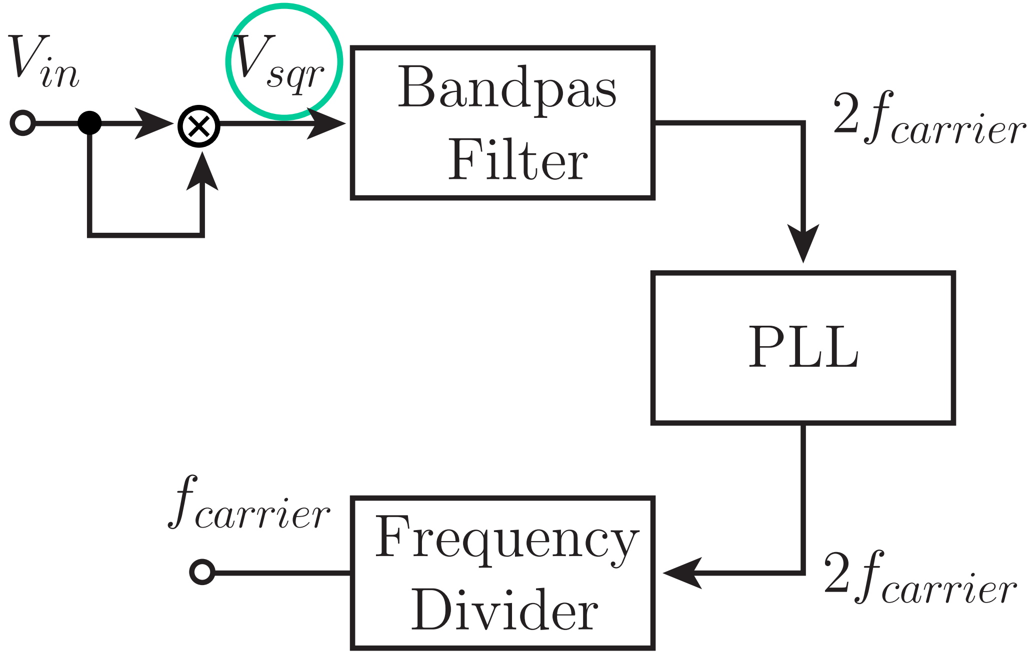
$$\rightarrow f_{signal}, \quad 2f_{carrier} - f_{signal}, \quad 2f_{carrier} + f_{signal}$$

$f_{carrier}$が未知の場合は？

2乗ループ回路



$$V_{sqr}(t) = V_{AM}{}^{2}(t)$$



$$
\begin{aligned}
V_{sqr}(t) = K_{sqr}K_{AM}{}^{2}V_{signal}{}^{2}[&\cos\{4\pi(f_{carrier} + f_{signal})t + \theta_{carrier}\} \\
&+ 2\cos(4\pi f_{carrier}t + \theta_{carrier}) \\
&+ 2\cos\{4\pi(f_{carrier} + f_{signal})t + \theta_{carrier}\} \\
&+ 2\cos(4\pi f_{signal}t + \theta_{carrier}) \\
&+ 2\cos\theta_{carrier}]/8
\end{aligned}
$$

周波数変調信号とその復調

搬送波：$V_{carrier}(t) = A_c \cos(2\pi f_{carrier} t + \theta_{carrier})$

周波数変調

$$f_{carrier} \to f_{inst}(t) = f_{carrier} + K_{FM} S_{org}(t)$$

$S_{org}(t)$: 変調波（信号）

$$S_{org}(t) = V_{signal} \cos(2\pi f_{carrier} t)$$

$$f_{inst}(t) = f_{carrier} + K_{FM} V_{signal} \cos(2\pi f_{signal} t)$$

$$\boxed{\Delta f = K_{FM} V_{signal}\ \text{:最大周波数偏移}}$$

瞬時位相

$$\phi_{inst} = 2\pi\int f_{inst}dt = 2\pi f_{carrier} + \frac{\Delta f}{f_{signal}}\sin(2\pi f_{signal}t) + \theta_{carrier}$$

$$K_{FM} = \frac{\Delta f}{f_{signal}} : \text{変調指数}$$

被変調波：$V_{FM}(t) = A_c \cos\{2\pi f_{carrier} + m_{FM}\sin(2\pi f_{signal}t) + \theta_{carrier}\}$

PLLの性質から $f_{vco} = K_{vco}V_C(t) + f_{free} = f_{carrier} + m_{FM}\sin(2\pi f_{signal}t)$

$$f_{carrier} = f_{free} \rightarrow V_C(t) = \frac{m_{FM}\sin(2\pi f_{signal}t)}{K_{vco}} \propto S_{org}$$

位相変調とその復調

搬送波：$V_{carrier}(t) = A_c \cos(2\pi f_{carrier} t + \theta_{carrier})$

位相変調

$$\theta_{carrier} \rightarrow m_{PF} S_{org}(t)$$

$S_{org}(t)$: 変調波（信号）

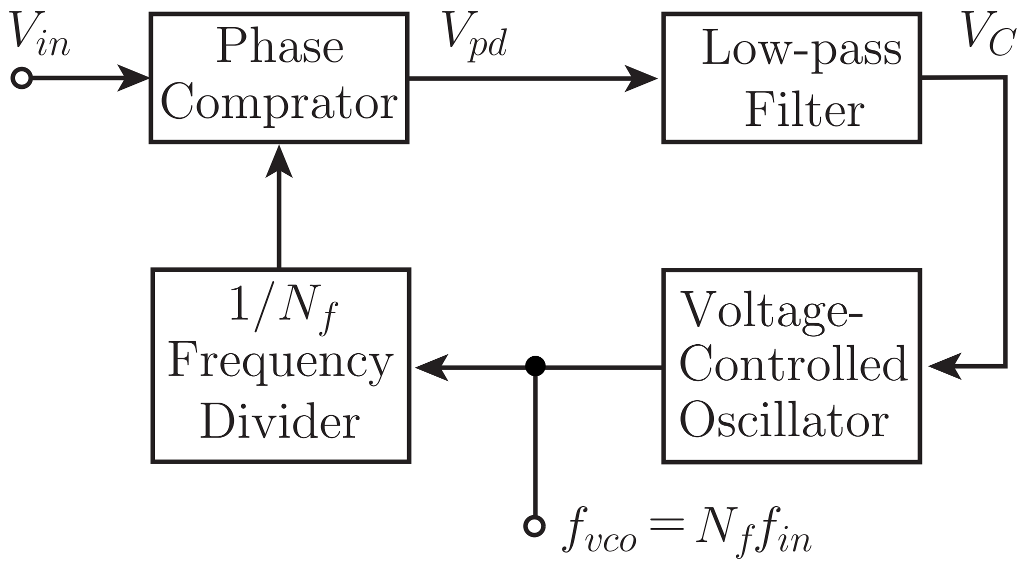
瞬時位相：$\phi_{inst}(t) = 2\pi f_{carrier} + m_{PF} S_{org}(t)$

瞬時位相の微分→瞬時周波数

瞬時周波数の積分→瞬時位相

PLLの$V_C(t)$（瞬時周波数）を積分

周波数シンセサイザの構成



f_{in}: 基準周波数

$f_{vco} = N_f f_{in}$: 出力周波数

N_fにより可変