

Fundamental Concepts of Fracture Mechanics

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Introduction

During World War II
5000 ships built



over 1000 ships: cracks by 1946

200 ships: serious damage

9 T-2 tankers

7 Liberty ships

} broken in two

What is Fracture Mechanics?

Strength of Materials

Yield $\sigma = \sigma_y$

Failure $\sigma = \sigma_B$

Fatigue $S - N$ Diagram, *Fatigue Limit*

Fracture Mechanics

Failure $K = K_C$ Stress Intensity Factor K
Fracture Toughness K_C

Fatigue $\frac{da}{dN} = C((\Delta K)^m - (\Delta K_{th})^m)K_C$

Fracture Mechanics is

Mechanics of Members with Cracks

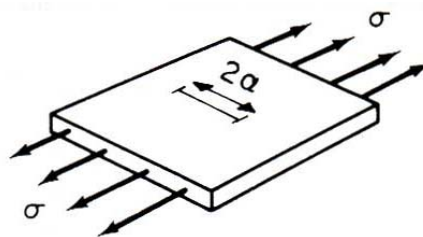
Stress Intensity Factor, K

Stress, σ

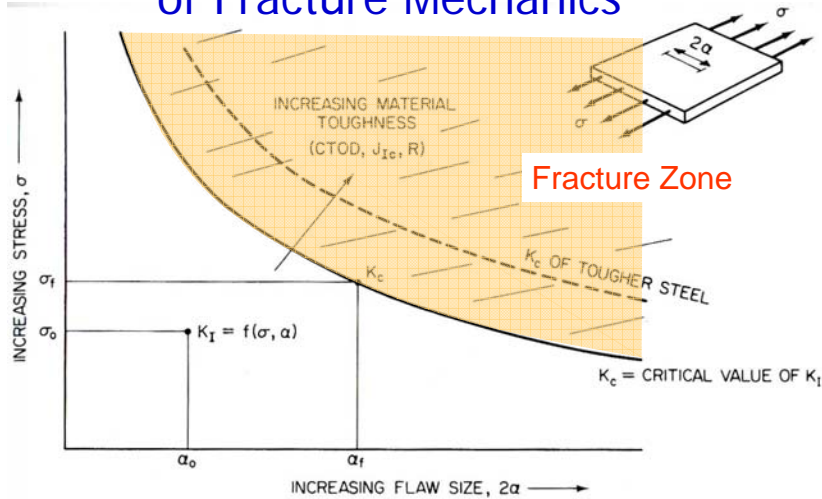
Crack Length, a

Fracture Toughness, K_C

Material Property



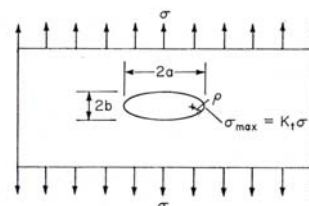
Fundamental Concepts of Fracture Mechanics



Relationship among a stress condition,
a crack size and fracture toughness

Types of Discontinuities

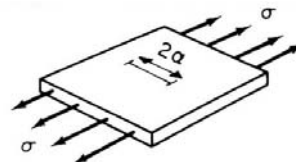
Notches



Imperfections

Sharp Tip

Cracks



History of Fracture Mechanics

1921 Griffith

Griffith's Formula

1948 Irwin

1957 Irwin

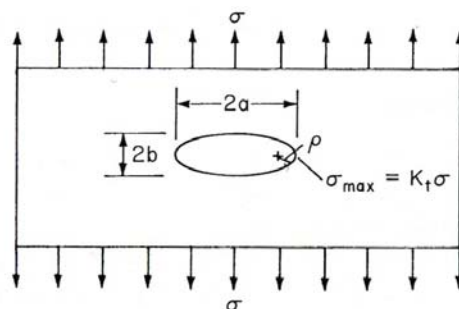
Stress Intensity Factor, K

.....

Stress Analysis of Members with Notches

Stress Concentration Factor, K_t

For An Elliptical Hole



$$K_T = \frac{\sigma_{\max}}{\sigma} = 1 + \frac{2a}{b}$$

$$\sigma_{\max} = \sigma \left(1 + \frac{2a}{b} \right)$$

$$\rho = \frac{b^2}{a} \Rightarrow b = \sqrt{a\rho}$$

$$\sigma_{\max} = \sigma \left(1 + 2\sqrt{\frac{a}{\rho}} \right) \Rightarrow = 2\sigma\sqrt{\frac{a}{\rho}} \quad (\rho \ll a)$$

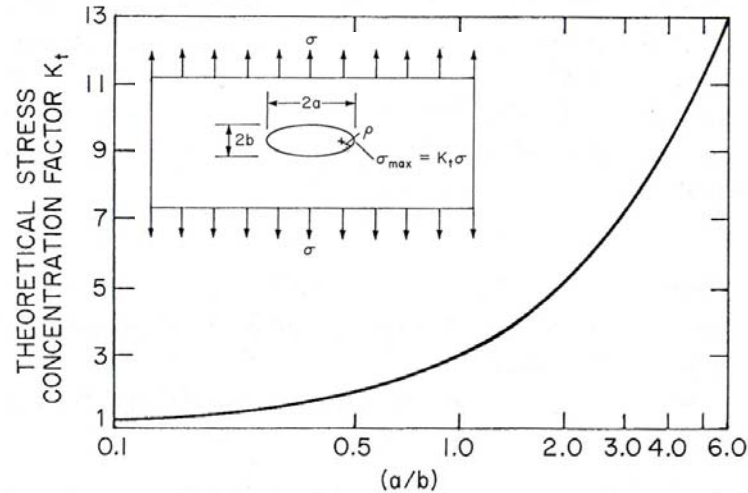
$$\sqrt{\frac{a}{\rho}} \rightarrow \infty \quad (\rho \rightarrow 0)$$

$$K_t \rightarrow \infty$$

Stress Analysis of Members with Notches

Stress Concentration Factor, K_t

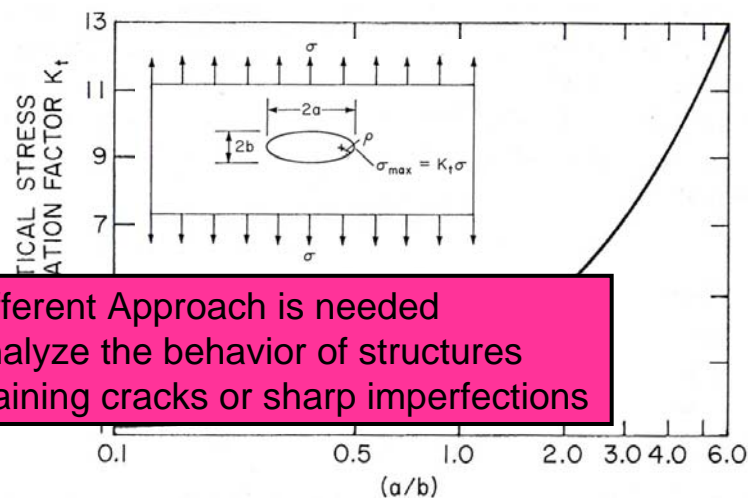
For An Elliptical Hole $K_t \rightarrow \infty$ Meaningless



Stress Analysis of Members with Notches

Stress Concentration Factor, K_t

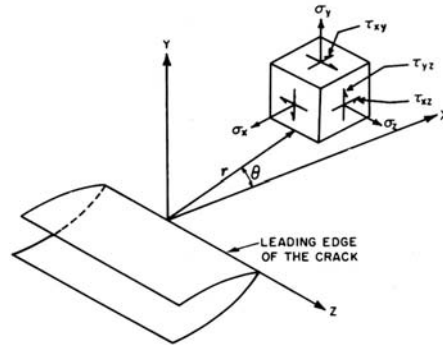
For An Elliptical Hole $K_t \rightarrow \infty$ Meaningless



A Different Approach is needed
to analyze the behavior of structures
containing cracks or sharp imperfections

Stress Analysis of Members with Notches

Stress Distribution
along X-axis
near the notch root



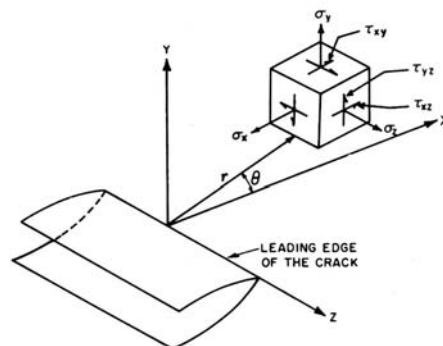
$$\frac{(\sigma_y)_{y=0}}{\sigma} = \sqrt{\frac{a}{2r + \rho}} \left(1 + \frac{\rho}{2r + \rho} \right) + \left(\frac{\rho}{2r + \rho} \right)$$

$$0 \leq r \ll a, \rho \ll a$$

$$\frac{(\sigma_x)_{y=0}}{\sigma} = \sqrt{\frac{a}{2r + \rho}} \left(1 - \frac{\rho}{2r + \rho} \right)$$

Stress Analysis of Members with Notches

Stress Distribution
along X-axis
near the notch root



$$\frac{(\sigma_y)_{y=0}}{\sigma} = \frac{(\sigma_x)_{y=0}}{\sigma} = \sqrt{\frac{a}{2r + \rho}}$$

$$\rho \rightarrow 0$$

$$\frac{(\sigma_y)_{y=0}}{\sigma} = \sqrt{\frac{a}{2r + \rho}} \left(1 + \frac{\rho}{2r + \rho} \right) + \left(\frac{\rho}{2r + \rho} \right)$$

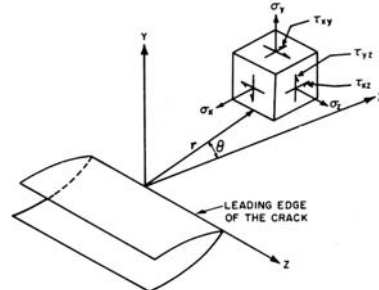
$$0 \leq r \ll a, \rho \ll a$$

$$\frac{(\sigma_x)_{y=0}}{\sigma} = \sqrt{\frac{a}{2r + \rho}} \left(1 - \frac{\rho}{2r + \rho} \right)$$

Stress Analysis of Members with Cracks

$$\frac{(\sigma_y)_{y=0}}{\sigma} = \frac{(\sigma_x)_{y=0}}{\sigma} = \sqrt{\frac{a}{2r + \rho}}$$

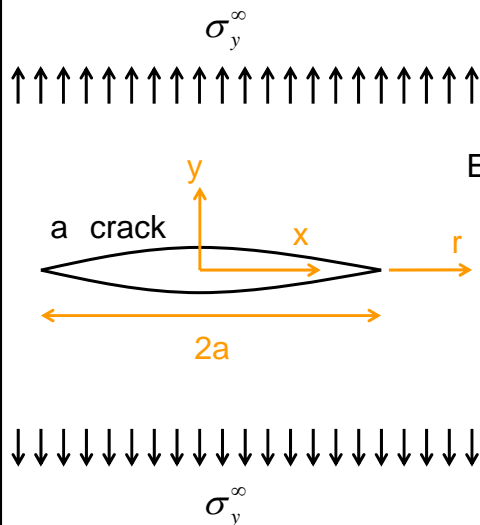
$$\rho \rightarrow 0$$



Stress near a crack tip

1. Singularity about $\frac{1}{\sqrt{r}}$
2. Intensity of Stress Singularity is Proportional to
far field stress σ_∞
square root of crack length a

Stress Analysis of Members with Cracks



Exact solution of stress near crack

$$\frac{(\sigma_y)_{y=0}}{\sigma} = \frac{|x|}{\sqrt{x^2 - a^2}} = \frac{a + r}{\sqrt{r(2a + r)}}$$

$$\frac{(\sigma_x)_{y=0}}{\sigma} = \frac{|x|}{\sqrt{x^2 - a^2}} - 1 = \frac{a + r}{\sqrt{r(2a + r)}} - 1$$

Stress Analysis of Members with Cracks

$$\frac{(\sigma_y)_{y=0}}{\sigma} = \frac{|x|}{\sqrt{x^2 - a^2}} = \frac{a+r}{\sqrt{r(2a+r)}}$$

$$\frac{(\sigma_x)_{y=0}}{\sigma} = \frac{|x|}{\sqrt{x^2 - a^2}} - 1 = \frac{a+r}{\sqrt{r(2a+r)}} - 1$$

Series Expansion $r/a < 1$

$$\frac{(\sigma_y)_{y=0}}{\sigma} = \sqrt{\frac{a}{2r}} + \frac{3}{4}\sqrt{\frac{r}{2a}} - \frac{5}{32}\left(\sqrt{\frac{r}{2a}}\right)^3 + \dots$$

$$\frac{(\sigma_x)_{y=0}}{\sigma} = \sqrt{\frac{a}{2r}} - 1 + \frac{3}{4}\sqrt{\frac{r}{2a}} - \frac{5}{32}\left(\sqrt{\frac{r}{2a}}\right)^3 + \dots$$

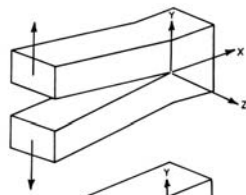
$r/a > 1$

$$\frac{(\sigma_y)_{y=0}}{\sigma_y^\infty} \Rightarrow 1 + \frac{1}{2}\left(\frac{a}{r}\right)^2$$

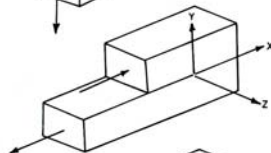
$$\frac{(\sigma_x)_{y=0}}{\sigma_y^\infty} \Rightarrow \frac{1}{2}\left(\frac{a}{r}\right)^2$$

Stress Analysis for Cracks in Elastic Solids

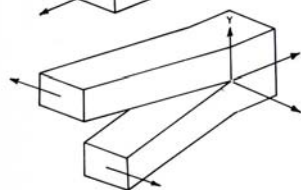
Three Types of Relative Movements of Two Crack Surfaces



MODE I : Opening Mode



MODE II : Sliding/Shear Mode



MODE III : Tearing Mode

Basic Types of Stress Fields
near Crack Tips

Important Basic Mode I

Most engineering situations corresponding to Mode I

$$\sigma_x = \frac{K_I}{(2\pi r)^{\frac{1}{2}}} \cos \frac{\theta}{2} \left[1 - \sin \frac{\theta}{2} \cdot \sin \frac{3\theta}{2} \right]$$

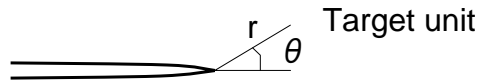
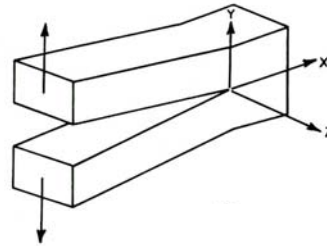
$$\sigma_y = \frac{K_I}{(2\pi r)^{\frac{1}{2}}} \cos \frac{\theta}{2} \left[1 + \sin \frac{\theta}{2} \cdot \sin \frac{3\theta}{2} \right]$$

$$\tau_{xy} = \frac{K_I}{(2\pi r)^{\frac{1}{2}}} \sin \frac{\theta}{2} \cdot \cos \frac{\theta}{2} \cdot \cos \frac{3\theta}{2}$$

$$u = \frac{K_I}{G} \left[\frac{r}{2\pi} \right]^{\frac{1}{2}} \cos \frac{\theta}{2} \left[1 - 2\nu + \sin^2 \frac{\theta}{2} \right]$$

$$v = \frac{K_I}{G} \left[\frac{r}{2\pi} \right]^{\frac{1}{2}} \sin \frac{\theta}{2} \left[1 - 2\nu - \cos^2 \frac{\theta}{2} \right]$$

$$w = 0$$



General Form of Stress Intensity Factor

The Applied Stress

The Crack Shape and Size

The Structural Configuration



Affect

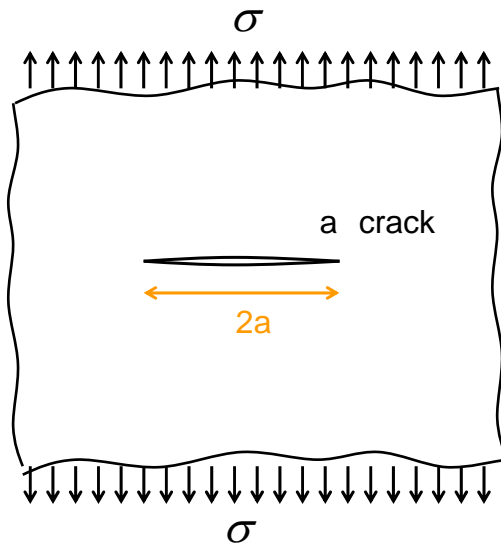
The Value of The Stress Intensity Factor, K

$$K = \sigma \sqrt{\pi a} \cdot f(g)$$

$f(g)$: Crack Geometry

Local Stress Field \longleftrightarrow Global Stress Field

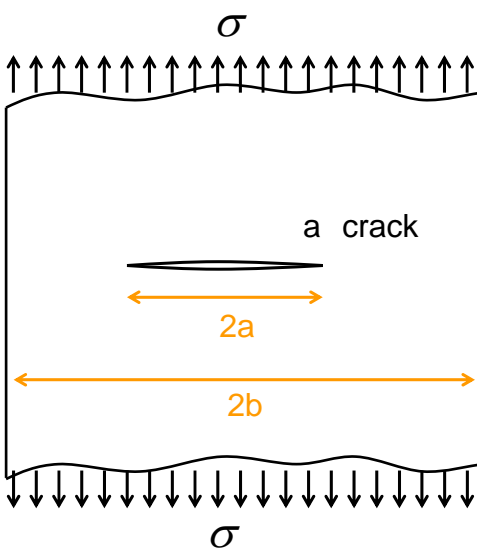
Stress Intensity Factor Equations (1)



A Through Thickness Crack
In an Infinite Plate subject
to Uniform Tensile Stress

$$K = \sigma \sqrt{\pi a}$$

Stress Intensity Factor Equations (2)

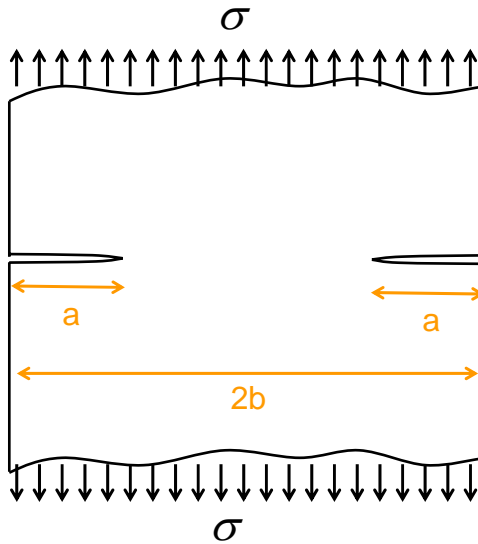


A Through Thickness Crack
In a Plate with Finite Width
Subject to
Uniform Tensile Stress

$$K = \sigma \sqrt{\pi a} \left(\frac{2b}{\pi a} \tan \frac{\pi a}{2b} \right)^{\frac{1}{2}}$$

$$= \sigma \sqrt{\pi a} \sqrt{\sec \left(\frac{\pi a}{2b} \right)}$$

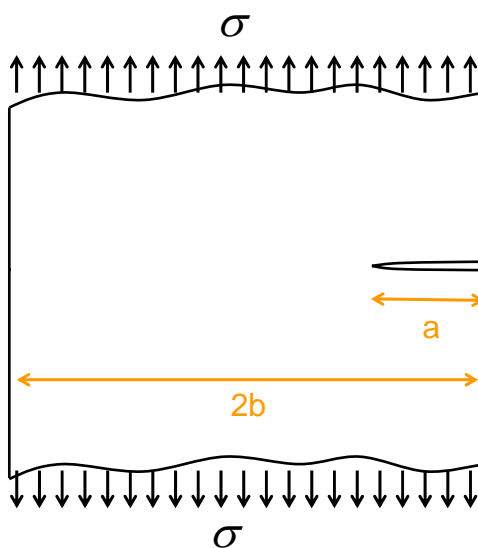
Stress Intensity Factor Equations (3)



Double-Edge Cracks
In a plate with finite width

$$K = \sigma \sqrt{\pi a} \cdot 1.12$$

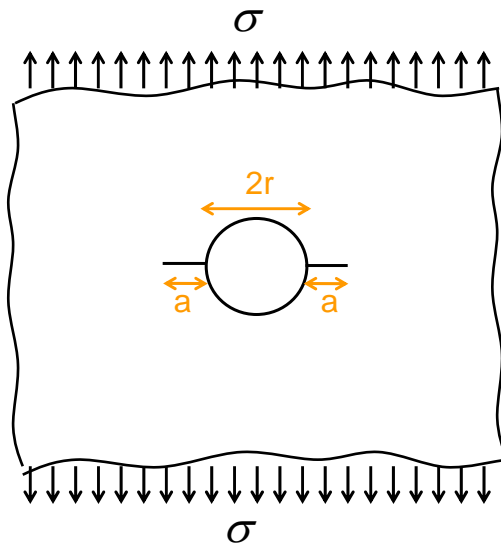
Stress Intensity Factor Equations (4)



Single-Edge Crack
In a plate with finite width

$$K = 1.12 \cdot \sigma \sqrt{\pi a} \cdot f\left(\frac{a}{b}\right)$$

Stress Intensity Factor Equations (5)



Cracks growing
from a round hole

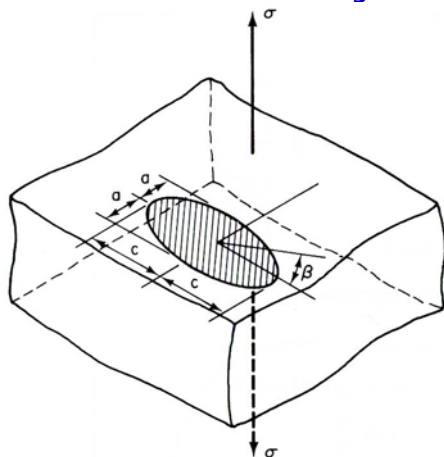
$$K = \sigma \sqrt{\pi a} \cdot f\left(\frac{a}{r}\right)$$

$$a \rightarrow 0 \quad f\left(\frac{a}{r}\right) \rightarrow 3$$

For short cracks

$$K = K_t \sigma \sqrt{\pi a}$$

Stress Intensity Factor Equations (6)



An Embedded elliptical crack
or a circular crack
In an Infinite Plate

$$K = \frac{\sigma \sqrt{\pi a}}{\Phi_0} \left(\sin^2 \beta + \frac{a^2}{c^2} \cos^2 \beta \right)^{\frac{1}{4}}$$

$$\Phi_0 = \int_0^{\pi/2} \left[1 - \left(\frac{c^2 - a^2}{c^2} \right) \sin^2 \theta \right]^{\frac{1}{2}} d\theta$$

$$K = \sigma \sqrt{\frac{\pi a}{Q}} \quad \text{for } \beta = \frac{\pi}{2}$$

$$Q = \Phi_0^2$$

$$K = 0.65 \sigma \sqrt{\pi a} = 1.15 \sigma \sqrt{a}$$

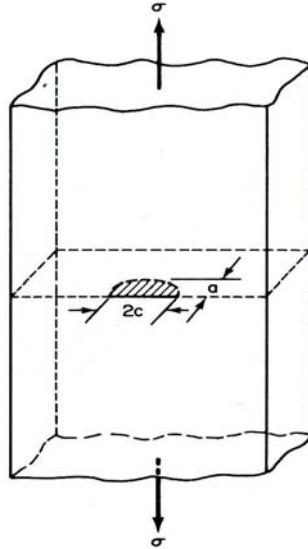
$$\text{for } \beta = \frac{\pi}{2}, a = c \quad (a \text{ circle})$$



$$K = \frac{2}{\sqrt{\pi}} \sigma \sqrt{a} = 1.13 \sigma \sqrt{a}$$

Exact expression

Stress Intensity Factor Equations (6)

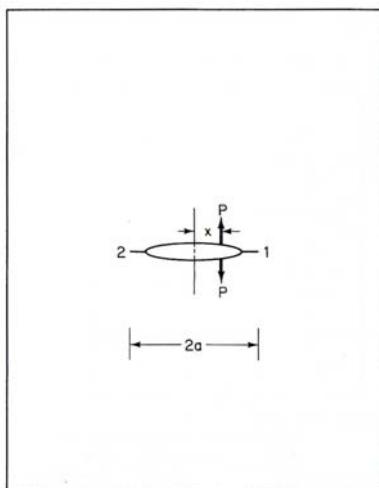


A Surface crack
or a circular crack
In an Infinite Plate

$$K = 1.12\sigma\sqrt{\pi\frac{a}{Q}} \cdot M_k$$

$$M_k = 1.0 + 1.2\left(\frac{a}{t} - 0.5\right)$$

Stress Intensity Factor Equations (7)



Cracks with wedge forces
and internal pressure

Eccentric line forces, P,
per unit thickness

$$K_{@1} = \frac{P}{\sqrt{\pi a}} \left(\frac{a+x}{a-x} \right)^{\frac{1}{2}}$$

$$K_{@2} = \frac{P}{\sqrt{\pi a}} \left(\frac{a-x}{a+x} \right)^{\frac{1}{2}}$$

$$K_{@1} = K_{@2} = \frac{P}{\sqrt{\pi a}} \quad \text{at } x=0$$

$$K = p\sqrt{\pi a} \quad : \text{ internal pressure}$$

Engineering Calculation of Stress Intensity Factor

Pedro Albrecht/ K. Yamada, ASCE, STR, Feb.1977

$$K = F(a)\sigma\sqrt{\pi a}$$

$$F(a) = F_e \cdot F_s \cdot F_w \cdot F_g$$

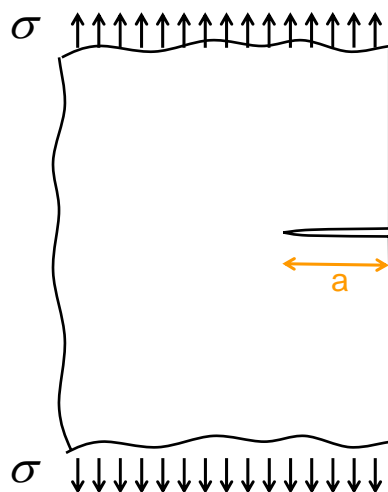
F_e : The Shape of Crack Front,
which is often assumed as an elliptical crack, correction

F_s : Effects of Free Surface, Front Free Surface Correction

F_w : Finite Width, The Back Surface Correction

F_g : Non-uniform Opening Stresses, Stress Gradient Correction

2 Dimensional Crack Problems(1)



The Edge Crack
in a semi-infinite plate

$$K = F(a)\sigma\sqrt{\pi a}$$

$$F(a) = F_s = 1.12$$

Front Free Surface Correction

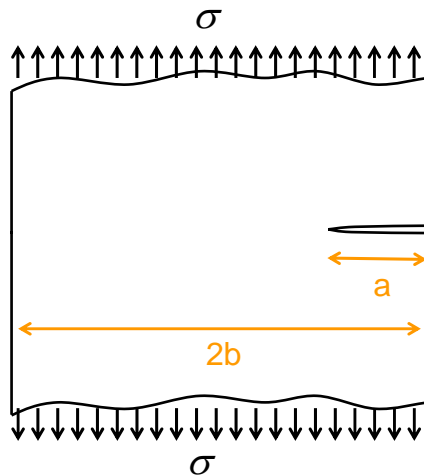


$$K = 1.12\sigma\sqrt{\pi a}$$

2 Dimensional Crack Problems(2)

Single-Edge Crack

In a plate with finite width



$$K = F(a)\sigma\sqrt{\pi a}$$

$$F(a) = F_s \cdot F_w$$

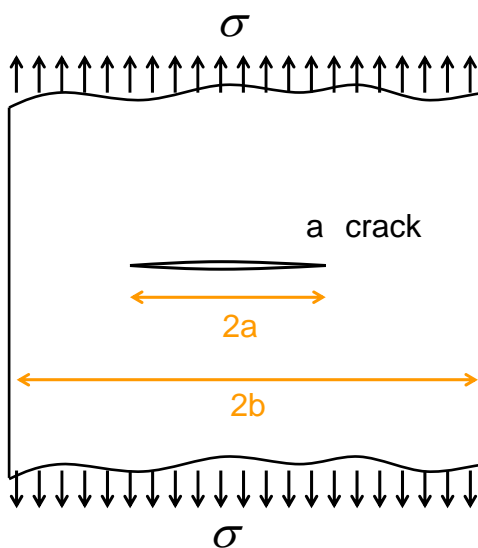
$$K = \underbrace{1.12}_{F_s} \sigma \sqrt{\pi a} \cdot \underbrace{f\left(\frac{a}{b}\right)}_{F_w}$$

Front Free Surface Correction Finite Width

2 Dimensional Crack Problems (3)

A Through Thickness Crack

In a Plate with Finite Width
Subject to
Uniform Tensile Stress

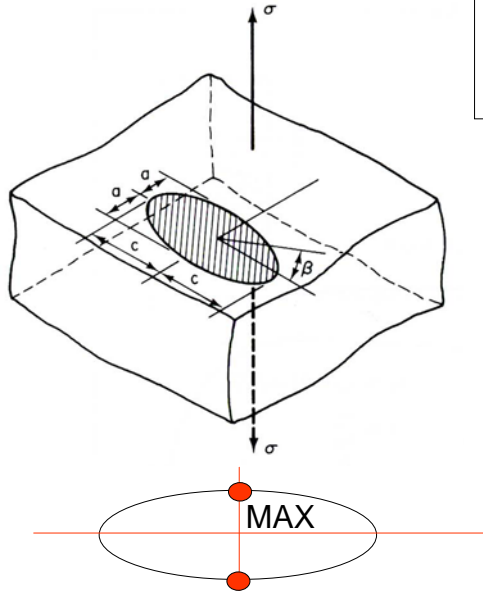


$$K = \sigma \sqrt{\pi a} \left(\frac{2b}{\pi a} \tan \frac{\pi a}{2b} \right)^{\frac{1}{2}}$$

$$= \sigma \sqrt{\pi a} \sqrt{\sec \left(\frac{\pi a}{2b} \right)}$$

Finite Width $F(a) = F_w$

3 Dimensional Crack Problems (1)



An Embedded elliptical crack
or a circular crack
In an Infinite Plate

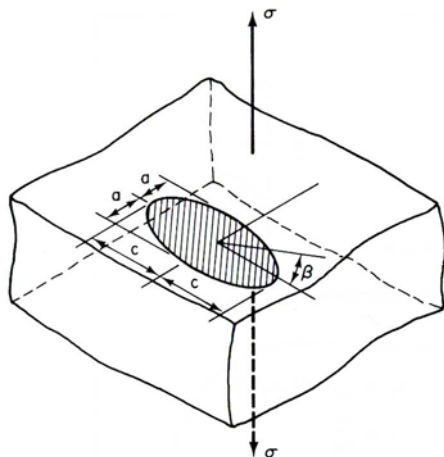
$$K = \frac{1}{\Phi_0} \left(\sin^2 \beta + \frac{a^2}{c^2} \cos^2 \beta \right)^{\frac{1}{4}} \cdot \sigma \sqrt{\pi a}$$

$$\Phi_0 = \int_0^{\frac{\pi}{2}} \left[1 - \left(\frac{c^2 - a^2}{c^2} \right) \sin^2 \theta \right]^{\frac{1}{2}} d\theta$$

$$K = \frac{1}{\Phi_0} \sigma \sqrt{\pi a} \quad \text{for } \beta = \frac{\pi}{2}$$

$$F_e \quad F(a) = F_e = \frac{1}{\Phi_0}$$

3 Dimensional Crack Problems (1)



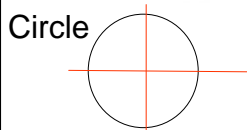
An Embedded elliptical crack
or a circular crack
In an Infinite Plate

$$K = \frac{1}{\Phi_0} \left(\sin^2 \beta + \frac{a^2}{c^2} \cos^2 \beta \right)^{\frac{1}{4}} \cdot \sigma \sqrt{\pi a}$$

$$\Phi_0 = \int_0^{\frac{\pi}{2}} \left[1 - \left(\frac{c^2 - a^2}{c^2} \right) \sin^2 \theta \right]^{\frac{1}{2}} d\theta$$

$$K = \frac{1}{\Phi_0} \sigma \sqrt{\pi a} \quad \text{for } \beta = \frac{\pi}{2}$$

$$F_e \quad F(a) = F_e = \frac{1}{\Phi_0} = \frac{2}{\pi}$$

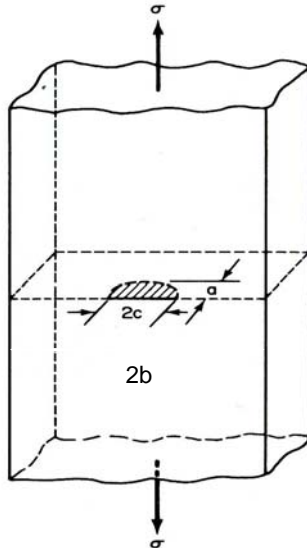


$$K = \frac{2}{\pi} \sigma \sqrt{\pi a}$$



for $a = c$

3 Dimensional Crack Problems (2)



A Surface crack
or a circular crack
In a Plate with Finite Width

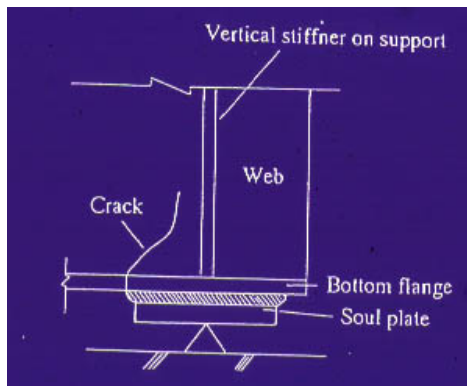
$$K = M_k \cdot \underbrace{\frac{1}{\Phi_0}}_{F_s} \cdot \underbrace{F_g}_{F_e} \sigma \sqrt{\pi a} \cdot \underbrace{\sqrt{\frac{2b}{\pi a} \tan \frac{\pi a}{2b}}}_{F_w}$$

$$M_k = \begin{cases} = 1.0 + 1.2 \left(\frac{a}{t} - 0.5 \right) & \text{for surface cracks} \\ = 1.12 & \text{for through thickness cracks} \end{cases}$$

Stress Gradient Correction
Correction for Stress Concentration

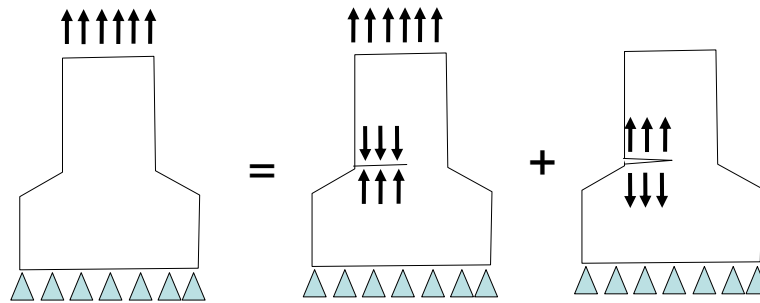
Geometry Correction Factor F_g

Cracks always occur at geometrical discontinuities
as cover plate ends



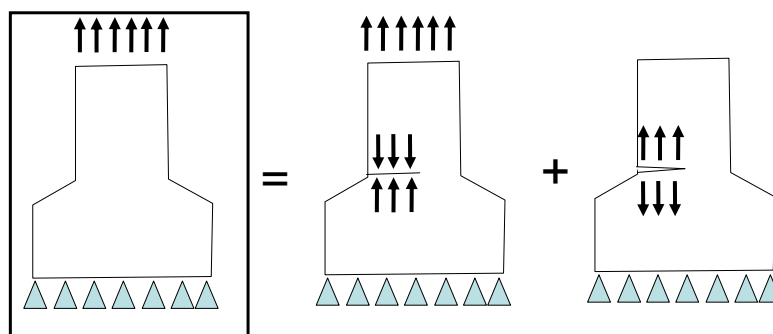
Calculation of F_g

Superimpose



Calculation of F_g

Superimpose



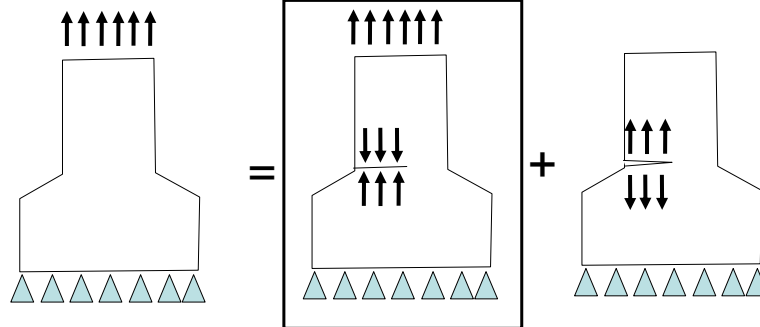
Step 1

Compute the actual stress
Along the line where the crack
Shall be inserted
By any suitable methods.

FEM

Calculation of F_g

Superimpose

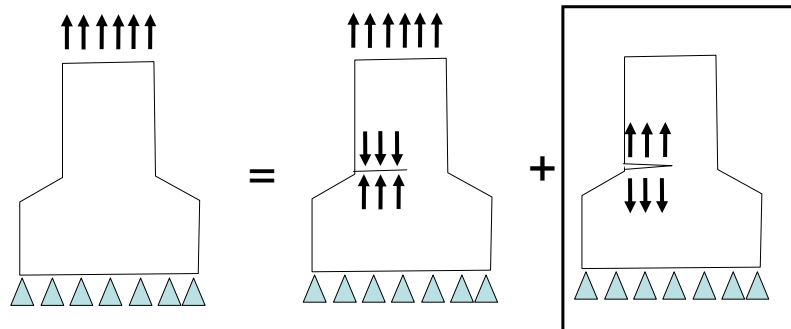


Step 2

Insert a crack of given length
Along the same line

Calculation of F_g

Superimpose

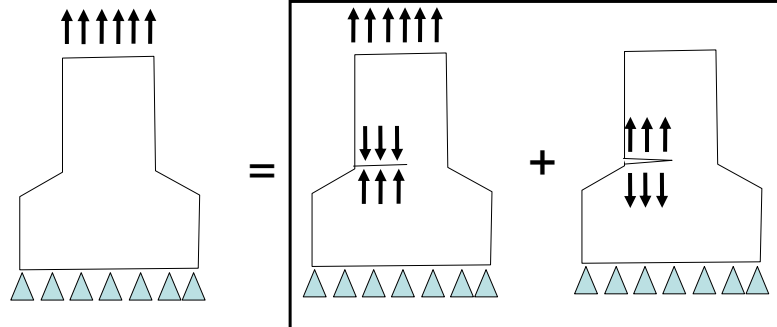


Step 3

Compute K by integrating away
the normal, determined in Step 1
stresses applied over the length
of the crack

Calculation of F_g

Superimpose



Step 4

Repeat steps 2 and 3
for any desired crack size

Calculation of F_g

Calculation of K in Step 3

Concentrated Forces

$$K = \frac{2P}{\sqrt{\pi a}} \frac{a}{\sqrt{a^2 - b^2}}$$

Distributed Forces

$$K = \sqrt{\pi a} \frac{2}{\pi} \int_0^a \frac{\sigma_b}{\sqrt{a^2 - b^2}} db$$

Calculation of F_g

Stress Distribution in Step 1

Closed form expressions defining the stress distribution
In the uncracked body are usually not available.

From FEM

$$K = \sqrt{\pi a} \frac{2}{\pi} \sum_{i=1}^n \sigma_{b_i} \int_{b_i}^{b_{i+1}} \frac{1}{\sqrt{a^2 - b^2}} db$$

In which the discrete stress σ_{b_i} is applied
over the element b_i and b_{i+1}

Calculation of F_g

Stress Distribution in Step 1

After factoring out the mean stress

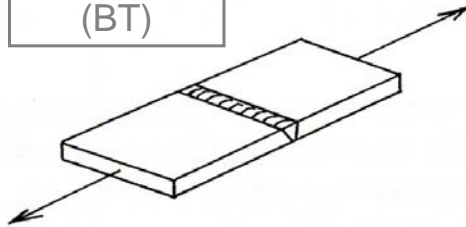
$$K = \sqrt{\pi a} \frac{2}{\pi} \sum_{i=1}^n \sigma_{b_i} \int_{b_i}^{b_{i+1}} \frac{1}{\sqrt{a^2 - b^2}} db$$



$$K = \sigma \sqrt{\pi a} \frac{2}{\pi} \sum_{i=1}^n \frac{\sigma_{b_i}}{\sigma} \left(\arcsin \frac{b_{i+1}}{a} - \arcsin \frac{b_i}{a} \right)$$

Examples of F_g

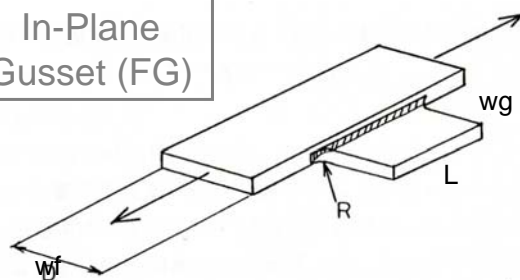
Butt Joint
(BT)



$$F_g = 1$$

Examples of F_g

In-Plane
Gusset (FG)

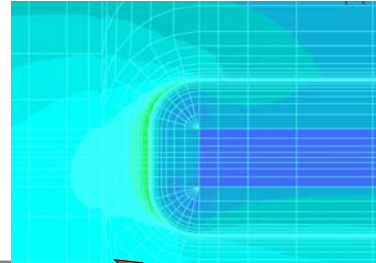
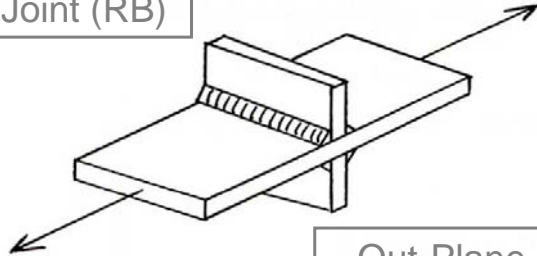


$$F_g = \frac{-1.115 \log\left(\frac{R}{w_f}\right) + 0.537 \log\left(\frac{L}{w_f}\right) + 0.1384 \log\left(\frac{w_g}{w_f}\right) + 0.6801}{1 + \frac{1}{1.158} \cdot \left(\frac{a}{w_f}\right)^{0.6051}}$$

By Zettlemoyer and Fisher

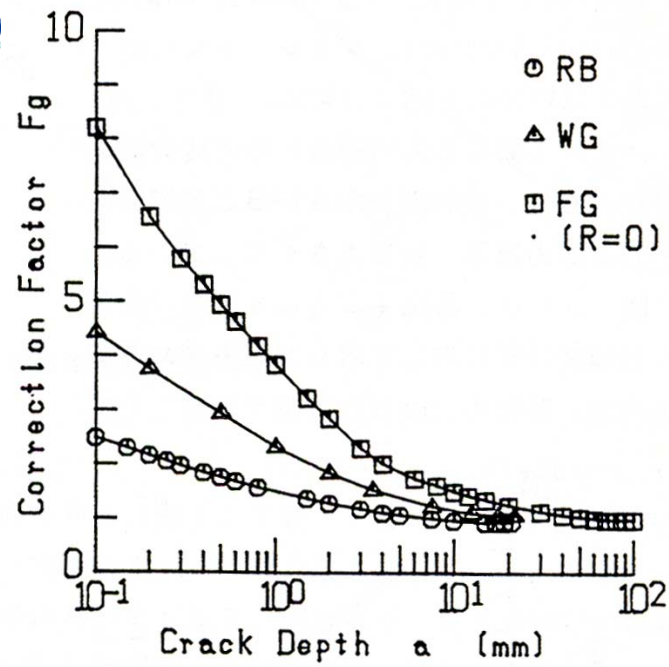
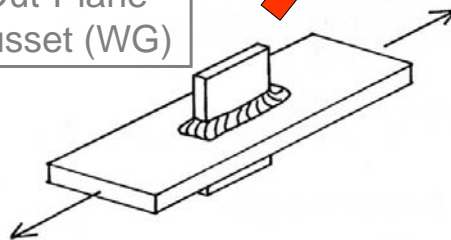
Examples of F_g

Cruciform
Joint (RB)



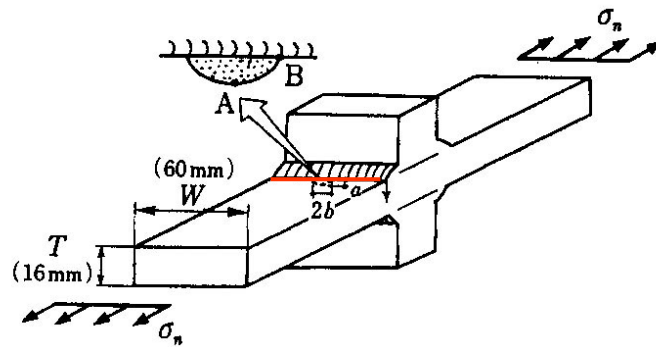
Stress Distribution
From FEM

Out-Plane
Gusset (WG)

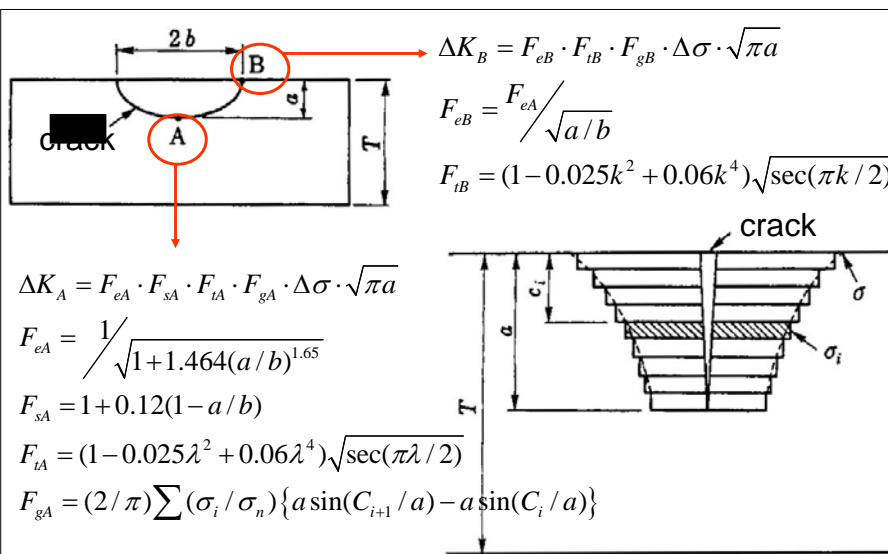


Examples of F_g

Cracks from Incomplete Penetration

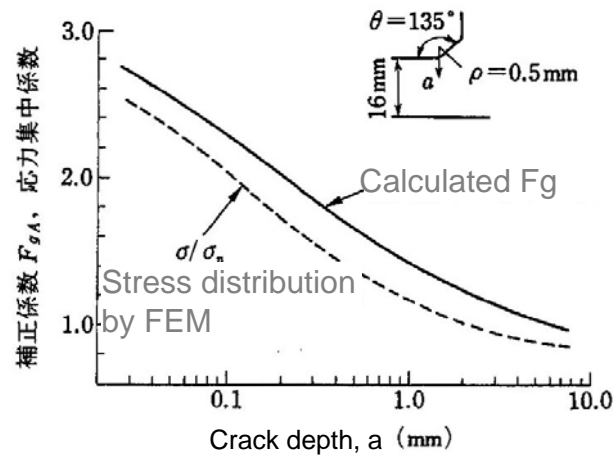


Model of Cracks

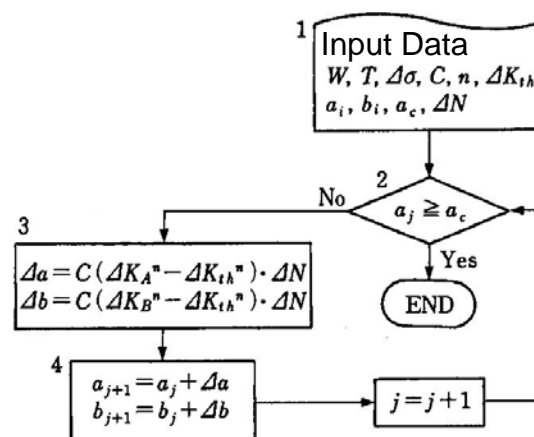


Calculation of F_g

Relationships of stress distribution and F_g

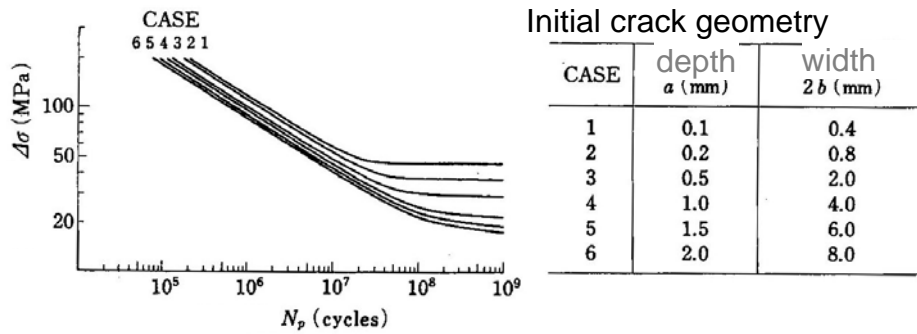


Calculation strategies

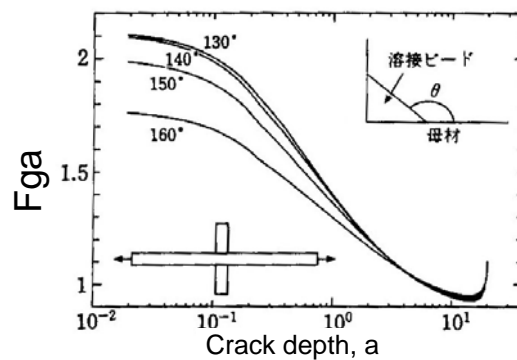
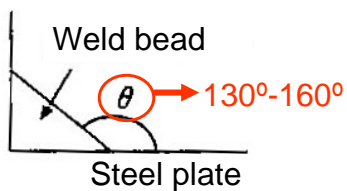
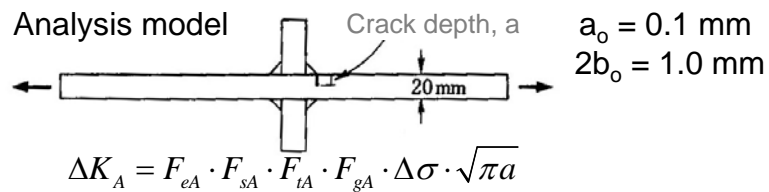


Results

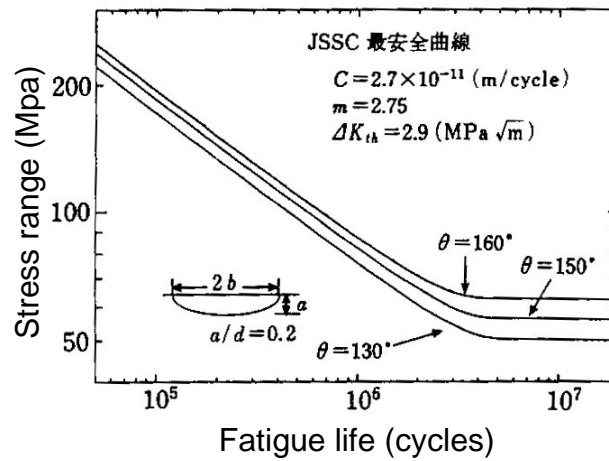
Variation of initial crack width and height



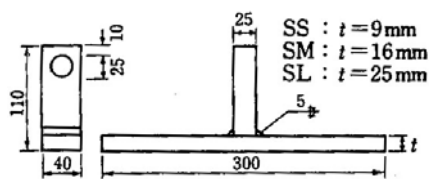
Effects of weld shape



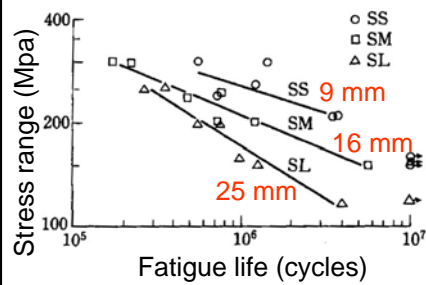
Effects of weld shape



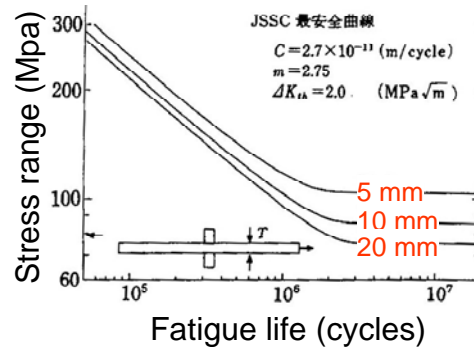
Effects of plate thickness



Experimental results



Computational results



Fracture Toughness

The Condition for Unstable Crack Growth

$$K = K_c$$

Fracture Toughness

The Critical Stress Intensity Factor
for Unstable Crack Growth

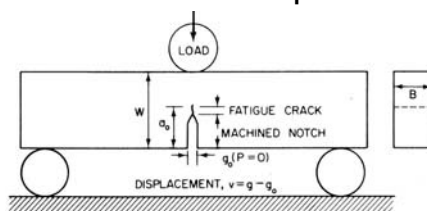
Mode I

$$K_I = K_{Ic}$$

Test Methods to Evaluate K_{Ic}

ASTM E-399

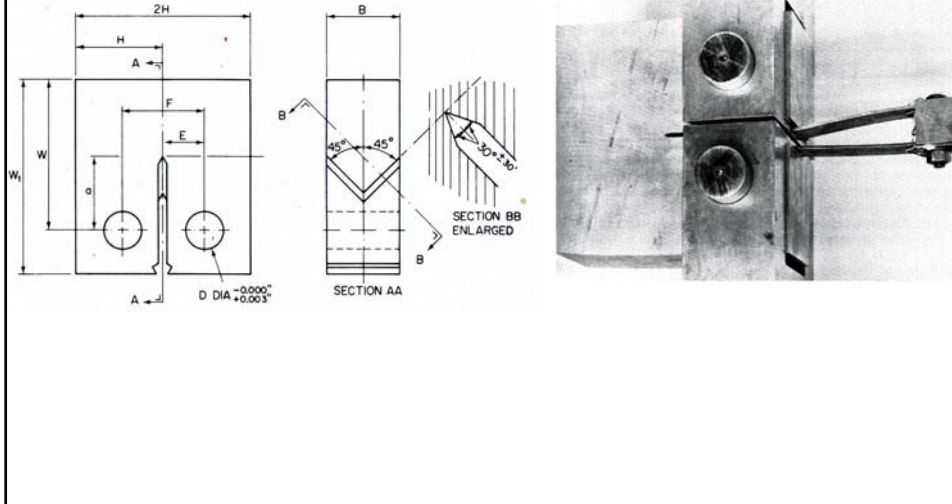
A Bend Specimen



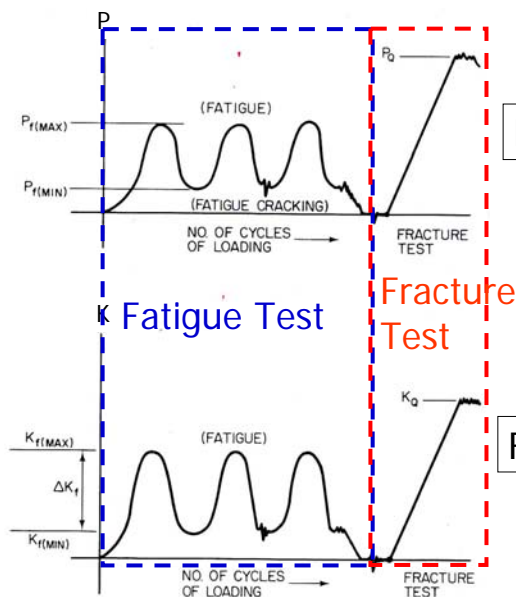
Test Methods to Evaluate K_{Ic}

ASTM E-399

Compact Tension Specimen



Test Methods to Evaluate K_{Ic}



The Testing Program

Fatigue Cracking

Small Scale Yielding
Condition

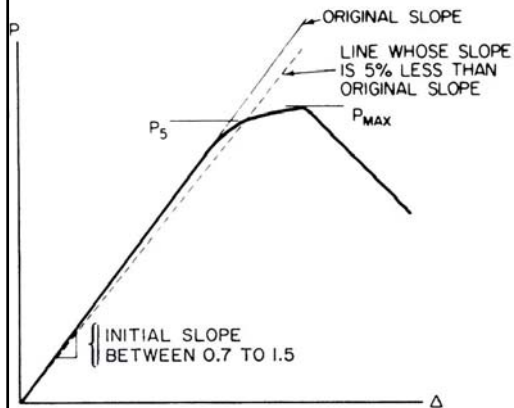


Fracture Test

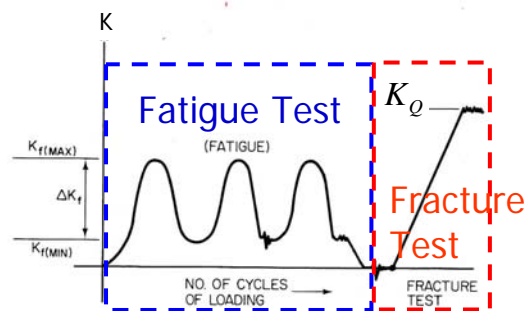
Plane Strain Condition

Test Methods to Evaluate K_{Ic}

Fracture Tests



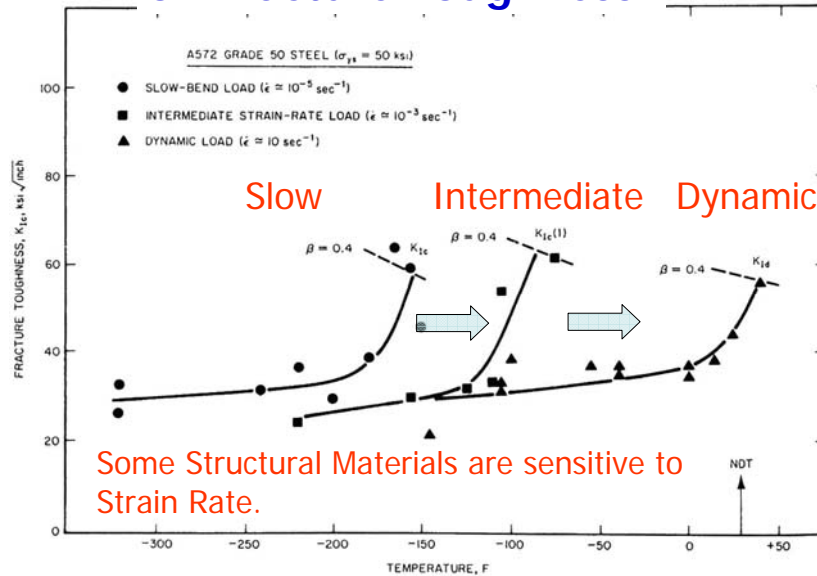
Test Methods to Evaluate K_{Ic}



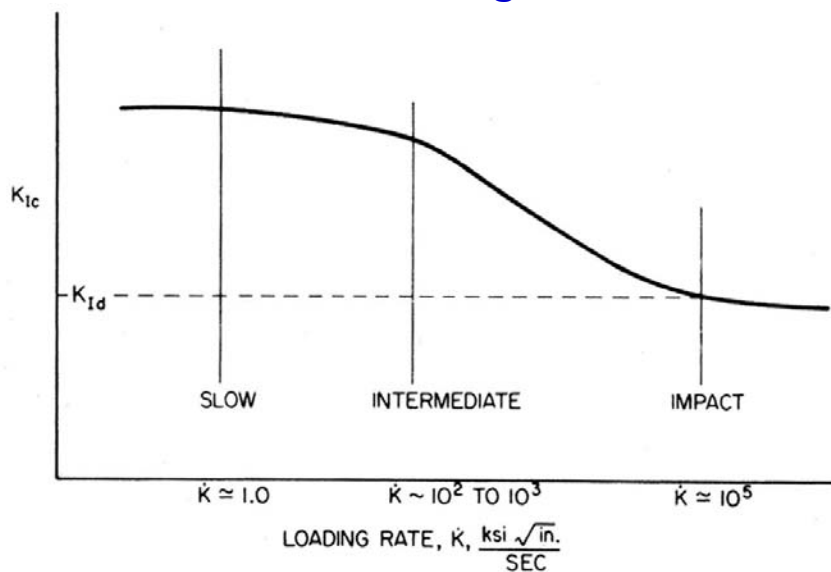
$$a \geq 2.5 \left(\frac{K_Q}{\sigma_{Yield}} \right)^2, B \geq 2.5 \left(\frac{K_Q}{\sigma_{Yield}} \right)^2, W \geq 5.0 \left(\frac{K_Q}{\sigma_{Yield}} \right)^2$$

$$K_Q \Rightarrow K_{Ic}$$

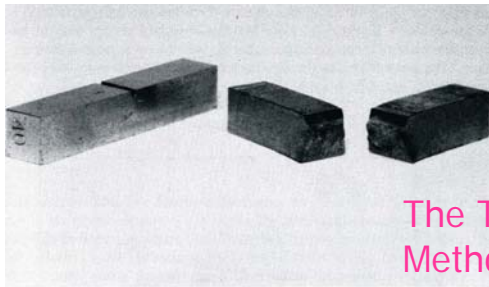
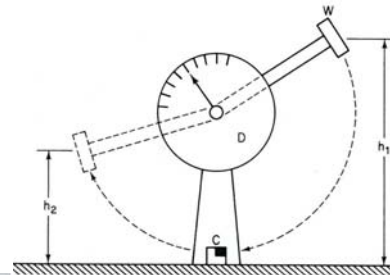
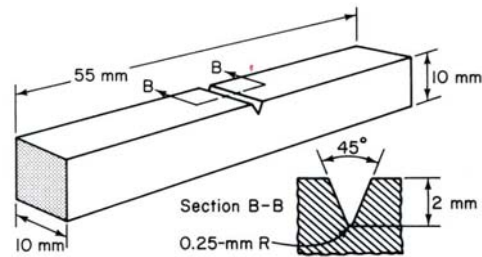
Effects of Loading Rate on Fracture Toughness



Effects of Loading Rate on Fracture Toughness



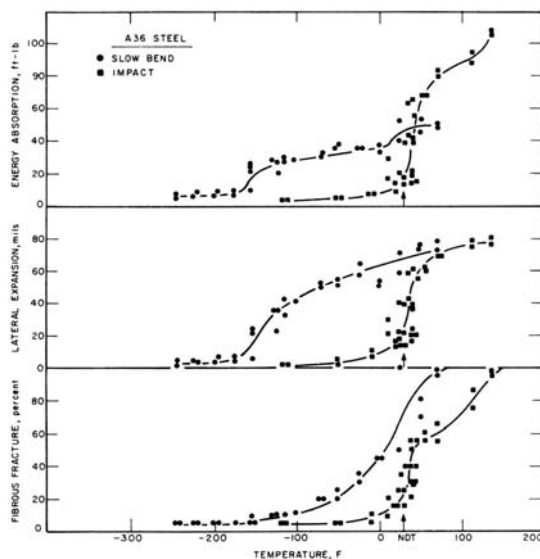
Charpy Impact Tests



Charpy V-Notch (CVN)
Impact Tests

The Traditional Evaluation
Method of Fracture Toughness

Charpy Impact Tests



To Prevent
Brittle Fracture

Absorbed Energy: E_v
Transition
Temperature: T_{tr}

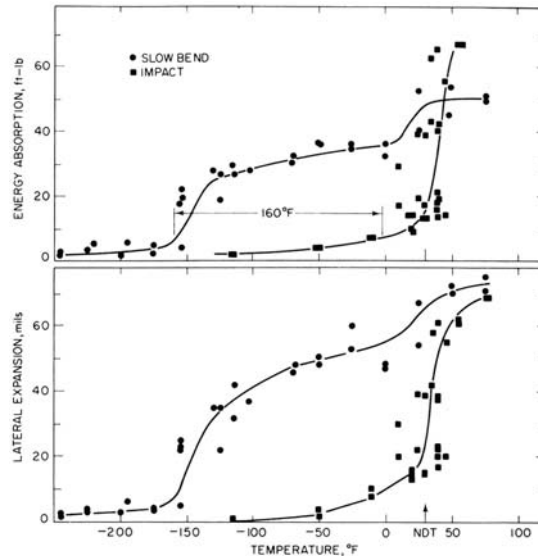
Absorbed Energy-Temperature

CVN Tests vs K1c Tests

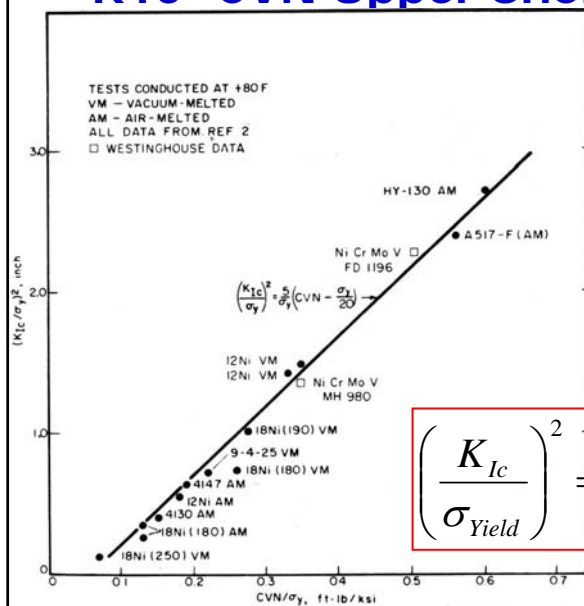
CVN Tests:
Dynamic Condition
Crack Initiation
+ Propagation



K1c Tests:
Slow Bending
Crack Initiation



K1c- CVN Upper Shelf Correlation



Proposed by
Barsom and Rolf

$$\left(\frac{K_{Ic}}{\sigma_{Yield}} \right)^2 = 5 \left(\frac{CVN}{\sigma_{Yield}} - 0.05 \right)$$