

低雑音増幅回路

通信機器システムの要

雑音の評価

雑音の定式化

瞬時値での評価→困難(不可)



統計的な評価

$$\text{2乗平均値} : \overline{v_n^2} = \frac{1}{\tau} \int_0^{\tau} v_n^2(t) dt$$

複数の雑音源の取り扱い

$$\begin{aligned} & \frac{1}{\tau} \int_0^{\tau} \{v_{n1}(t) + v_{n2}(t)\}^2 dt \\ &= \frac{1}{\tau} \left[\int_0^{\tau} v_{n1}^2(t) dt + 2 \int_0^{\tau} v_{n1}(t) v_{n2}(t) dt + \int_0^{\tau} v_{n2}^2(t) dt \right] \\ &= \overline{v_{n1}^2} + \frac{2}{\tau} \int_0^{\tau} v_{n1}(t) v_{n2}(t) dt + \overline{v_{n2}^2} \end{aligned}$$

$v_{n1}(t)$ と $v_{n2}(t)$ が全く独立：無相関

$$\int_0^{\tau} v_{n1}(t) v_{n2}(t) dt = 0$$

信号対雑音比

$$SNR = \frac{\overline{v_S^2}}{v_N^2}$$

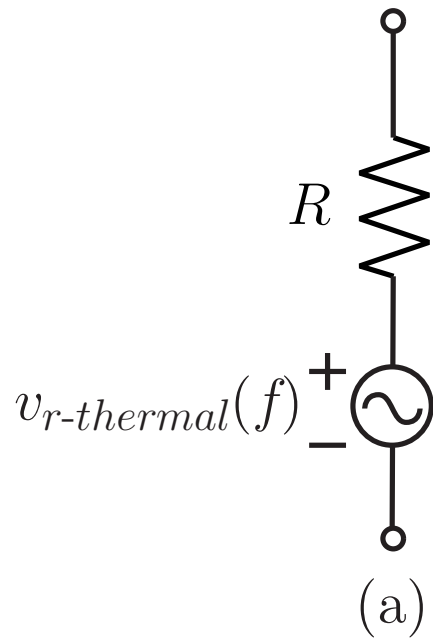
S/N比(Signal-to-Noise Ratio, SNR)

雑音源を考慮した回路素子モデル

熱雑音, $1/f$ 雑音, ショット雑音

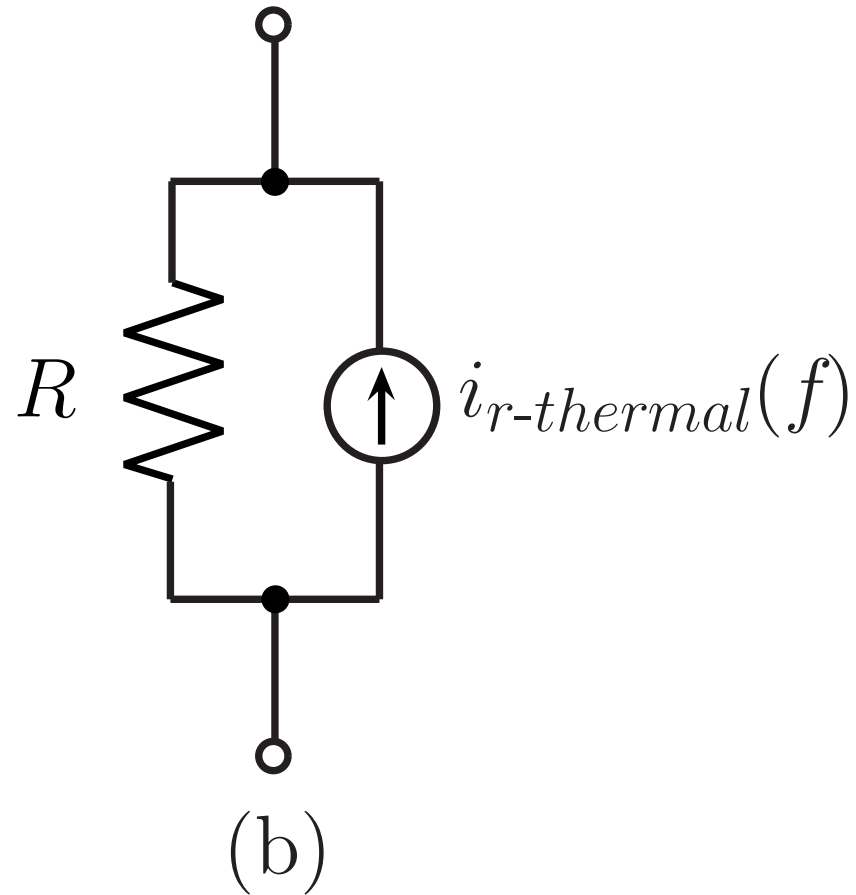
熱雑音: キャリアが移動する際のランダムな動きによって
生じる雑音

抵抗の熱雑音



$$\overline{v_{r-thermal}^2(f)} = 4kTR [\text{V}^2/\text{Hz}]$$

電源の等価性($v_{r-thermal}=Ri_{r-thermal}$)から



$$\overline{i_{r-thermal}^2(f)} = 4kT \frac{1}{R} [\text{A}^2/\text{Hz}]$$

MOSTランジスタの熱雑音

キャリアがチャネルを通過する際に発生

$$\overline{v_{mos-thermal}^2} = 4kT\gamma_n \frac{1}{g_m}$$

$$\overline{i_{mos-thermal}^2} = 4kT\gamma_n g_m$$

$$\gamma_n \approx \frac{2}{3} \quad (\text{最小線幅が短くなると増加})$$

バイポーラトランジスタの熱雑音

$$\text{ベース広がり抵抗： } \overline{v_{b-thermal}(f)^2} = 4kTr_b$$

エミッタ抵抗, コレクタ・エミッタ間抵抗: 仮想の抵抗



熱雑音の発生無し

スペクトラム強度が一定: 白色雑音

1/f雑音: シリコンの汚れや結晶欠陥によりキャリアが捕らわれたり, 捕らわれたキャリアが離されたりを繰り返すというランダムな過程によって生じる雑音

直流電流がないと発生しない

$$\overline{v_{mos-1/f}^2(f)} = \frac{\alpha_{1/f}}{C_{OX}WLf}$$

$$\overline{i_{b-1/f}^2(f)} = \frac{K_{1/f}I_B^a}{f}$$

ショット雑音：

電流＝キャリアによる電流パルスの和の平均値
電流の平均値からの揺らぎ

$$\overline{i_{b-shot}^2(f)} = 2qI_B$$

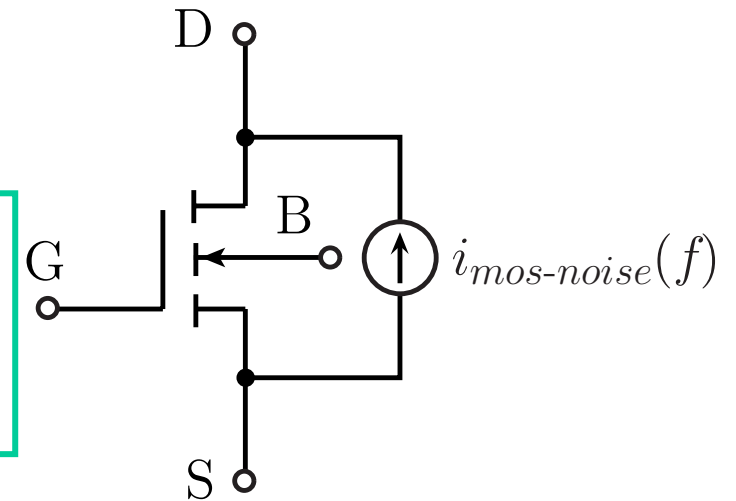
$$\overline{i_{c-shot}^2(f)} = 2qI_C$$

$$1\text{nA}@1\text{GHz} \rightarrow 1.6 \times 10^{-19} \text{C} \times 6.25 \text{個}$$

雑音源を含むトランジスタモデル

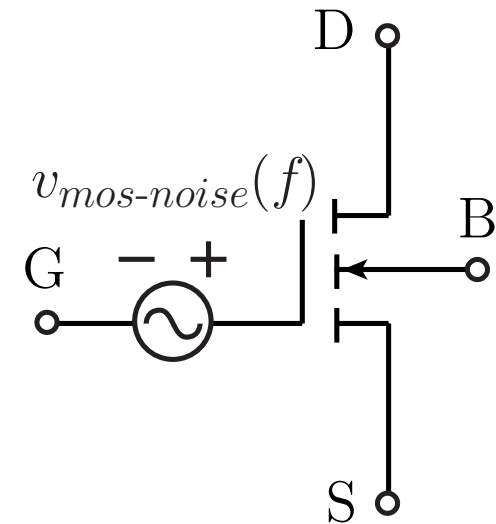
MOSトランジスタモデル

$$\overline{i_{mos-noise}^2(f)} = 4kT\gamma_n g_m + \frac{\alpha_{1/f} g_m^2}{C_{OX}WLf}$$



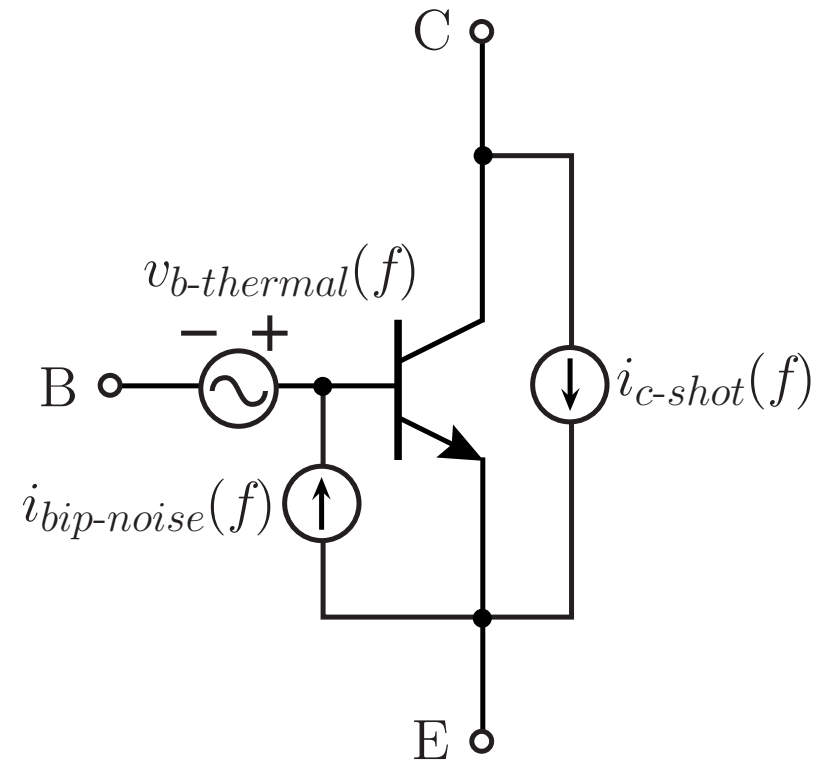
(a)

$$\overline{v_{mos-noise}^2(f)} = 4kT\gamma_n \frac{1}{g_m} + \frac{\alpha_{1/f}}{C_{OX}WLf}$$



(b)

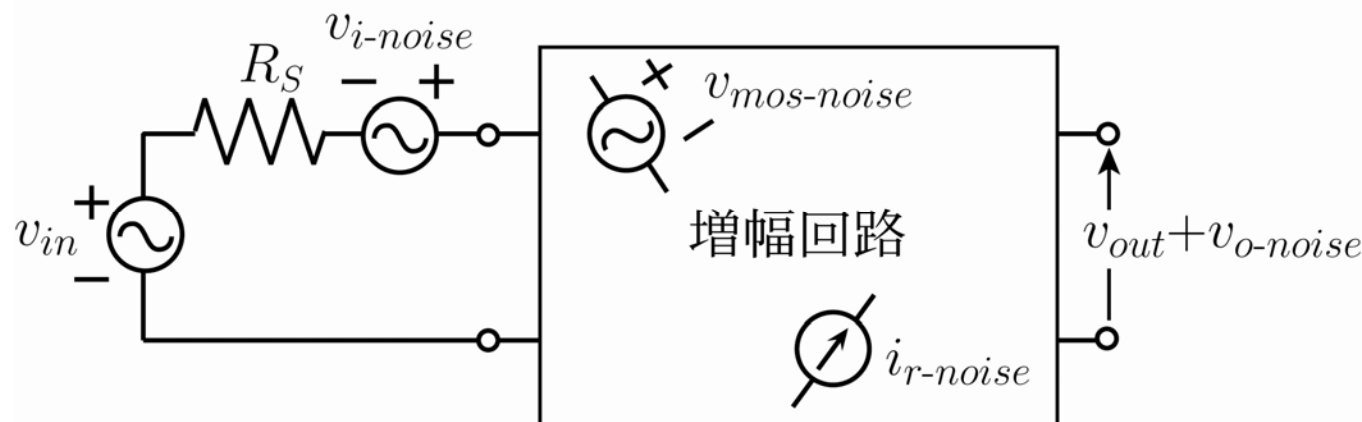
バイポーラトランジスタモデル



$$\overline{i_{bip-noise}^2(f)} = \frac{K_{1/f} I_B^a}{f} + 2qI_B$$

増幅回路と雑音

雑音係数と雑音指数

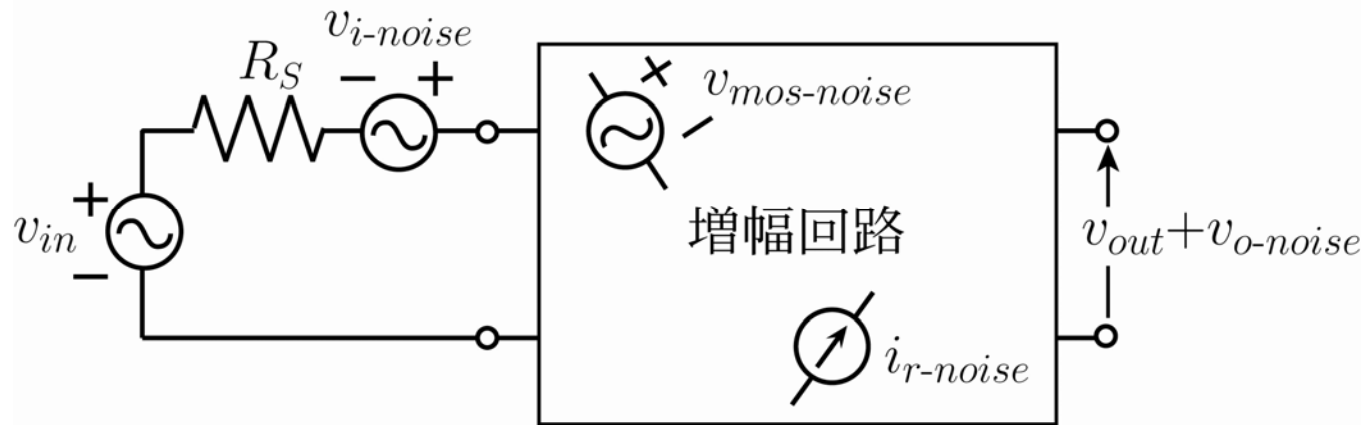


$$SNR_{in} = \frac{\overline{v_{in}^2}}{\overline{v_{i-noise}^2}}$$

$$SNR_{out} = \frac{A^2 \overline{v_{in}^2}}{\overline{v_{o-noise}^2}}$$

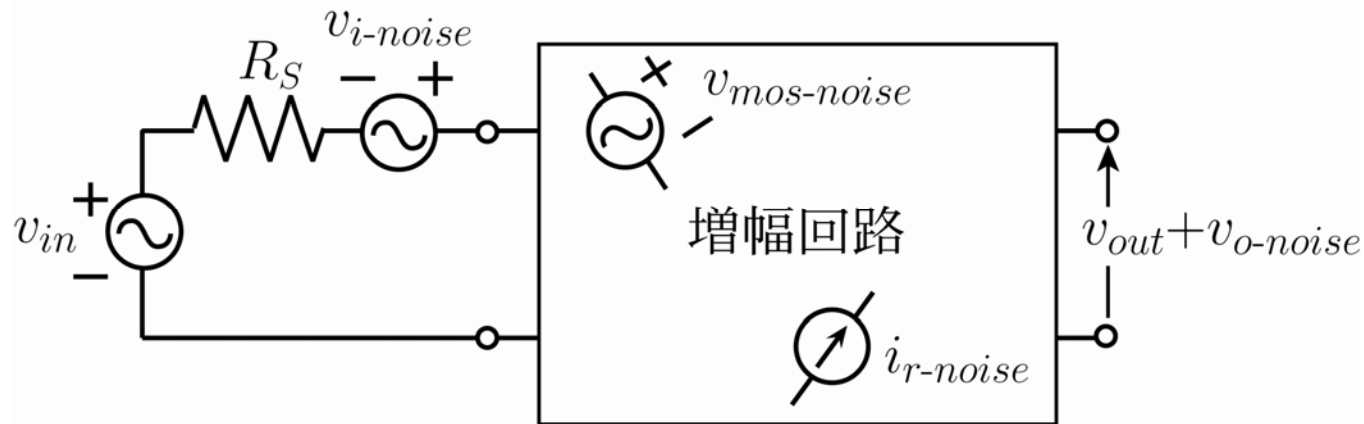
$$F = \frac{SNR_{in}}{SNR_{out}} : \text{雑音係数}$$

$$NF = 10 \log F : \text{雑音指数}$$



$$\overline{v_{o-noise}^2} = A^2 \overline{v_{i-noise}^2} + \overline{v_{inner-noise}^2}$$

$v_{mos-noise}$ や $i_{r-noise}$ などの増幅回路内部の雑音に起因する成分



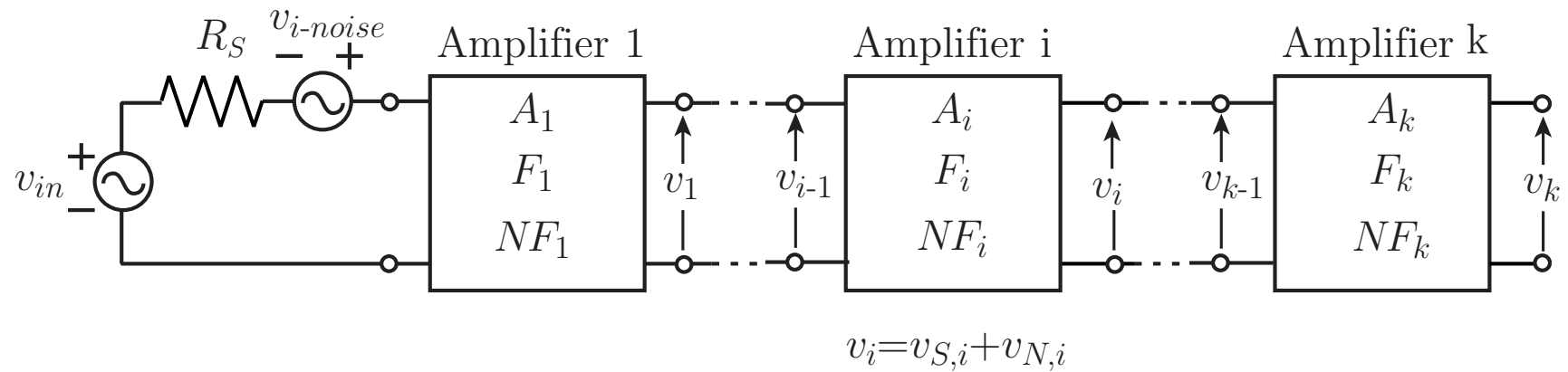
$$\overline{v_{o-noise}^2} = A^2 \overline{v_{i-noise}^2} + \overline{v_{inner-noise}^2}$$

$$SNR_{out} = \frac{A^2 \overline{v_{in}^2}}{\overline{v_{o-noise}^2}} = \frac{\overline{v_{in}^2}}{\overline{v_{i-noise}^2} + \overline{v_{inner-noise}^2} / A^2}$$

$$SNR_{in} = \frac{\overline{v_{in}^2}}{\overline{v_{i-noise}^2}} \geq SNR_{out}$$

$$F = 1 + \frac{\overline{v_{inner-noise}^2}}{A^2 \overline{v_{i-noise}^2}}$$

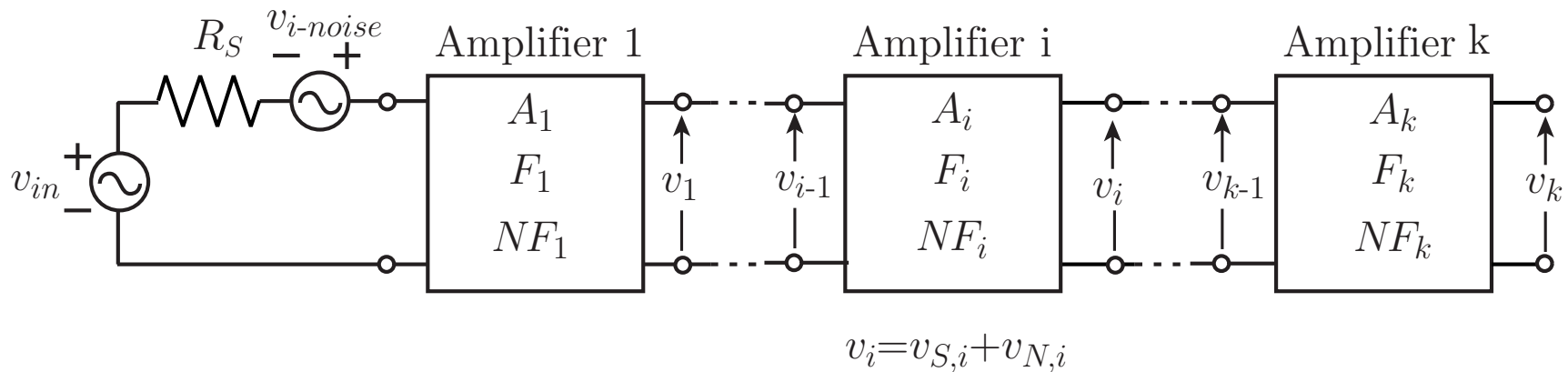
縦続接続型システムの評価



$$\overline{v_{o-noise}^2} = A^2 \overline{v_{i-noise}^2} + \overline{v_{inner-noise}^2}$$

$$F = 1 + \frac{\overline{v_{inner-noise}^2}}{A^2 \overline{v_{i-noise}^2}}$$

$$\overline{v_{N,i}^2} = A_i^2 F_i \overline{v_{N,i-1}^2}$$

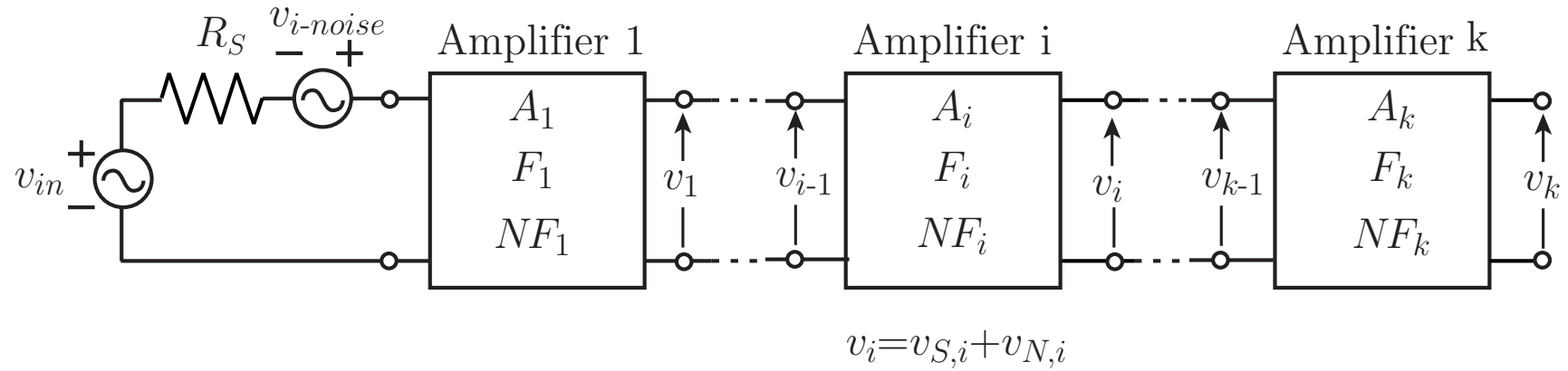


$$\overline{v_{N,i}^2} = A_i^2 F_i \overline{v_{N,i-1}^2}$$

$$\overline{v_{S,i}^2} = A_i^2 \overline{v_{S,i-1}^2}$$

$$F_{total} = \frac{\overline{v_{i-noise}^2}}{\overline{v_{S,k}^2}} = \frac{\overline{v_{i-noise}^2}}{\overline{A_1^2 A_2^2 \dots A_k^2 v_{in}^2}} = \underline{\underline{F_1 F_2 \dots F_k}}$$

$$\overline{v_{N,k}^2} = \overline{A_1^2 F_1 A_2^2 F_2 \dots A_k^2 F_k v_{i-noise}^2}$$



$$\overline{v_{N,i}^2} = A_i^2 \overline{v_{N,i-1}^2} + \overline{v_{inner-noise,i}^2}$$

$$\begin{aligned} \overline{v_{N,k}^2} &= A_k^2 A_{k-1}^2 \dots A_1^2 \overline{v_{i-noise}^2} \\ &+ A_k^2 A_{k-1}^2 \dots A_2^2 \overline{v_{inner-noise,1}^2} \\ &+ A_k^2 A_{k-1}^2 \dots A_3^2 \overline{v_{inner-noise,2}^2} + \dots \\ &+ A_k^2 \overline{v_{inner-noise,k-1}^2} + \overline{v_{inner-noise,k}^2} \end{aligned}$$

$$\begin{aligned}
\overline{v_{N,k}^2} &= A_k^2 A_{k-1}^2 \cdots A_1^2 \overline{v_{i-noise}^2} \\
&+ A_k^2 A_{k-1}^2 \cdots A_2^2 \overline{v_{inner-noise,1}^2} \\
&+ A_k^2 A_{k-1}^2 \cdots A_3^2 \overline{v_{inner-noise,2}^2} + \cdots \\
&+ A_k^2 \overline{v_{inner-noise,k-1}^2} + \overline{v_{inner-noise,k}^2}
\end{aligned}$$

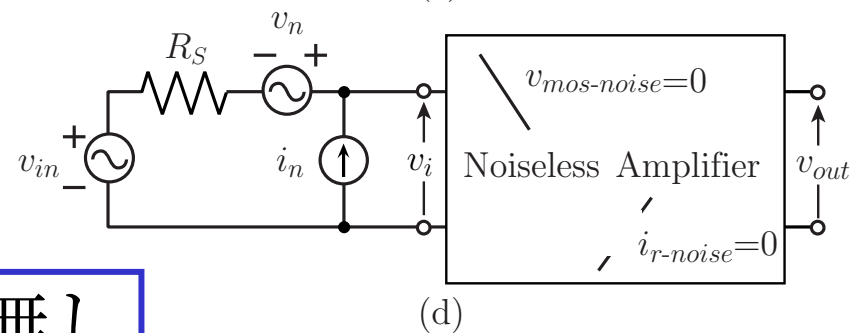
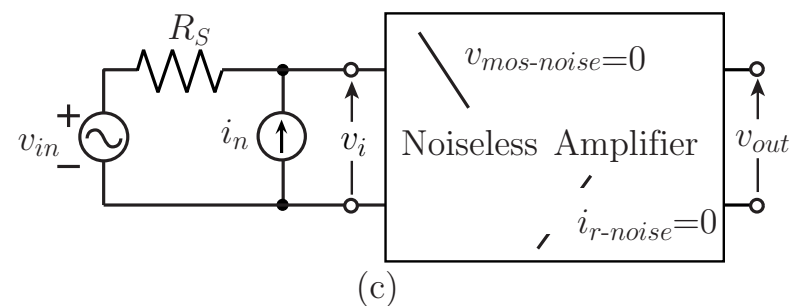
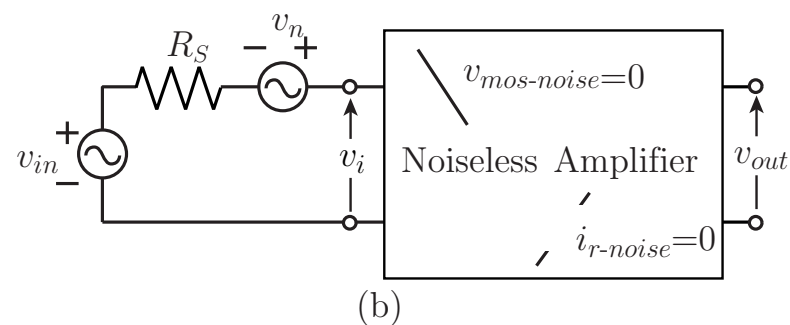
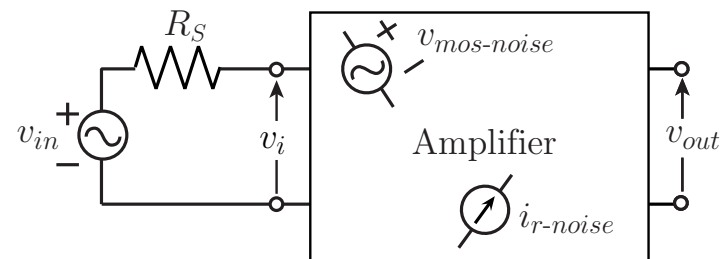
$$\begin{aligned}
F_{total} &= \frac{\overline{v_{in}^2}}{\overline{v_{i-noise}^2}} \cdot \frac{\overline{v_{N,k}^2}}{\overline{v_{S,k}^2}} = 1 + \frac{\overline{v_{inner-noise,1}^2}}{A_1^2 \overline{v_{i-noise}^2}} + \frac{\overline{v_{inner-noise,2}^2}}{A_1^2 A_2^2 \overline{v_{i-noise}^2}} \\
&+ \cdots + \frac{\overline{v_{inner-noise,k-1}^2}}{A_1^2 A_2^2 \cdots A_{k-1}^2 \overline{v_{i-noise}^2}} + \frac{\overline{v_{inner-noise,k}^2}}{A_1^2 A_2^2 \cdots A_k^2 \overline{v_{i-noise}^2}}
\end{aligned}$$

低雑音増幅回路の設計

増幅回路内部で発生する
雑音の等価表現

(b): R_S が無限大のとき矛盾
(c): R_S が零のとき矛盾

(d): 任意の R_S について矛盾無し

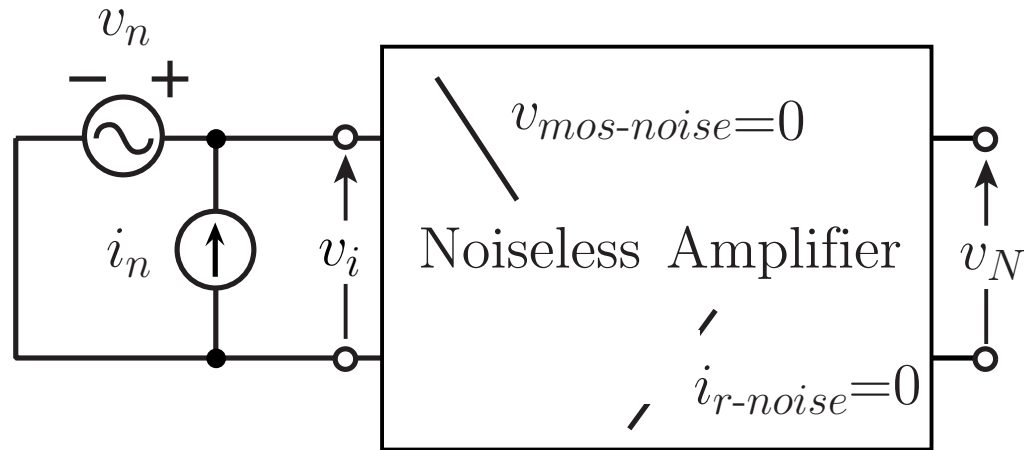


増幅回路の出力雑音のみ考慮

v_n の求め方

$$v_n = \frac{v_N}{A}$$

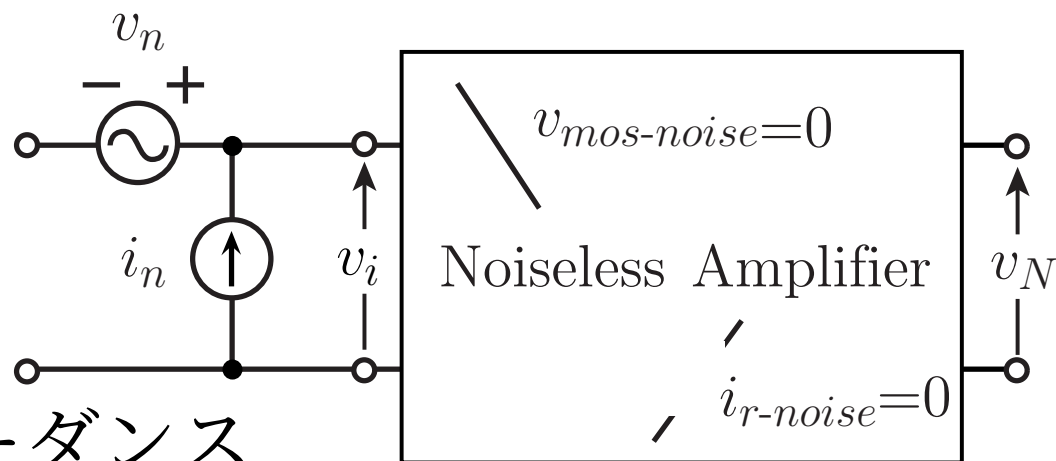
A : 電圧利得



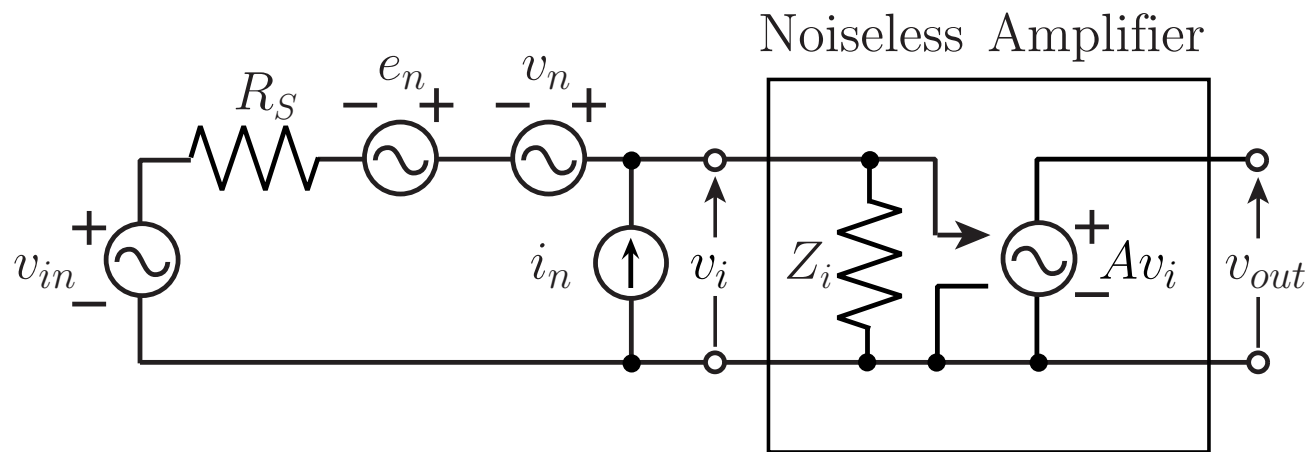
i_n の求め方

$$i_n = \frac{v_N}{Z_T}$$

Z_T : 伝達インピーダンス

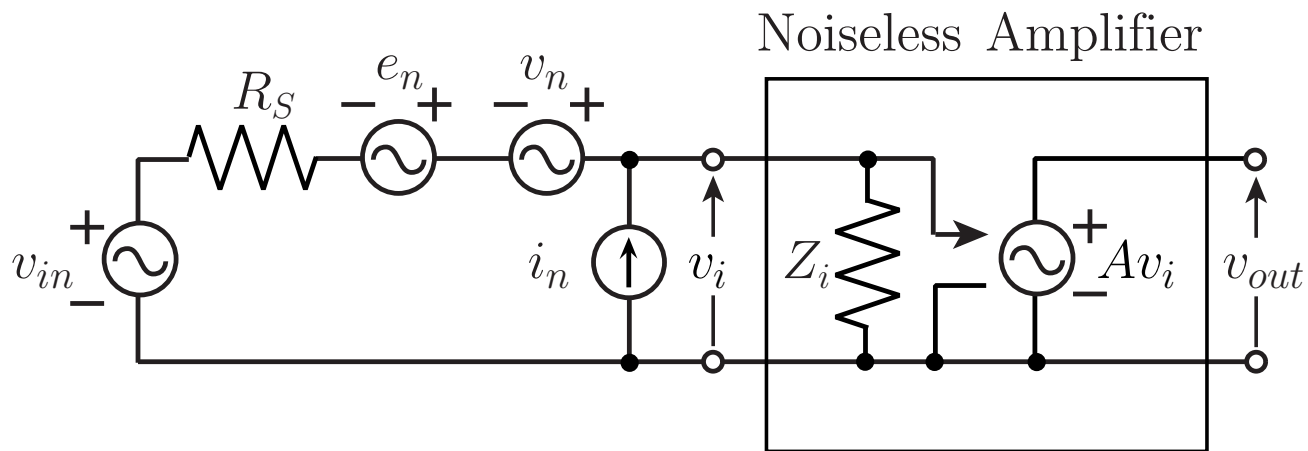


雜音整合



$$\overline{e_n^2} = 4kTR_S$$

$$v_S = \frac{AZ_i}{R_S + Z_i} v_{in}$$



e_n と v_n , e_n と i_n は無相関

v_n と i_n も無相関と仮定

$$\overline{v_N^2} = \frac{A^2 Z_i^2}{|R_S + Z_i|^2} (\overline{e_n^2} + \overline{v_n^2} + R_S^2 \overline{i_n^2})$$

$$\overline{e_n^2} = 4kTR_S \qquad v_S = \frac{AZ_{in}}{R_S + Z_{in}} v_{in}$$

$$\overline{v_N^2} = \frac{A^2 Z_{in}^2}{|R_S + Z_{in}|^2} (\overline{e_n^2} + \overline{v_n^2} + R_S^2 \overline{i_n^2})$$

$$SNR_{in} = \frac{\overline{v_{in}^2}}{\overline{e_n^2}}$$

$$SNR_{out} = \frac{\overline{v_S^2}}{\overline{v_N^2}} = \frac{\overline{v_{in}^2}}{\overline{e_n^2} + \overline{v_n^2} + R_S^2 \overline{i_n^2}}$$

$$F = \frac{SNR_{in}}{SNR_{out}} = \frac{\overline{e_n^2} + \overline{v_n^2} + R_S^2 \overline{i_n^2}}{\overline{e_n^2}} = 1 + \frac{\overline{v_n^2} + R_S^2 \overline{i_n^2}}{4kTR_S}$$

$$F=1+\frac{\overline{v_n^2}+R_S^2\overline{i_n^2}}{4kTR_S}$$

相加・相乗平均の定理

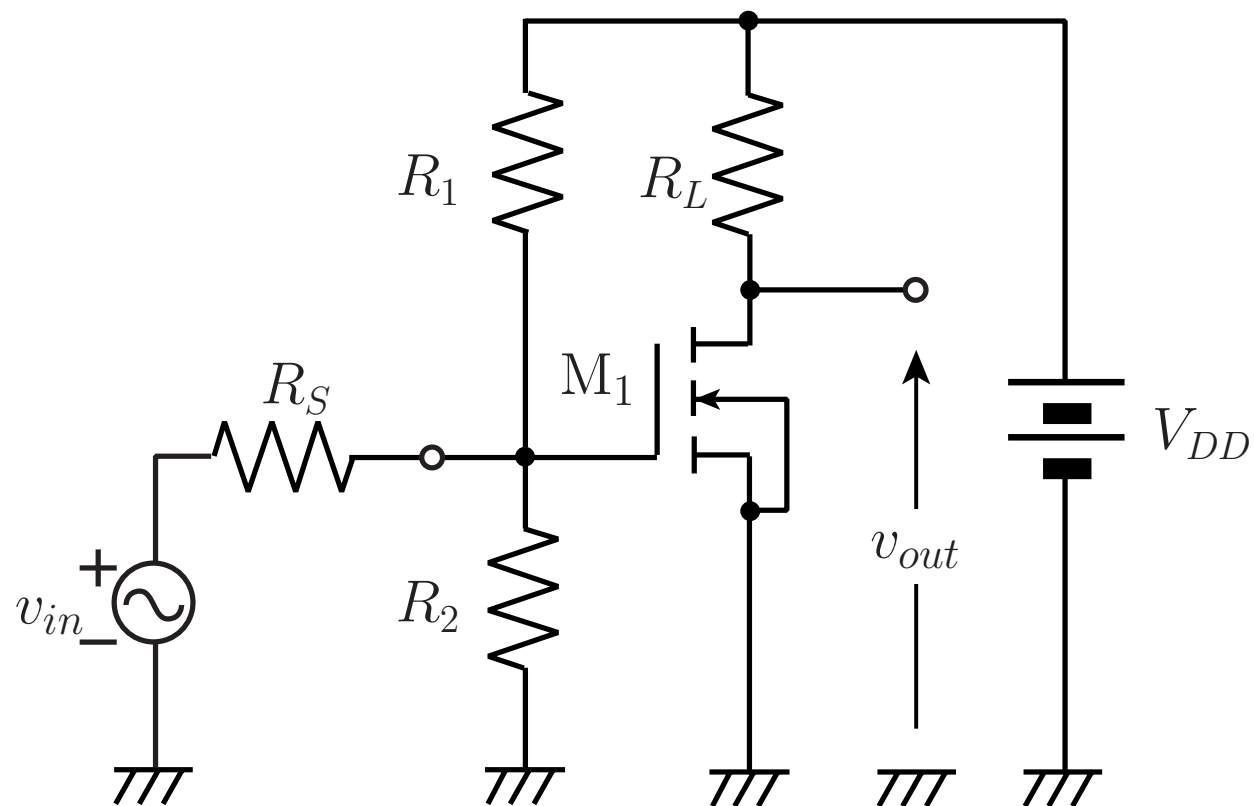
$$a+b\geq 2\sqrt{ab}$$

$$\frac{\overline{v_n^2}}{R_S}=R_S\overline{i_n^2} \text{ のとき } F \text{ が最小}$$

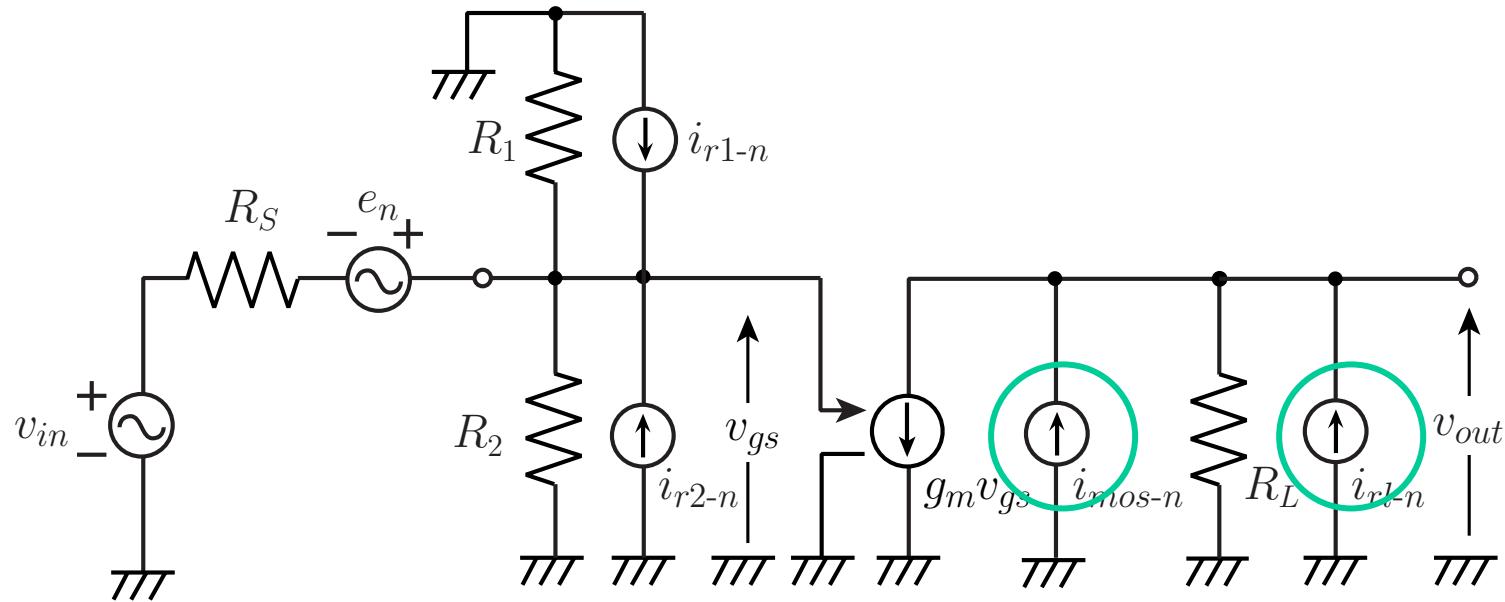
$$R_S=\sqrt{\frac{\overline{v_n^2}}{\overline{i_n^2}}}$$

実際の増幅回路の雑音解析例

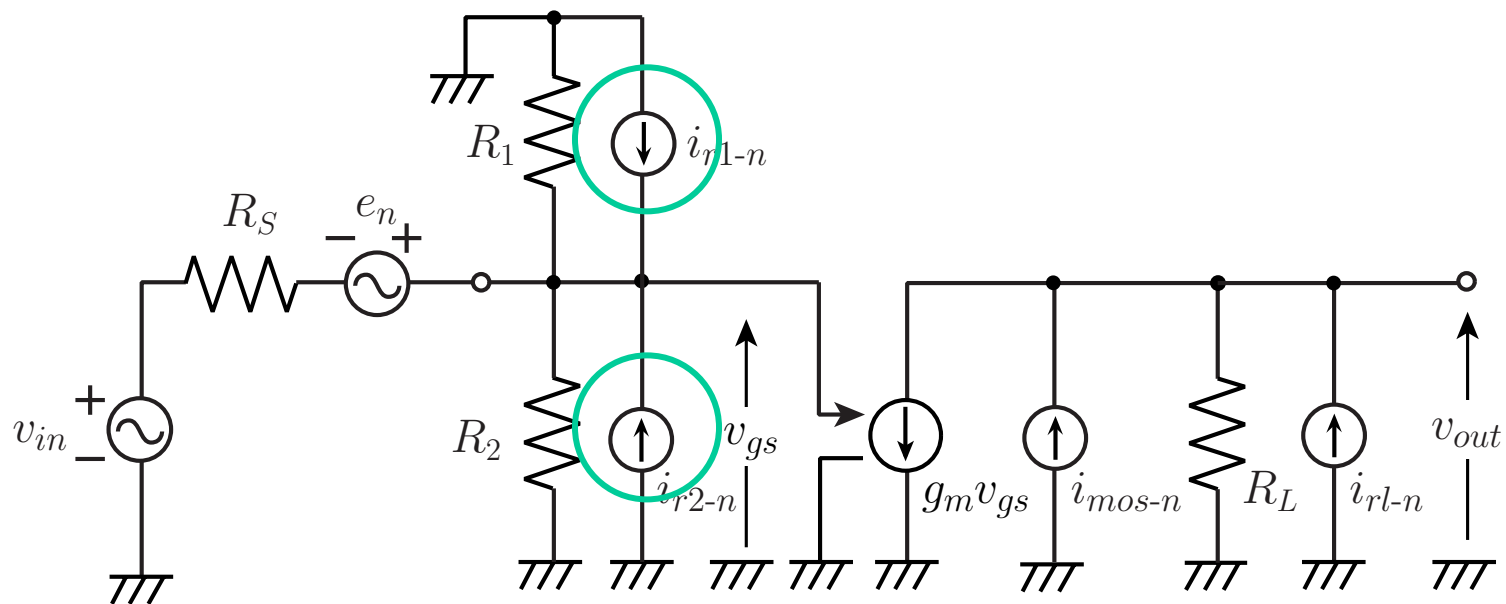
ソース接地増幅回路



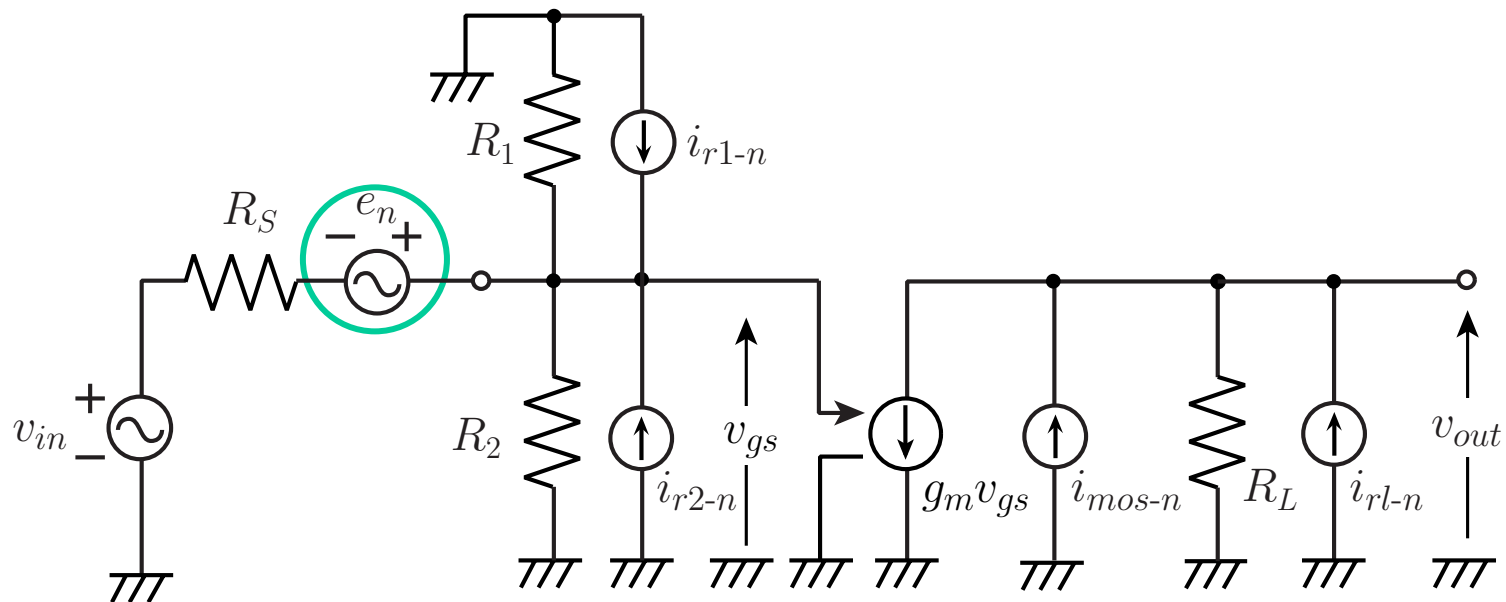
ソース接地増幅回路の小信号モデル



$$\overline{v_{o-n1}^2} = R_L^2 (\overline{i_{rl-n}^2} + \overline{i_{mos-n}^2})$$



$$\overline{v_{o-n}^2} = \left| g_m R_L \frac{R_1 R_2 R_S}{R_1 R_2 + R_2 R_S + R_S R_1} \right|^2 (\overline{i_{r1-n}^2} + \overline{i_{r2-n}^2})$$



$$\overline{v_{o-n}^2} = \left| g_m R_L \frac{R_1 R_2}{R_1 R_2 + R_2 R_S + R_S R_1} \right|^2 \overline{e_n^2}$$

$$\begin{aligned}
\overline{v_N^2} &= \overline{v_{o-n1}^2 + v_{o-n2}^2 + v_{o-n3}^2} \\
&= R_L^2 (\overline{i_{rl-n}^2 + i_{mos-n}^2}) \\
&\quad + \left| g_m R_L \frac{R_1 R_2 R_S}{R_1 R_2 + R_2 R_S + R_S R_1} \right|^2 (\overline{i_{r1-n}^2 + i_{r2-n}^2}) \\
&\quad + \left| g_m R_L \frac{R_1 R_2}{R_1 R_2 + R_2 R_S + R_S R_1} \right|^2 \overline{e_n^2} \\
\overline{v_S^2} &= \left| g_m R_L \frac{R_1 R_2}{R_1 R_2 + R_2 R_S + R_S R_1} \right|^2 \overline{v_{in}^2} \\
SNR_{in} &= \frac{\overline{v_{in}^2}}{\overline{e_n^2}}
\end{aligned}$$

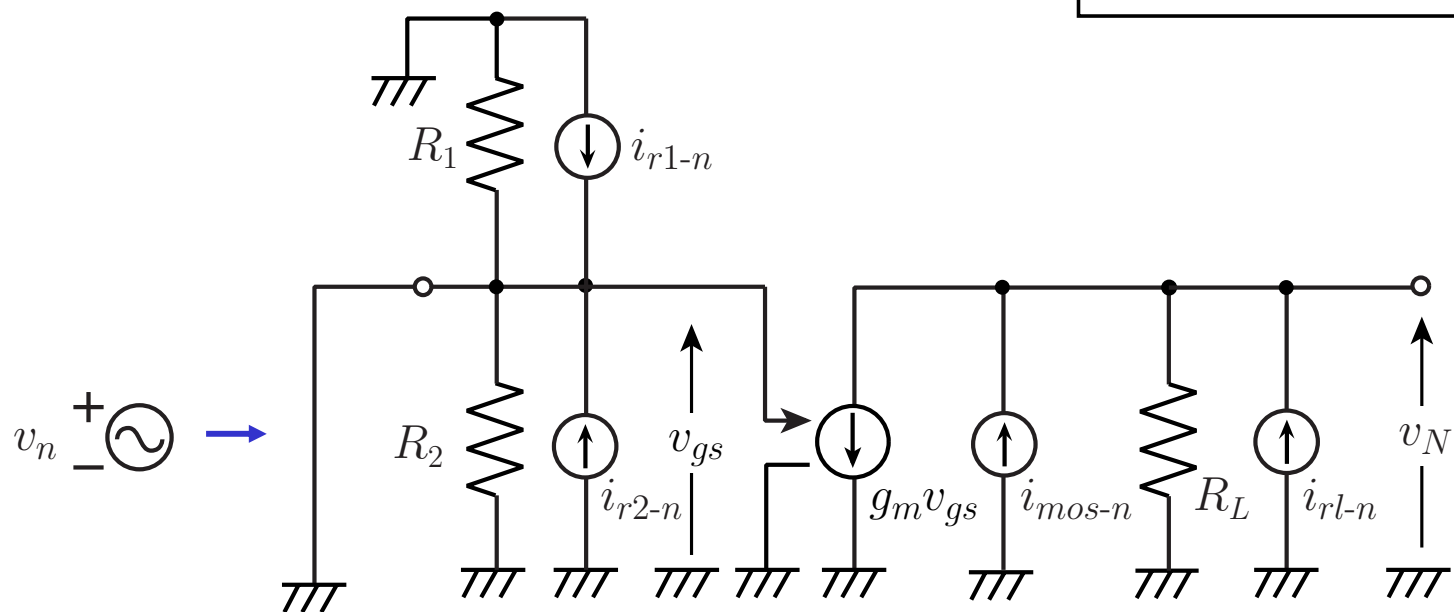
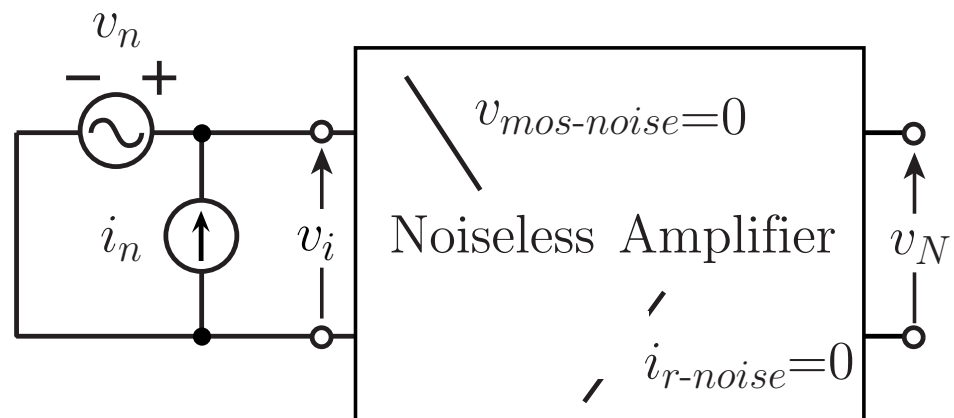
$$\overline{v_N^2} = \overline{v_{o-n1}^2} + \overline{v_{o-n2}^2} + \overline{v_{o-n3}^2}$$

$$\overline{v_S^2} = \left| g_m R_L \frac{R_1 R_2}{R_1 R_2 + R_2 R_S + R_S R_1} \right|^2 \overline{v_{in}^2}$$

$$SNR_{in} = \frac{\overline{v_{in}^2}}{\overline{e_n^2}} \quad \overline{e_n^2} = 4kTR_S$$

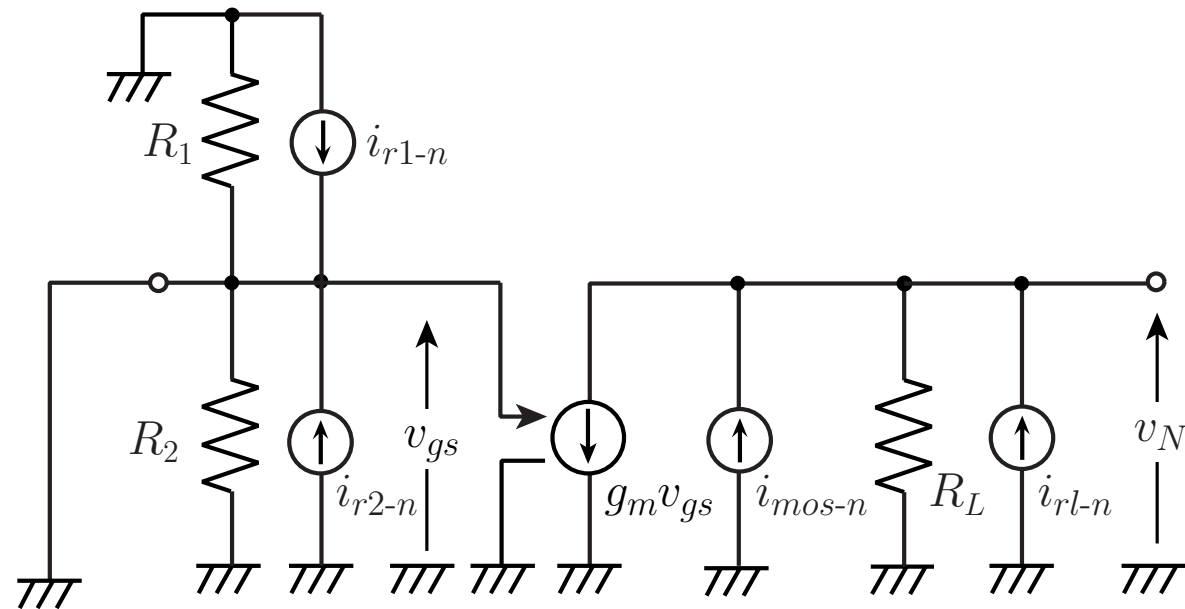
$$\begin{aligned} F &= \frac{SNR_{in}}{SNR_{out}} \\ &= 1 + \frac{1}{4kTR_S} \left\{ \left| \frac{R_1 R_2 + R_2 R_S + R_S R_1}{g_m R_1 R_2} \right|^2 (\overline{i_{rl-n}^2} + \overline{i_{mos-n}^2}) \right. \\ &\quad \left. + R_S^2 (\overline{i_{r1-n}^2} + \overline{i_{r2-n}^2}) \right\} \end{aligned}$$

v_n の求め方



$$A = \frac{v_{out}}{v_n} = -g_m R_L$$

$$\overline{v_n^2} = \frac{\overline{v_N^2}}{A^2}$$

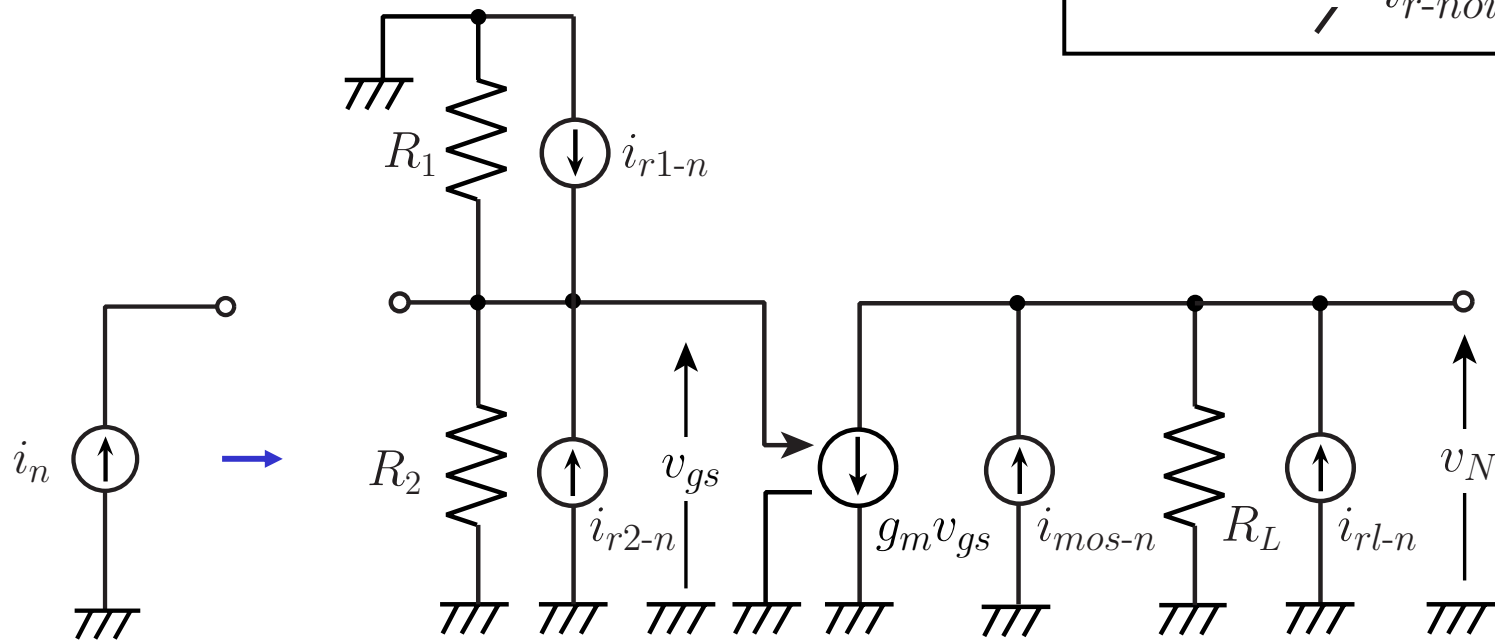
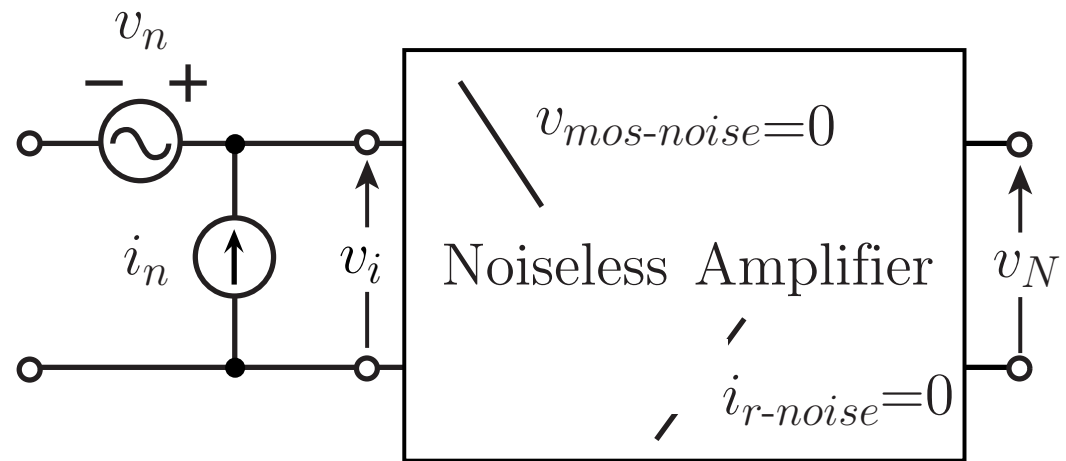


$$\overline{v_N^2} = R_L^2 (\overline{i_{rl-n}^2} + \overline{i_{mos-n}^2})$$

$$A = \frac{v_{out}}{v_n} = -g_m R_L$$

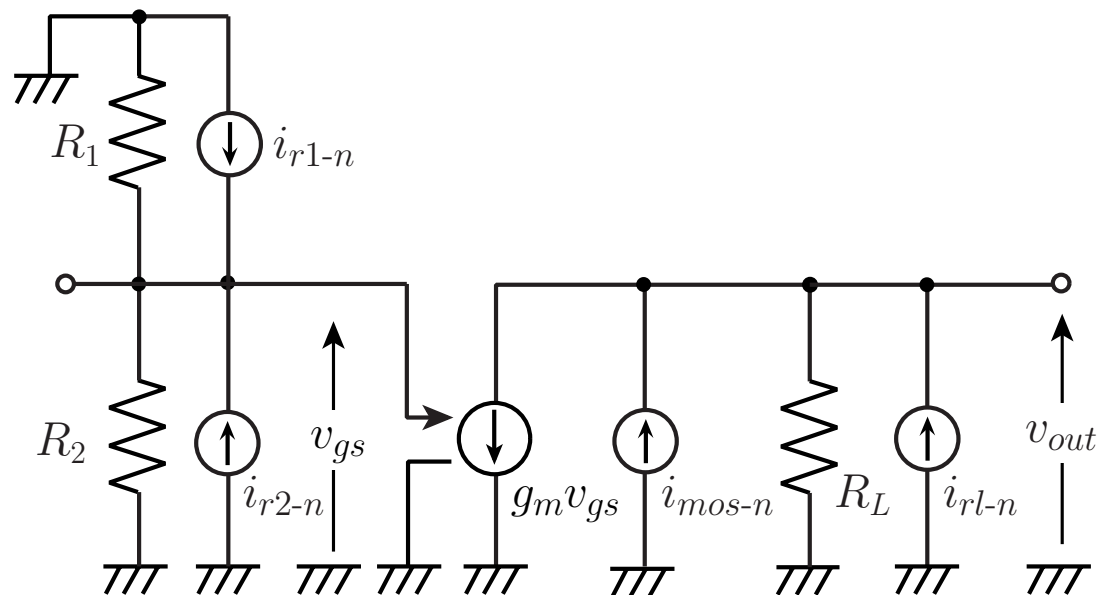
$$\overline{v_n^2} = \frac{1}{2} (\overline{i_{rl-n}^2} + \overline{i_{mos-n}^2})$$

i_n の求め方



$$Z_T = -g_m R_L \frac{R_1 R_2}{R_2 + R_1}$$

$$\overline{i_n^2} = \frac{\overline{v_N^2}}{Z_T^2}$$

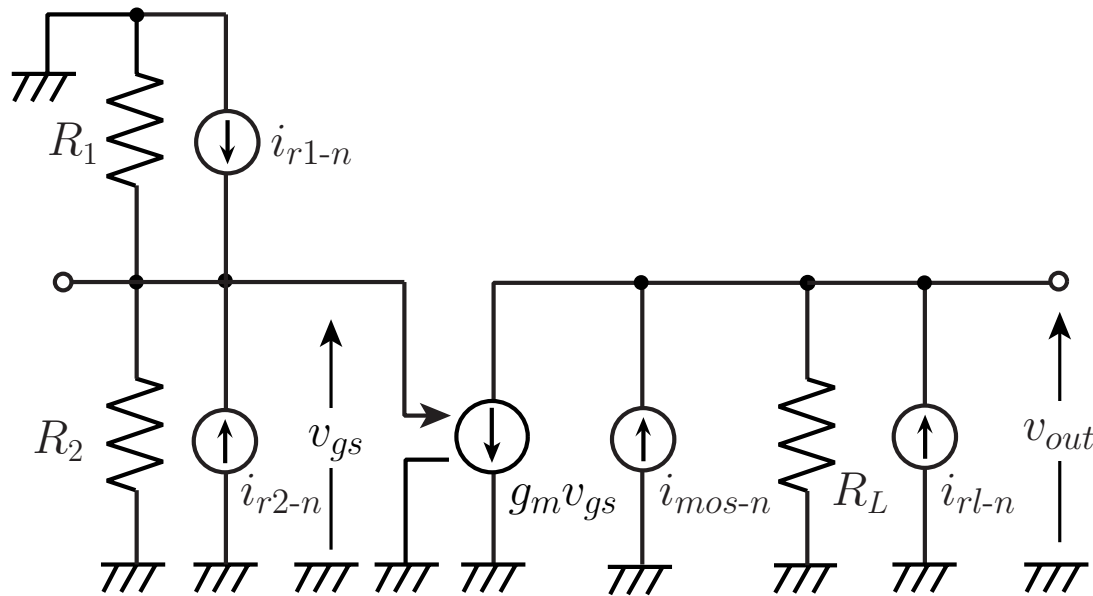


$$\overline{v_{o-n1}^2} = R_L^2 (\overline{i_{rl-n}^2} + \overline{i_{mos-n}^2})$$

$$\overline{v_{o-n2}^2} = \left| g_m R_L \frac{R_1 R_2}{R_2 + R_1} \right|^2 (\overline{i_{r1-n}^2} + \overline{i_{r2-n}^2})$$

$$\overline{v_{out}^2} = \overline{v_{o-n1}^2} + \overline{v_{o-n2}^2}$$

$$\overline{v_N^2} = R_L^2 (\overline{i_{rl-n}^2} + \overline{i_{mos-n}^2}) + \left| g_m R_L \frac{R_1 R_2}{R_2 + R_1} \right|^2 (\overline{i_{r1-n}^2} + \overline{i_{r2-n}^2})$$



$$i_n = \frac{v_N}{Z_T}$$

$$Z_T = -g_m R_L \frac{R_1 R_2}{R_2 + R_1}$$

$$\overline{i_n^2} = \left| \frac{R_1 + R_2}{g_m R_2 R_1} \right|^2 (\overline{i_{rl-n}^2} + \overline{i_{mos-n}^2}) + (\overline{i_{r1-n}^2} + \overline{i_{r2-n}^2})$$

$$\overline{v_n^2} = \frac{1}{2g_m} (\overline{i_{rl-n}^2} + \overline{i_{mos-n}^2})$$

$$\overline{i_n^2} = \left| \frac{R_1 + R_2}{g_m R_2 R_1} \right|^2 (\overline{i_{rl-n}^2} + \overline{i_{mos-n}^2}) + (\overline{i_{r1-n}^2} + \overline{i_{r2-n}^2})$$

$$F = 1 + \frac{\overline{v_n^2} + R_S^2 \overline{i_n^2}}{4kTR_S}$$

$$F = 1 + \frac{1}{4kTR_S} \left[\left\{ \frac{1}{g_m^2} + \left| \frac{(R_1 + R_2)R_S}{g_m R_1 R_2} \right|^2 \right\} (\overline{i_{rl-n}^2} + \overline{i_{mos-n}^2}) + R_S^2 (\overline{i_{r1-n}^2} + \overline{i_{r2-n}^2}) \right]$$

v_n と i_n の間の相関も考慮

$$F=1+\frac{1}{4kTR_S}\left\{\left|\frac{R_1R_2+R_2R_S+R_SR_1}{g_mR_1R_2}\right|^2\overline{(i_{rl-n}^2+i_{mos-n}^2)}+R_S^2\overline{(i_{r1-n}^2+i_{r2-n}^2)}\right\}$$

v_n と i_n は無相関と仮定

$$F=1+\frac{1}{4kTR_S}\left[\left\{\frac{1}{g_m^2}+\left|\frac{(R_1+R_2)R_S}{g_mR_1R_2}\right|^2\right\}\overline{(i_{rl-n}^2+i_{mos-n}^2)}+R_S^2\overline{(i_{r1-n}^2+i_{r2-n}^2)}\right]$$

低雑音増幅回路の設計(1)

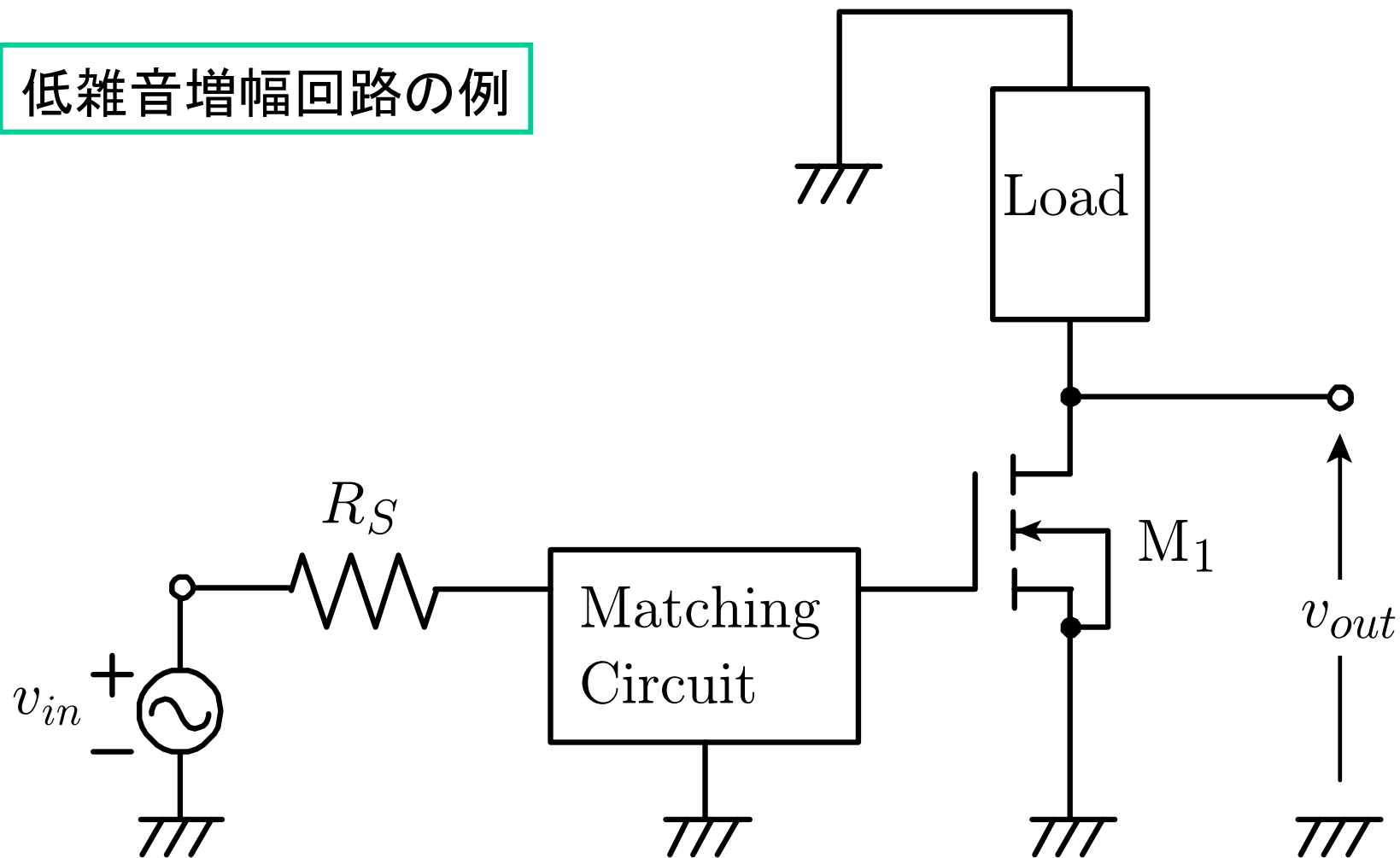
通信機器用低雑音増幅回路

高周波(数GHz)

狭帯域

LC共振特性の応用

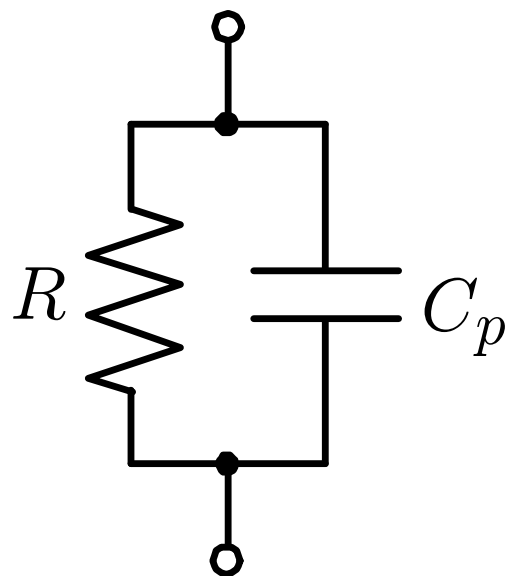
低雑音増幅回路の例



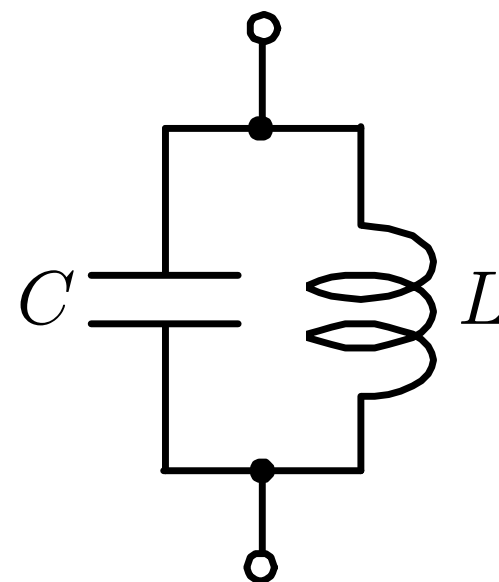
バイアス回路は省略

低雑音増幅回路における負荷の例

抵抗(寄生容量付き)

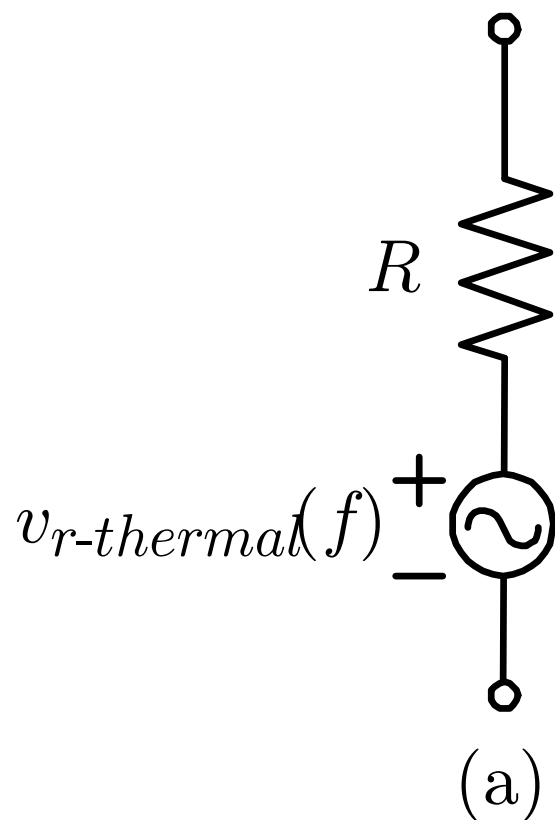


LC共振回路

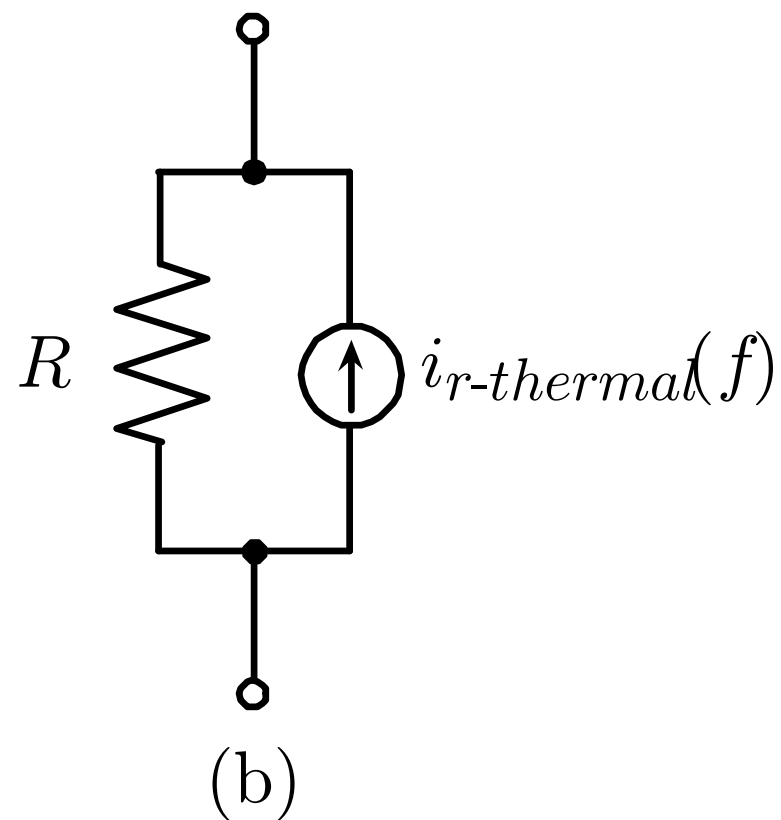


LC共振回路は無雑音?

抵抗の熱雑音

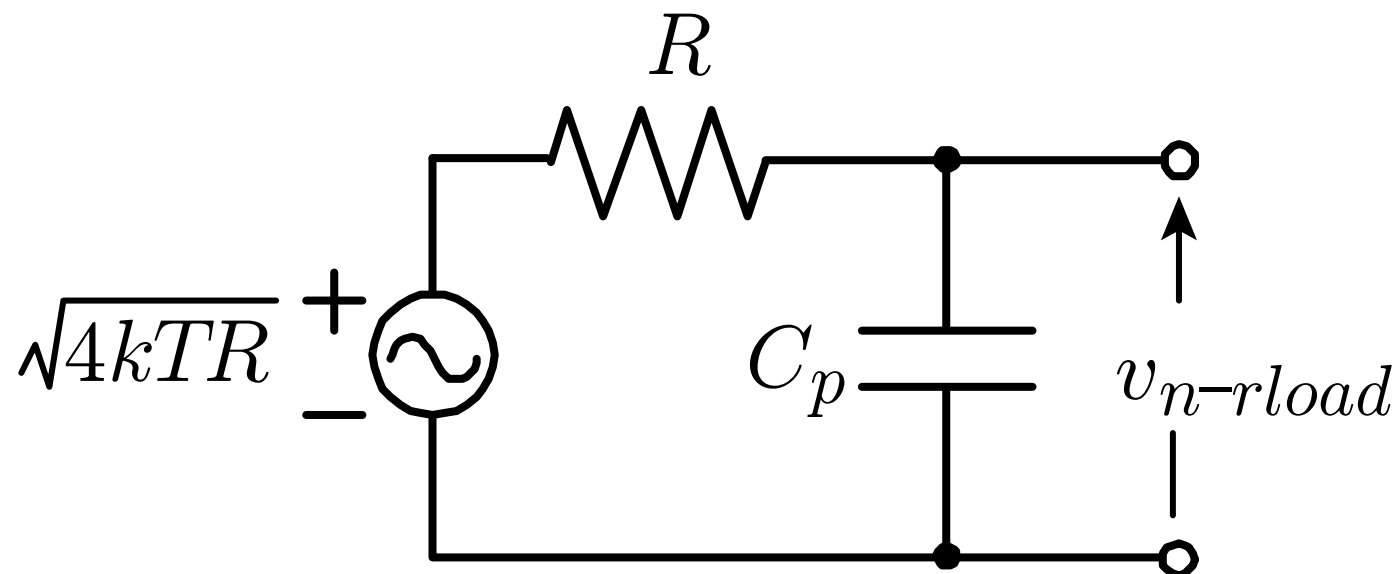


$$\overline{v_{r-thermal}^2(f)} = 4kTR$$



$$\overline{i_{r-thermal}^2(f)} = 4kT \frac{1}{R}$$

負荷抵抗が発生する雑音



$$T_{rload}(s) = \frac{1}{1+sC_pR} \Rightarrow |T_{rload}(j\omega)|^2 = \frac{1}{1+\omega^2 C_p^2 R^2}$$

$$T_{rload}(s) = \frac{1}{1+sC_p R} \Rightarrow |T_{rload}(j\omega)|^2 = \frac{1}{1+\omega^2 C_p^2 R^2}$$

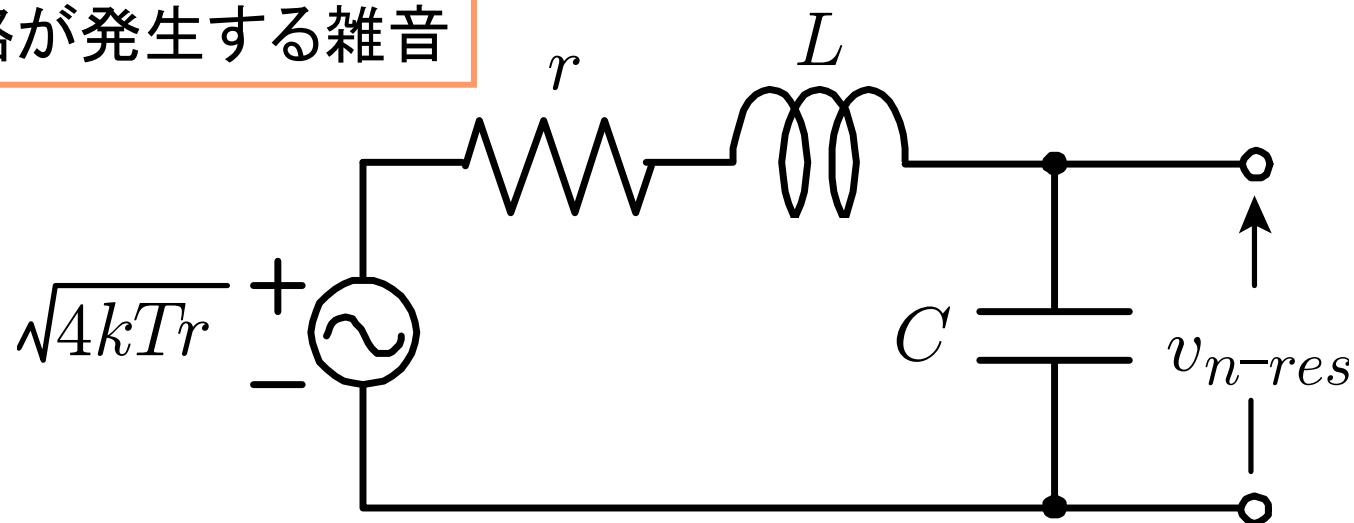
$$\overline{v_{n-rload}^2} = \int_0^\infty |T_{rload}(j\omega)|^2 \frac{4kTR}{2\pi} d\omega$$

$$= \int_0^\infty \frac{1}{1+\omega^2 C_p^2 R^2} \frac{4kTR}{2\pi} d\omega$$

$$= \frac{2kT}{C_p^2 R \pi} \int_0^\infty \frac{1}{\omega^2 + \frac{1}{C_p^2 R^2}} d\omega = \frac{2kT}{C_p^2 R \pi} \left[C_p R \tan^{-1}(C_p R \omega) \right]_0^\infty$$

$$= \frac{2kT}{C_p^2 R \pi} \cdot C_p R \left(\frac{\pi}{2} - 0 \right) = \frac{kT}{\underline{C_p}}$$

LC共振回路が発生する雑音



$$T_{res}(s) = \frac{\frac{1}{LC}}{s^2 + s\frac{r}{L} + \frac{1}{LC}}$$

$$\Rightarrow |T_{res}(j\omega)|^2 = \frac{\frac{1}{L^2 C^2}}{\left(\omega^2 - \frac{1}{LC}\right)^2 + \left(\frac{r}{L}\right)^2 \omega^2}$$

$$T_{res}(s) = \frac{\frac{1}{LC}}{s^2 + s\frac{r}{L} + \frac{1}{LC}}$$

$$\Rightarrow |T_{res}(j\omega)|^2 = \frac{\frac{1}{L^2 C^2}}{\left(\omega^2 - \frac{1}{LC}\right)^2 + \left(\frac{r}{L}\right)^2 \omega^2}$$

$$\overline{v_{n-res}^2} = \int_0^\infty |T_{res}(j\omega)|^2 \frac{4kTr}{2\pi} d\omega$$

$$\overline{v_{n-res}^2} = \int_0^\infty \frac{\frac{1}{L^2 C^2}}{\left(\omega^2 - \frac{1}{LC}\right)^2 + \left(\frac{r}{L}\right)^2 \omega^2} \cdot \frac{4kTr}{2\pi} d\omega$$

$$\int_0^\infty \frac{x^2}{\left(x^2 - a^2\right)^2 + b^2 x^2} dx = \frac{\pi}{2b}$$

ただし, $0 < \frac{b}{2} < a$ でなければならない.

$$\omega = \frac{1}{x} \text{ と置 } \Leftrightarrow d\omega = -\frac{1}{x^2} dx$$

$$\begin{aligned}
\overline{v_{n-res}^2} &= \frac{2kTr}{\pi L^2 C^2} \int_0^\infty \frac{1}{\left(\omega^2 - \frac{1}{LC}\right)^2 + \left(\frac{r}{L}\right)^2 \omega^2} d\omega \\
&= \frac{2kTr}{\pi L^2 C^2} \int_\infty^0 \frac{1}{\left(\frac{1}{x^2} - \frac{1}{LC}\right)^2 + \left(\frac{r}{L}\right)^2 \frac{1}{x^2}} \cdot \frac{-1}{x^2} dx \\
&= \frac{2kTr}{\pi L^2 C^2} \int_0^\infty \frac{1}{\left(\frac{1}{x^2} - \frac{1}{LC}\right)^2 x^2 + \left(\frac{r}{L}\right)^2} dx \\
&= \frac{2kTr}{\pi L^2 C^2} \int_0^\infty \frac{x^2}{\left(1 - \frac{1}{LC} x^2\right)^2 + \left(\frac{r}{L}\right)^2 x^2} dx
\end{aligned}$$

$$\overline{v_{n-res}^2} = \frac{2kTr}{\pi L^2 C^2} \int_0^\infty \frac{x^2}{\left(1 - \frac{1}{LC}x^2\right)^2 + \left(\frac{r}{L}\right)^2 x^2} dx$$

$$= \frac{2kTr}{\pi} \int_0^\infty \frac{x^2}{(x^2 - LC)^2 + (Cr)^2 x^2} dx$$

$$\int_0^\infty \frac{x^2}{(x^2 - a^2)^2 + b^2 x^2} dx = \frac{\pi}{2b}$$

ただし, $0 < \frac{b}{2} < a$ でなければならない.

$$\overline{v_{n-res}^2} = \frac{2kTr}{\pi} \int_0^\infty \frac{x^2}{(x^2 - LC)^2 + (Cr)^2 x^2} dx$$

$$a = \sqrt{LC} \quad b = Cr \text{ より}$$

$$\sqrt{\frac{L}{C}} > \frac{r}{2} \text{ が成り立てばよい}$$

$$\overline{v_{n-res}^2} = \frac{2kTr}{\pi} \cdot \frac{\pi}{2Cr} = \frac{kT}{C}$$

LC共振回路と抵抗負荷が発生する雑音の比較

- 雑音電力はいずれの場合も容量値のみに依存
- $C=C_p$ であれば雑音電力は等しい

信号増幅周波数の上限に関する考察

条件(1): $C=C_p$

(負荷の雑音電力が等しい)

条件(2): LC 共振回路の共振時の抵抗 $R_{res}=R$

(増幅利得が等しい)

$$Z_{res}(s) = \frac{sL+r}{s^2LC+sCr+1} = R_{res}$$

$$\Rightarrow j\omega_{res}L+r = j\omega_{res}CR_{res}r + R_{res}(1-\omega_{res}^2LC)$$

$$j\omega_{res}L + r = j\omega_{res}CR_{res}r + R_{res}(1 - \omega_{res}^2 LC)$$

$$\Rightarrow L = CR_{res}r, \quad r = R_{res}(1 - \omega_{res}^2 LC)$$

$$\omega_{res} = \sqrt{\frac{1}{LC} \left(1 - \frac{r}{R_{res}}\right)} = \sqrt{\frac{1}{C^2 R_{res} r} \left(1 - \frac{r}{R_{res}}\right)} = \frac{1}{CR_{res}} \sqrt{\frac{R_{res}}{r} - 1}$$

$$Q_L = \frac{\omega_{res}L}{r} = \left(\frac{1}{CR_{res}} \sqrt{\frac{R_{res}}{r} - 1} \right) CR_{res} = \sqrt{\frac{R_{res}}{r} - 1}$$

$$\omega_{res} = \frac{1}{CR_{res}} Q_L$$

$$\omega_{res} = \frac{1}{CR_{res}} Q_L$$

抵抗負荷の場合の遮断角周波数： $\omega_C = \frac{1}{C_p R}$

$C = C_p$ かつ $R_{res} = R$ ならばLC共振回路負荷の場合の
信号増幅周波数の上限は Q_L 倍

増幅利得に関する考察

$$LC共振回路負荷の場合：R_{res} = \frac{1}{C\omega_{res}} Q_L$$

$$抵抗負荷の場合：R = \frac{1}{C_p\omega_C}$$

$C=C_p$ かつ $\omega_{res}=\omega_C$ ならば

LC 共振回路負荷の場合の増幅利得は
抵抗負荷の場合の Q_L 倍

雑音に関する考察

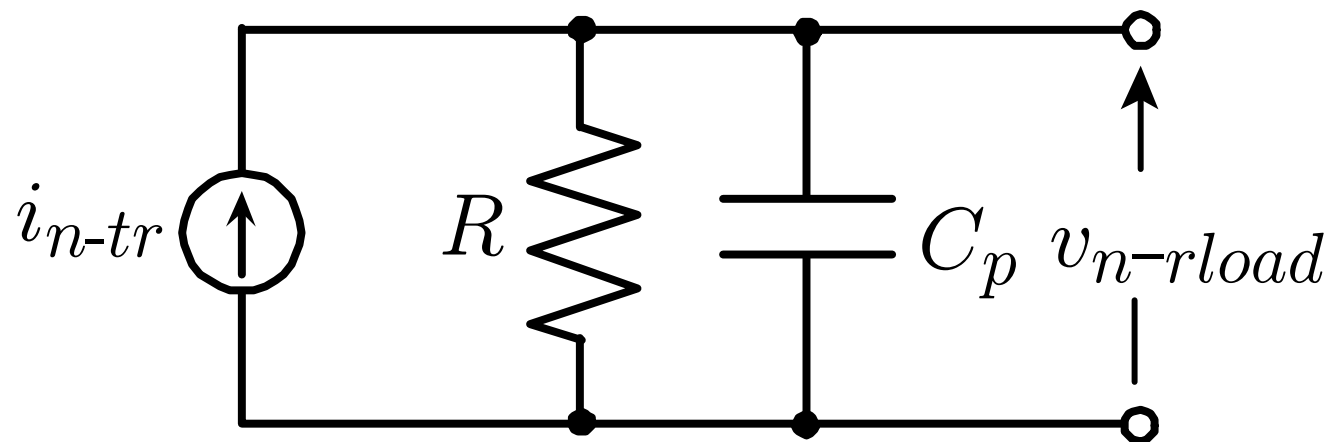
$$LC共振回路負荷の場合：C = \frac{1}{R_{res}\omega_{res}}Q_L$$

$$抵抗負荷の場合：C_p = \frac{1}{R\omega_C}$$

$\omega_{res} = \omega_C$ かつ $R_{res} = R$ ならば
LC共振回路負荷の場合の雑音は
抵抗負荷の場合の $\frac{1}{Q_L}$ 倍

トランジスタからの雑音の影響

抵抗負荷の場合



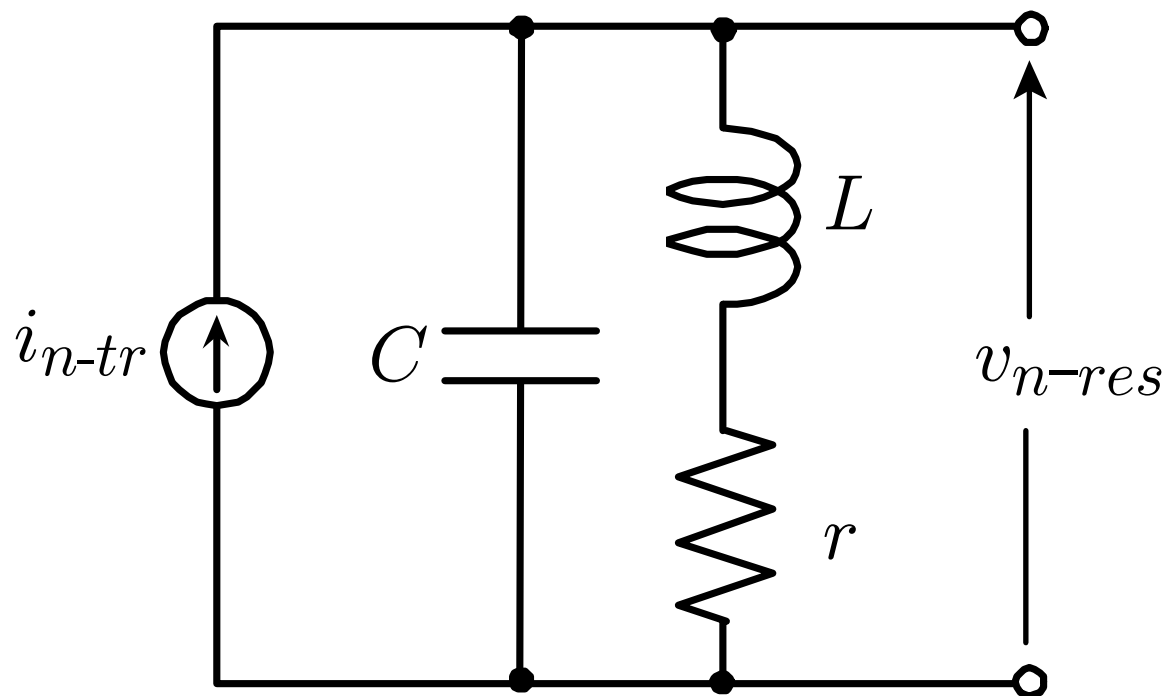
$$\overline{v_{n-rload}^2} = \int_0^\infty \frac{R^2}{1 + \omega^2 C_p^2 R^2} \overline{i_{n-tr}^2} d\omega$$

$$\overline{v_{n-rload}^2} = \int_0^\infty \frac{R^2}{1 + \omega^2 C_p^2 R^2} \overline{i_{n-tr}^2} d\omega$$

$$= \frac{\overline{i_{n-tr}^2}}{C_p^2} \int_0^\infty \frac{1}{\omega^2 + \frac{1}{C_p^2 R^2}} d\omega = \frac{\overline{i_{n-tr}^2}}{C_p^2} \left[C_p R \tan^{-1}(C_p R \omega) \right]_0^\infty$$

$$= \frac{\overline{i_{n-tr}^2}}{C_p^2} \cdot C_p R \frac{\pi}{2} = \frac{R\pi}{\underline{2C_p}} \overline{i_{n-tr}^2}$$

LC共振回路負荷の場合



$$Z_{res}(s) = \frac{sL + r}{s^2 LC + sCr + 1}$$

$$\overline{v_{n-res}^2} = \int_0^\infty \frac{\omega^2 L^2 + r^2}{(1 - \omega^2 LC)^2 + \omega^2 C^2 r^2} \overline{i_{n-tr}^2} d\omega$$

$$\begin{aligned}
\overline{v_{n-res}}^2 &= \int_0^\infty \frac{\omega^2 L^2 + r^2}{\left(1 - \omega^2 LC\right)^2 + \omega^2 C^2 r^2} \overline{i_{n-tr}}^2 d\omega \\
&= \int_0^\infty \frac{\omega^2 L^2 \overline{i_{n-tr}}^2}{\left(1 - \omega^2 LC\right)^2 + \omega^2 C^2 r^2} d\omega + \int_0^\infty \frac{r^2 \overline{i_{n-tr}}^2}{\left(1 - \omega^2 LC\right)^2 + \omega^2 C^2 r^2} d\omega \\
&= \frac{1}{C^2} \int_0^\infty \frac{\omega^2 \overline{i_{n-tr}}^2}{\left(\omega^2 - \frac{1}{LC}\right)^2 + \frac{r^2}{L^2} \omega^2} d\omega + \int_0^\infty \frac{r^2 \overline{i_{n-tr}}^2 x^2}{\left(x^2 - LC\right)^2 + C^2 r^2 x^2} dx \\
&= \left(\frac{1}{C^2} \frac{L\pi}{2r} + r^2 \frac{\pi}{2Cr} \right) \overline{i_{n-tr}}^2 = \left(\frac{L}{Cr} + r \right) \frac{\pi}{2C} \overline{i_{n-tr}}^2
\end{aligned}$$

$$\overline{v_{n-res}}^2 = \left(\frac{L}{Cr} + r \right) \frac{\pi}{2C} \overline{i_{n-tr}}^2$$

$$\overline{v_{n-rload}}^2 = R \frac{\pi}{2C_p} \overline{i_{n-tr}}^2$$

$$j\omega_{res}L + r = j\omega_{res}CR_{res}r + R_{res}(1 - \omega_{res}^2LC)$$

$$\Rightarrow L = CR_{res}r, \quad r = R_{res}(1 - \omega_{res}^2LC)$$

$$\overline{v_{n-res}}^2 = (R_{res} + r) \frac{\pi}{2C} \overline{i_{n-tr}}^2$$

負荷の雑音レベルと増幅利得が等しい場合

$$Q_L = \sqrt{\frac{R_{res}}{r} - 1} \text{より } r = \frac{R_{res}}{1 + Q_L^2}$$

$Q_L=3$ のとき, $R_{res}+r$ は R の1.1倍=0.41dB

$Q_L=5$ のとき, $R_{res}+r$ は R の1.04倍=0.16dB

$Q_L=10$ のとき, $R_{res}+r$ は R の1.01倍=0.043dB

最大利得周波数：LC共振回路負荷=抵抗負荷 $\times Q_L$

増幅利得と最大利得周波数が等しい場合

$$C = \frac{1}{R_{res} \omega_{res}} Q_L \qquad C_p = \frac{1}{R \omega_C}$$

$$\overline{v_{n-res}}^2 = (R_{res} + r) \frac{\pi}{2C} \overline{i_{n-tr}}^2 = \frac{R_{res} (R_{res} + r)}{2Q_L} \pi \overline{i_{n-tr}}^2$$

$$\overline{v_{n-rload}}^2 = R \frac{\pi}{2C_p} \overline{i_{n-tr}}^2 = \frac{R_{res}^2 \omega_{res}}{2} \pi \overline{i_{n-tr}}^2$$

最大利得周波数と負荷の雑音レベルが等しい場合

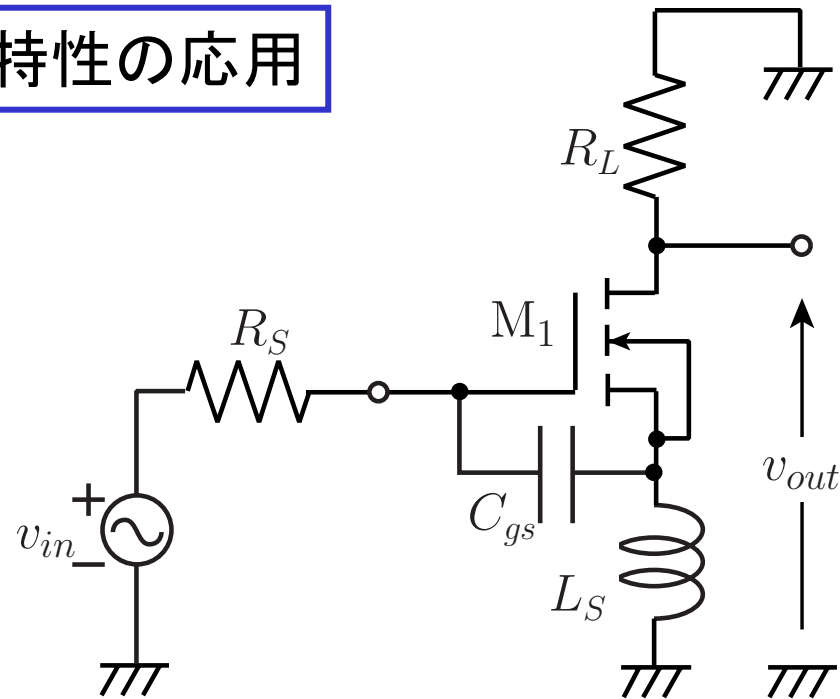
$$R_{res} = \frac{1}{\omega_{res} C} Q_L \quad R = \frac{1}{\omega_C C_p}$$

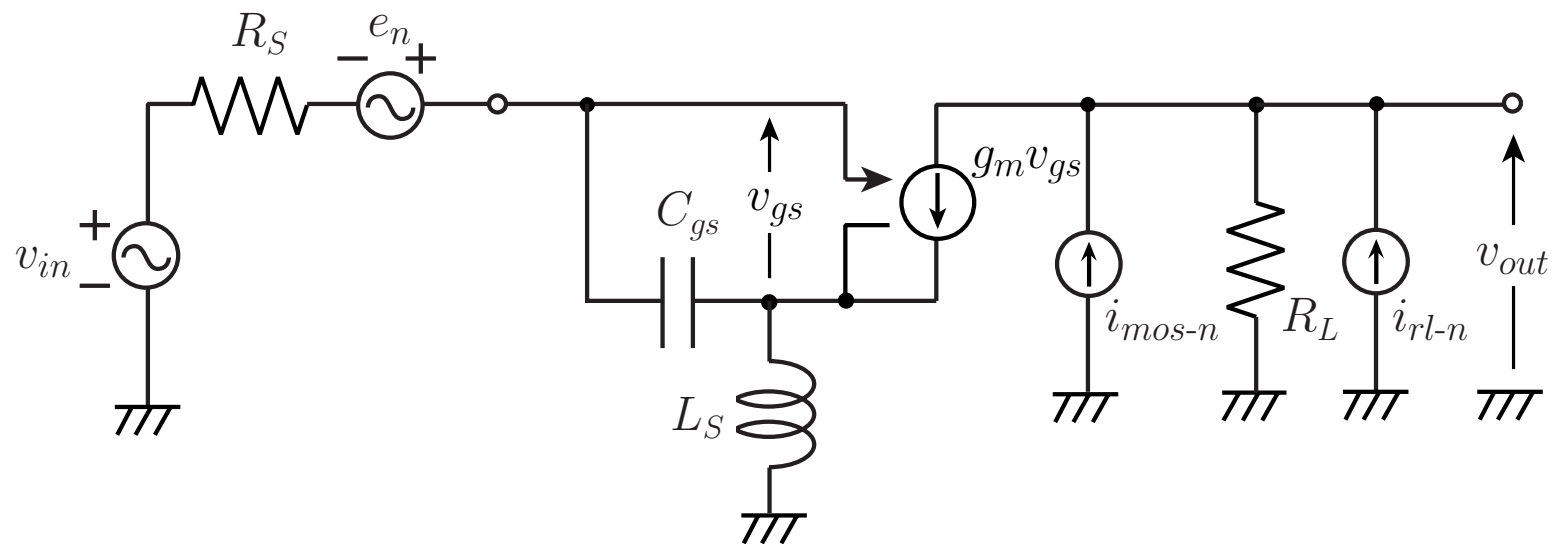
$$\overline{v_{n-res}}^2 = (R_{res} + r) \frac{\pi}{2C} \overline{i_{n-tr}}^2 = \left(\frac{Q_L}{\omega_{res} C} + r \right) \frac{\pi}{2C} \overline{i_{n-tr}}^2$$

$$\overline{v_{n-rload}}^2 = R \frac{\pi}{2C_p} \overline{i_{n-tr}}^2 = \frac{1}{\omega_{res} C} \frac{\pi}{2C} \overline{i_{n-tr}}^2$$

低雑音増幅回路の設計(2)

LC共振特性の応用





$$\overline{v_N^2} = R_L^2 (\overline{i_{rl-n}^2} + \overline{i_{mos-n}^2})$$

$$A = \frac{-g_m R_L}{1 + s L_S (g_m + s C_{gs})}$$

$$Z_T = \frac{g_m}{s C_{gs}} R_L$$

$$\overline{v_N^2} = R_L^2 (\overline{i_{rl-n}^2} + \overline{i_{mos-n}^2})$$

$$A = \frac{-g_m R_L}{1 + sL_S(g_m + sC_{gs})}$$

$$Z_T = \frac{g_m}{sC_{gs}} R_L$$

$$\overline{v_n^2} = \left| \frac{1 + sL_S(g_m + sC_{gs})}{g_m} \right|^2 (\overline{i_{rl-n}^2} + \overline{i_{mos-n}^2})$$

$$\overline{i_n^2} = \left| \frac{sC_{gs}}{g_m} \right|^2 (\overline{i_{rl-n}^2} + \overline{i_{mos-n}^2})$$

$$\overline{v_n^2} = \left| \frac{1 + sL_S(g_m + sC_{gs})}{g_m} \right|^2 (\overline{i_{rl-n}^2} + \overline{i_{mos-n}^2})$$

$$\overline{i_n^2} = \left| \frac{sC_{gs}}{g_m} \right|^2 (\overline{i_{rl-n}^2} + \overline{i_{mos-n}^2})$$

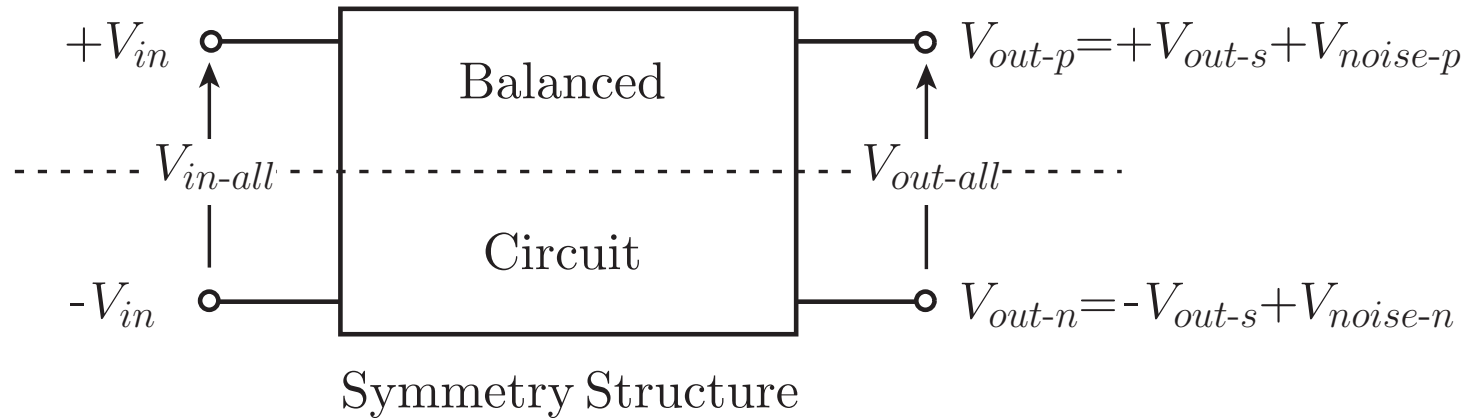
$$\sqrt{\frac{\overline{v_n^2}}{\overline{i_n^2}}} = \left| \frac{1 + sL_S(g_m + sC_{gs})}{sC_{gs}} \right|$$

$$\left| \frac{1 + sL_S(g_m + sC_{gs})}{sC_{gs}} \right| = R_S$$

↑

$$f = \frac{1}{2\pi} \sqrt{\frac{1}{L_S C_{gs}}} \text{ のとき } \frac{L_S g_m}{C_{gs}} \text{ の純抵抗}$$

平衡型構成の利用



$$V_{noise-d} = \frac{V_{noise-p} - V_{noise-n}}{2}$$

$$V_{noise-c} = \frac{V_{noise-p} + V_{noise-n}}{2}$$

$$V_{noise-p} = +V_{noise-d} + V_{noise-c}$$

$$V_{noise-n} = -V_{noise-d} + V_{noise-c}$$

$$V_{out-all} = V_{out-p} - V_{out-n} = 2V_{out-s} + 2V_{noise-d}$$

不平衡型回路のS/N比

$$V_{out} = V_{out-s} + V_{noise}$$

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T V_{out-s}^2 dt = \overline{V_{out-s}}^2 \quad \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T V_{noise}^2 dt = \overline{V_{noise}}^2$$

$$SNR_{imbal} = \frac{\overline{V_{out-s}}^2}{\overline{V_{noise}}^2}$$

平衡型回路のSNR

$$V_{out-all} = V_{out-p} - V_{out-n} = 2(V_{out-s} + V_{noise-d})$$

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T V_{out-s}^2 dt = \overline{V_{out-s}}^2$$

$$\overline{V_{noise-d}}^2 = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T V_{noise-d}^2 dt = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T V_{noise-c}^2 dt = \frac{1}{2} \overline{V_{noise}}^2$$

$$\text{S/N比の改善 : } SNR_{bal} = \frac{\overline{V_{out-s}}^2}{\overline{V_{noise-d}}^2} = 2 \frac{\overline{V_{out-s}}^2}{\overline{V_{noise}}^2}$$

ただし、回路規模は2倍