化学生態システム解析

東京工業大学大学院　化学環境学専攻

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1. 非等温反応装置，熱交換器
2. 無次元数，対流伝熱（強制対流，自然対流）
3. 放射伝熱
4. 沸騰伝熱，凝縮伝熱，フィン

非等温反応装置の設計

反応装置のエネルギー収支

（流入流体によるエネルギーの流入速度）-（流出流体によるエネルギーの流出速度）
+（周囲より系へのエネルギーの流入速度）-（系によって周囲にされる仕事率）
=（系内のエネルギーの蓄積速度）

反応装置においては，一般的なエネルギー収支式を*エンタルピー*を用いる式に変換できる。

$$\Sigma E_{j0}F_{j0} - \Sigma E_j F_j + Q - w = \frac{\mathrm{d}E_{sys}}{\mathrm{d}t}$$

流れ系の場合： $w = -\Sigma PF_{j0}V_{j0} + \Sigma PF_j V_j + w_s$

$$E_j = U_j + M_j u_j^{\,2}/2 + M_j gz \qquad H_j = U_j + PV_j$$

$$\therefore \frac{\mathrm{d}E_{sys}}{\mathrm{d}t} = \Sigma\left(H_{j0} + \frac{M_j u_{j0}^{\,2}}{2} + M_j gZ_0\right)F_{j0} + \Sigma\left(H_j + \frac{M_j u_j^{\,2}}{2} + M_j gZ\right)F_j + Q - w_s$$

非等温反応装置の設計

反応装置のエネルギー収支

$$\therefore \frac{\mathrm{d}E_{sys}}{\mathrm{d}t} = \Sigma\left(H_{j0} + \frac{M_j u_{j0}^{\ 2}}{2} + M_j g Z_0\right)F_{j0} + \Sigma\left(H_j + \frac{M_j u_j^{\ 2}}{2} + M_j g Z\right)F_j + Q - w_s$$

E_{sys}は各成分のエネルギーの総和　$E_{sys} = \Sigma E_j n_j$

運動エネルギーと位置エネルギーはエンタルピーと比べると無視できる。　$E_j \approx U_j$

$$E_{sys} \approx \Sigma n_j U_j = \Sigma n_j \left(H_j - PV_j\right) = \Sigma n_j H_j - P\Delta V$$

定容系：$-P\Delta V$は消去できる。

非定容系：$\Sigma n_j H_j >> P\Delta V_j$

$$\frac{\mathrm{d}E_{sys}}{\mathrm{d}t} \approx \frac{\mathrm{d}\left(\Sigma n_j H_j\right)}{\mathrm{d}t}$$

$$\frac{\mathrm{d}\left(\Sigma n_j H_j\right)}{\mathrm{d}t} = \Sigma F_{j0} H_{j0} - \Sigma F_j H_j + Q$$

一般的なエネルギー収支式ではなく，化学反応装置では*エンタルピー収支式*を用いる。

非等温反応装置の設計

非等温液相回分反応器

$$\frac{\mathrm{d}\left(\Sigma n_j H_j\right)}{\mathrm{d}t} = \Sigma F_{j0} H_{j0} - \Sigma F_j H_j + Q$$

$$\Sigma F_{j0} H_{j0} = 0$$

$$\Sigma F_j H_j = 0$$

$$q = UA\left(T_s - T\right)$$

$$\therefore \frac{\mathrm{d}\left(\Sigma n_j H_j\right)}{\mathrm{d}t} = UA\left(T_S - T\right)$$

$$\frac{\mathrm{d}\left(\Sigma n_j H_j\right)}{\mathrm{d}t} = \Sigma \frac{\mathrm{d}\left(n_j H_j\right)}{\mathrm{d}t} = \Sigma n_j \frac{\mathrm{d}H_j}{\mathrm{d}t} + \Sigma \frac{\mathrm{d}n_j}{\mathrm{d}t} H_j$$

定圧下で相変化がない場合

$$\frac{\mathrm{d}H_j}{\mathrm{d}t} = \left(\frac{\partial H_j}{\partial T}\right)_P \frac{\mathrm{d}T}{\mathrm{d}t} = C_{p\,j} \frac{\mathrm{d}T}{\mathrm{d}t}$$

$$\Sigma n_j \frac{\mathrm{d}H_j}{\mathrm{d}t} = \left(\Sigma n_j C_{p\,j}\right) \frac{\mathrm{d}T}{\mathrm{d}t} = V\rho \overline{C_{p\,m}} \frac{\mathrm{d}T}{\mathrm{d}t}$$

回分反応器の物質収支式

$$\frac{\mathrm{d}n_j}{\mathrm{d}t} = r_j V$$

$$\Sigma \frac{\mathrm{d}n_j}{\mathrm{d}t} H_j = r_A V\left(-\Delta H_R\right)$$

$$V\rho \overline{C_{p\,m}} \frac{\mathrm{d}T}{\mathrm{d}t} = \left(-r_A\right) V\left(-\Delta H_R\right) + UA\left(T_S - T\right)$$

（顕熱の蓄積速度）＝（反応に伴う発熱速度）＋（周囲からの加熱速度）

非等温反応装置の設計

非等温管型反応器

$$\frac{\mathrm{d}\left(\Sigma n_j H_j\right)}{\mathrm{d}t} = \Sigma F_{j0} H_{j0} - \Sigma F_j H_j + Q$$

$$0 = \Sigma F_j H_j - \left(\Sigma F_j H_j + \frac{\mathrm{d}\left(\Sigma F_j H_j\right)}{\mathrm{d}z}\mathrm{d}z\right) + UA_h \mathrm{d}z\left(T_s - T\right)$$

$$\frac{\mathrm{d}\left(\Sigma F_j H_j\right)}{\mathrm{d}z} = UA_h\left(T_s - T\right)$$

$$\Sigma F_j \frac{\mathrm{d}H_j}{\mathrm{d}z} + \Sigma \frac{\mathrm{d}F_j}{\mathrm{d}z} H_j = UA_h\left(T_s - T\right)$$

$$\Sigma F_j \frac{\mathrm{d}H_j}{\mathrm{d}z} = \Sigma F_j \left(\frac{\partial H_j}{\partial T}\right)_P \frac{\mathrm{d}T}{\mathrm{d}z} = \left(\Sigma F_j C_{pj}\right)\frac{\mathrm{d}T}{\mathrm{d}z}$$

$$\Sigma F_j Cp_j = Su\rho \overline{Cp_m}$$

単位時間に通過する反応混合物の熱容量

$$\Sigma F_j \frac{\mathrm{d}H_j}{\mathrm{d}z} = Su\rho \overline{C_{pm}} \frac{\mathrm{d}T}{\mathrm{d}z}$$

管型反応器の物質収支式

$$\frac{\mathrm{d}F_j}{\mathrm{d}z} = Sr_j$$

$$\Sigma \frac{\mathrm{d}F_j}{\mathrm{d}z} H_j = S\Sigma r_j H_j = S\left(-r_A\right)\Delta H_R$$

$$Su\rho \overline{C_{pm}} \frac{\mathrm{d}T}{\mathrm{d}z} + S\left(-r_A\right)\Delta H_R = UA_h\left(T_S - T\right)$$

非等温反応装置の設計

非等温連続槽型反応器

$$\frac{\mathrm{d}\left(\Sigma n_j H_j\right)}{\mathrm{d}t} = \Sigma F_{j0} H_{j0} - \Sigma F_j H_j + Q$$

$$\Sigma F_{j0} H_{j0} - \Sigma F_j H_j + UA\left(T_s - T\right) = 0$$

連続槽型反応器の物質収支

$$F_{j0} - F_j + r_j V = 0$$

$$\Sigma F_{j0} H_{j0} - \Sigma\left(F_{j0} + r_j V\right) H_j + UA\left(T_s - T\right)$$

$$= -\Sigma F_{j0}\left(H_j - H_{j0}\right) - V\Sigma r_j H_j + UA\left(T_s - T\right) = 0$$

相変化がない液相反応

$$\Sigma F_{j0}\left(H_j - H_{j0}\right) = \Sigma F_{j0}\overline{C_{p\,j}}\left(T - T_0\right)$$

$$= v\rho\overline{C_{p\,j}}\left(T - T_0\right)$$

$$V\Sigma r_j H_j = V\left(-r_A\right)\Delta H_R$$

$$V\left(-r_A\right)\left(-\Delta H_R\right) = v\rho\overline{C_{p\,m}}\left(T - T_0\right) + UA\left(T - T_s\right)$$

Heat Transfer

伝導 *Conduction*

分子運動によるエネルギー伝達

Fourier's Law

$$q = k\frac{\partial T}{\partial x}$$

伝熱

対流 *Convection*

流体中のマクロな流れによるエネルギー伝達

Newton's Law of Cooling

$$q = h\Delta T$$

放射 *Radiation*

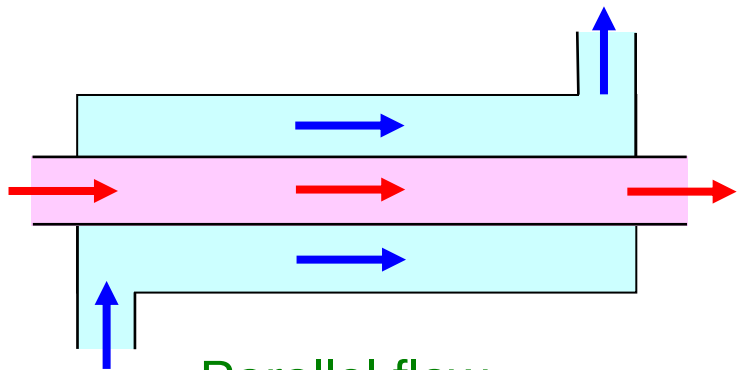
電磁波によるエネルギー伝達

Stefan-Boltzmann Law

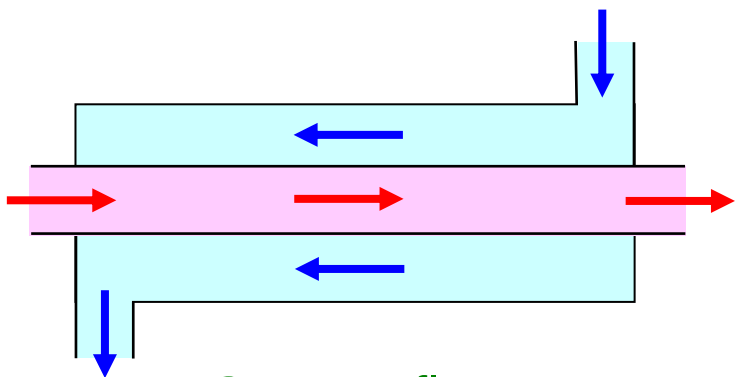
$$q = \sigma T^4$$

Heat Exchanger

- Equipment to transfer thermal energy from high temperature fluid to low temperature fluid.
- Shell and tube type is typical.



Parallel flow



Counter flow

shell-and-tube heat exchanger with one shell pass and one tube pass

Heat Exchanger

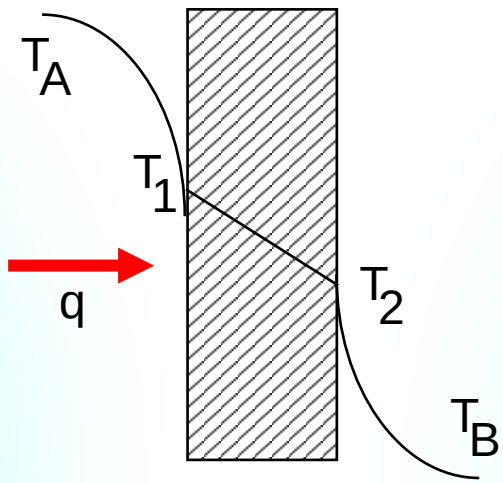
Overall Heat Transfer Coefficient

Plate

$$q = h_1 (T_A - T_1)$$

$$q = h_2 (T_2 - T_B)$$

$$q = k \frac{T_1 - T_2}{\Delta x}$$



Pipe

$$Q = 2 \pi r_i L h_i (T_A - T_i)$$

$$Q = \frac{2 \pi L k}{\ln (r_o/r_i)} (T_i - T_o)$$

$$Q = 2 \pi r_o L h_o (T_o - T_B)$$

$$q \left(\frac{1}{h_1} + \frac{\Delta x}{k} + \frac{1}{h_2} \right) = T_A - T_B$$

$$q = \frac{T_A - T_B}{\left(\frac{1}{h_1} + \frac{\Delta x}{k} + \frac{1}{h_2} \right)} = U (T_A - T_B)$$

$$\frac{1}{U} = \frac{1}{h_1} + \frac{\Delta x}{k} + \frac{1}{h_2}$$

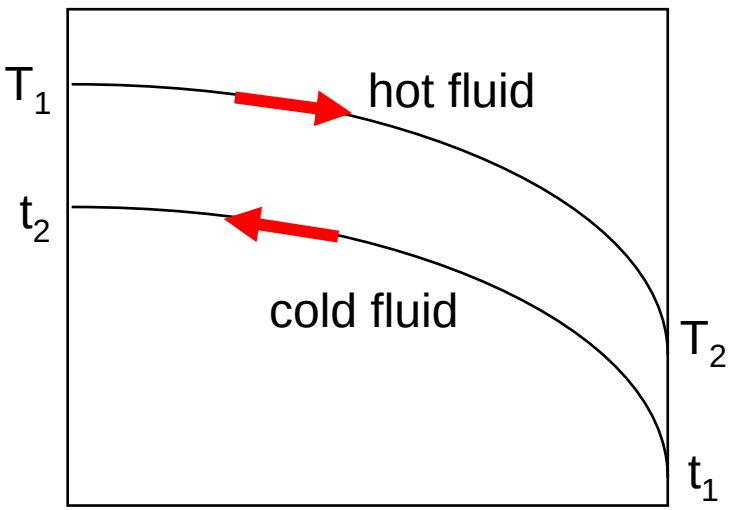
$$q = \frac{Q}{A_i} = \frac{T_A - T_B}{\left(\frac{1}{h_i} + \frac{A_i \ln (r_o/r_i)}{2 \pi L k} + \frac{A_i}{A_o} \frac{1}{h_o} \right)} = U_i (T_A - T_B)$$

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{A_i \ln (r_o/r_i)}{2 \pi L k} + \frac{A_i}{A_o} \frac{1}{h_o}$$

$$\frac{1}{U_o} = \frac{A_o}{A_i} \frac{1}{h_i} + \frac{A_o \ln (r_o/r_i)}{2 \pi L k} + \frac{1}{h_o}$$

Heat Exchanger

Log Mean Temperature Difference (Counter Flow)



$$dQ = U\, dA\ \Delta t, \quad \Delta t = T - t$$

$$dQ = \dot{M}\, C\, dT = \dot{m}\, c\, dt$$

$$d(\Delta t) = d(T - t) = dQ \left(\frac{1}{\dot{M}C} - \frac{1}{\dot{m}c}\right)$$

$$\frac{d(\Delta t)}{\Delta t} = U\, dA \left(\frac{1}{\dot{M}C} - \frac{1}{\dot{m}c}\right)$$

$$\ln \frac{\Delta t_1}{\Delta t_2} = U\, A \left(\frac{1}{\dot{M}C} - \frac{1}{\dot{m}c}\right), \quad \Delta t_1 = T_1 - t_2, \quad \Delta t_2 = T_2 - t_1$$

$$Q = \dot{M}\, C\, (T_1 - T_2) = \dot{m}\, c\, (t_2 - t_1)$$

$$\ln \frac{\Delta t_1}{\Delta t_2} = U\, A \left(\frac{T_1 - T_2}{Q} - \frac{t_2 - t_1}{Q}\right) = \frac{U\, A}{Q} (\Delta t_1 - \Delta t_2)$$

$$Q = U\, A\, \frac{(\Delta t_1 - \Delta t_2)}{\ln (\Delta t_1/\Delta t_2)} = U\, A\ \Delta t_{lm}$$

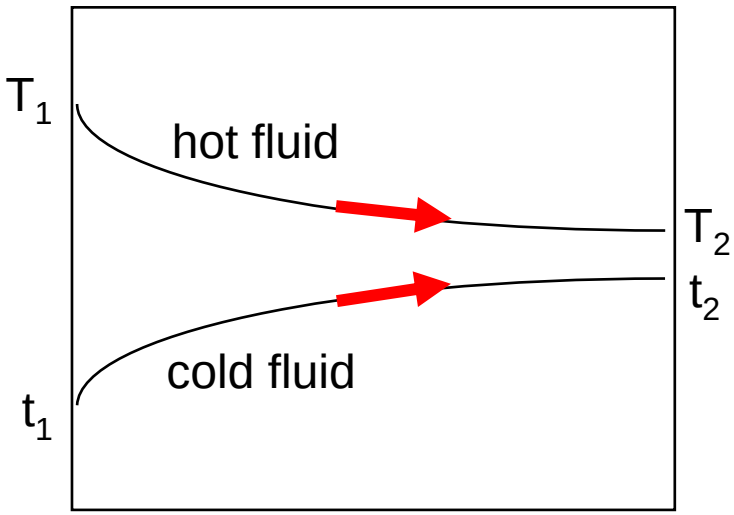
$$\Delta t_{lm} = \frac{(\Delta t_1 - \Delta t_2)}{\ln (\Delta t_1/\Delta t_2)}$$

if $(T_1 - T_2)$, $(t_1 - t_2) << (T_2 - t_1)$

$$Q = U\, A \left(\frac{T_1 + T_2}{2} - \frac{t_1 + t_2}{2}\right)$$

Heat Exchanger

Log Mean Temperature Difference (Parallel Flow)



$$dQ = U\, dA\ \Delta t, \quad \Delta t = T - t$$

$$dQ = \dot{M}\, C\, dT = -\dot{m}\, c\, dt$$

$$d(\Delta t) = d(T - t) = dQ \left(\frac{1}{\dot{M}C} + \frac{1}{\dot{m}c}\right)$$

$$\frac{d(\Delta t)}{\Delta t} = U\, dA \left(\frac{1}{\dot{M}C} + \frac{1}{\dot{m}c}\right)$$

$$\ln \frac{\Delta t_1}{\Delta t_2} = U\, A \left(\frac{1}{\dot{M}C} + \frac{1}{\dot{m}c}\right), \quad \Delta t_1 = T_1 - t_1, \quad \Delta t_2 = T_2 - t_2$$

$$Q = \dot{M}\, C\, (T_1 - T_2) = \dot{m}\, c\, (t_2 - t_1)$$

$$\ln \frac{\Delta t_1}{\Delta t_2} = U\, A \left(\frac{T_1 - T_2}{Q} + \frac{t_2 - t_1}{Q}\right) = \frac{U\, A}{Q} (\Delta t_1 - \Delta t_2)$$

$$Q = U\, A\, \frac{(\Delta t_1 - \Delta t_2)}{\ln (\Delta t_1/\Delta t_2)} = U\, A\, \Delta t_{lm}$$

$$\Delta t_{lm} = \frac{(\Delta t_1 - \Delta t_2)}{\ln (\Delta t_1/\Delta t_2)}$$

same as the counter flow

Non-Dimension Number

Reynolds number, $Re = \rho\, u\, L / \mu$　**慣性力／粘性力**

（粘性によって移動するFlux）に対する（流れによって運ばれる運動量Flux）の比である。流れにより運ばれる運動量Fluxは ρu^2，粘性によって移動する運動量Fluxは τ_w であり，移動面に沿ったせん断層中の速度勾配を u/L とすれば，レイノルズ数の定義を得る。

$$\mathrm{Re} = \frac{\rho u^2}{\mu(u/L)} = \frac{\rho u L}{\mu}$$

Nusselt number, $Nu = h\, L / k$　**対流による熱移動／熱伝導による熱移動**

流体と伝熱面との間の対流伝熱速度を表す。

Prandtl number, $Pr = Cp\, \mu / k$

運動量の輸送（動粘性係数）／熱の輸送（熱拡散率）

物性値のみで決まる。

速度境界層と温度境界層の厚さの相対的な割合を表す。Pr<1ならば，速度境界層よりも温度境界層が厚くなる。

Grashof number, $Gr = g\, \rho^2\, \beta\, \Delta T\, L^3 / \mu^2$　**浮力／粘性力**

速度場および温度場への体膨張による浮力の影響を示す。

Gr/Re^2 は浮力と慣性力の比を表す。

Forced Convection – *External Flow*

Flat plate in parallel flow

$Re_{cr} = 3.2 \times 10^5$

laminar flow

$$Nu_x = \frac{h_x x}{k} = 0.332\ Re_x^{1/2}\ Pr^{1/3} \qquad \overline{h_L} = 2h_{x=L} \qquad \overline{Nu_L} = \frac{\overline{h_L}\,L}{k} = 0.664\ Re_L^{1/2}\ Pr^{1/3}$$

turbulent flow

$$Nu_x = \frac{h_x x}{k} = 0.0297\ Re_x^{4/5}\ Pr^{1/3} \qquad \overline{h_L} = \frac{5}{4} h_{x=L} \qquad \overline{Nu_L} = \frac{\overline{h_L}\,L}{k} = 0.0371\ Re_L^{4/5} Pr^{1/3}$$

Forced Convection – *External Flow*

Entry length $$\frac{x_{fd}}{D} \approx 0.05\, Re_D$$

Laminar flow $$Re_{cr} = 2.3 \times 10^3$$

$$Nu_D = \frac{h\,D}{k} = 3.66 \qquad T_s = \text{constant} \qquad Nu_D = \frac{h\,D}{k} = 4.36 \qquad q_s = \text{constant}$$

Turbulent flow $$Nu_D = \frac{h\,D}{k} = 0.023\, Re_D{}^{4/5}\, Pr^{1/3}$$

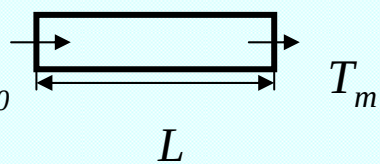
$$Re_D \geq 10{,}000, \;\; L/D \geq 10$$

Colburn's equation $$St = \frac{Nu}{Re\,Pr}, \qquad j = St\,Pr^{2/3} = \frac{Nu}{Re\,Pr^{1/3}} = \frac{0.023}{Re^{0.2}}$$

Forced Convection – *Internal Flow*

Mixed Mean Temperature

$$T_m = \frac{\int_0^R \rho C_p u T (2\pi r\, dr)}{\int_0^R \rho C_p u (2\pi r\, dr)}$$



Reynolds Analogy

$$\frac{C_f}{2} = j = St\, Pr^{2/3} = \frac{Nu_D}{Re_D\, Pr} Pr^{2/3} = \frac{0.023}{Re^{0.2}}$$

For Pr=1

$$\left[\frac{\partial\{(T-T_0)/(T_\infty - T_0)\}}{\partial y}\right]_0 = \left\{\frac{\partial(u/u_\infty)}{\partial y}\right\}_0$$

$$c_f = \frac{\tau_0}{(1/2)\rho u_\infty^{\ 2}} = \frac{\mu(\partial u/\partial y)_0}{(1/2)\rho u_\infty^{\ 2}}$$

$$St = \frac{h}{\rho u_\infty C_p}, \quad Pr = 1 \rightarrow C_p \mu = k \qquad St = \frac{C_f}{2}$$

Graetz Number

$$Q = \frac{\pi}{4} D^2 u_m \rho C_p (T_0 - T_m)$$

$$h_m = \frac{Q}{\pi D L\left(\frac{T_o + T_m}{2} - T_w\right)} = \frac{D u_m \rho C_p}{2L}\left[\frac{1-\left(\frac{\theta_m}{\theta_0}\right)}{1+\left(\frac{\theta_m}{\theta_0}\right)}\right]$$

$$Nu_m = \frac{2\dot{m} C_p}{\pi k L}\left[\frac{1-\left(\frac{\theta_m}{\theta_0}\right)}{1+\left(\frac{\theta_m}{\theta_0}\right)}\right] = f\left(\frac{\dot{m} C_p}{\pi k L}\right)$$

$$Gz = \frac{\dot{m} C_p}{k L} = \frac{\rho u_m \frac{\pi}{4} D^2}{k L} C_p = \frac{\pi}{4} Re\, Pr \frac{D}{L}$$

Gz<20: Fully developed temperature distribution

Natural Convection

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}-g+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial p}{\partial x}=-\rho_\infty g \quad \text{for outside boundary layer } (u=0)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{g}{\rho}(\rho_\infty-\rho)+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}$$

$$\beta=-\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_p\approx-\frac{1}{\rho}\frac{\rho_\infty-\rho}{T_\infty-T} \quad \therefore(\rho_\infty-\rho)\approx\rho\beta(T-T_\infty)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta(T-T_\infty)+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}$$

$$x^*=\frac{x}{L},\ y^*=\frac{y}{L},\ u^*=\frac{u}{u_0},\ v^*=\frac{v}{u_0},\ T^*=\frac{T-T_\infty}{T_s-T_\infty}$$

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=\frac{g\beta(T_s-T_\infty)L}{u_0^{\ 2}}T^*+\frac{1}{Re}\frac{\partial^2 u^*}{\partial y^{*2}}$$

$$\frac{g\beta(T_s-T_\infty)L}{u_o^{\ 2}}=\frac{Gr}{Re^2} \quad \left(Gr=\frac{g\beta(T_s-T_\infty)L^3}{\nu^2}\right)$$

Combined free and forced convection

A general criterion for determining whether free convection effects dominate:

$$\frac{Gr}{Re^2}>10$$

$$Nu=C(Gr\,Pr)^m, \qquad Ra=Gr\,Pr$$

Radiation Heat Transfer

Planck's law $$E_{\lambda B} = \frac{C_1 \lambda^{-5}}{\exp(C_2/\lambda T) - 1} \qquad C_1 = 2\pi c_0^{\,2} h \qquad C_2 = c_0 h/k$$

Stefan-Boltzmann law $$E_B = \int_0^\infty E_{\lambda B}\, d\lambda = \sigma T^4 \qquad \sigma = 5.670 \times 10^{-8}\ \mathrm{W/(m^2K^4)}$$

Emissivity $$\varepsilon = \frac{E}{E_B} \qquad \varepsilon_\lambda = \frac{E_\lambda}{E_{\lambda B}}$$

Kirchhoff's law $$\varepsilon_\lambda = \alpha_\lambda$$

View factor, Geometrical factor $$A_i F_{ij} = A_j F_{ji} \qquad \sum_{j=1}^{N} F_{ij} = 1$$

Blackbody radiation exchange

$$Q_{1\to 2} = \sigma(T_1^4 - T_2^{\,4}) A_1 F_{12}$$

$$Q_{1\to 2,3,4\ldots} = A_1 F_{12}\sigma(T_1^4 - T_2^{\,4}) + A_1 F_{13}\sigma(T_1^4 - T_3^{\,4}) + \cdots$$

$$= \left(A_1 F_{11} + A_1 F_{12} + A_1 F_{13} + \cdots\right)\sigma T_1^4 - A_1 F_{11}\sigma T_1^4 - A_1 F_{12}\sigma T_2^{\,4} - A_1 F_{13}\sigma T_3^{\,4} + \cdots$$

$$= A_1 \sigma T_1^4 - \left(A_1 F_{11}\sigma T_1^4 + A_2 F_{21}\sigma T_2^{\,4} + A_3 F_{31}\sigma T_3^{\,4} + \cdots\right)$$

Radiation Heat Transfer

Radiation exchange between gray surface

$$W_i A_i = \varepsilon_i \sigma T_i^4 A_i + (1-\varepsilon_i)\sum_{k=1}^{n} W_k A_k F_{ki}$$

$$Q_i = W_i A_i - \sum_{k=1}^{n} W_k A_k F_{ki}$$

$$A_k F_{ki} = A_i F_{ik}$$

$$A_k F_{ki} = A_i F_{ik}$$

$$W_i A_i = \varepsilon_i \sigma T_i^4 A_i + (1-\varepsilon_i)\sum_{k=1}^{n} W_k A_i F_{ik}$$

$$Q_i = W_i A_i - \sum_{k=1}^{n} W_k A_i F_{ik}$$

$$W_i = \varepsilon_i \sigma T_i^4 + (1-\varepsilon_i)\sum_{k=1}^{n} W_k F_{ik}$$

$$q_{i\rightarrow} = \frac{Q_i}{A_i} = W_i - \sum_{k=1}^{n} W_k F_{ik}$$

Radiation exchange between parallel plate

$$F_{11} = 0, F_{12} = 0, F_{21} = 1, F_{22} = 0$$

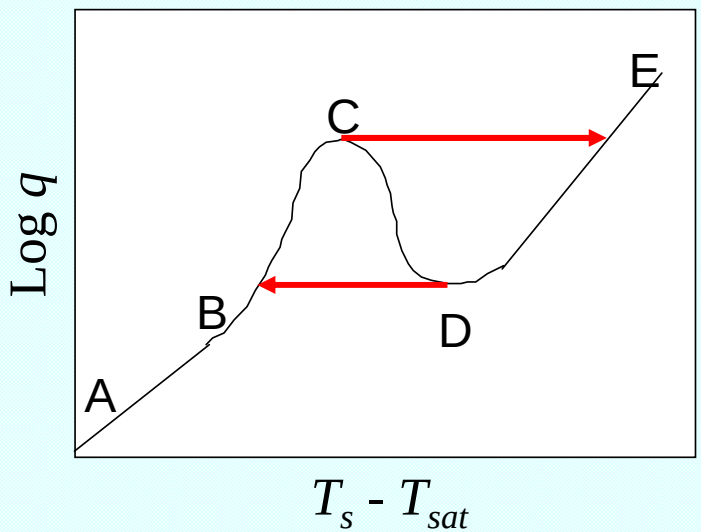
$$W_1 = \varepsilon_1 \sigma T_1^4 + (1-\varepsilon_1) W_2, \quad W_2 = \varepsilon_2 \sigma T_2^4 + (1-\varepsilon_2) W_1$$

$$W_1 = \frac{\varepsilon_1 \sigma T_1^4 + (1-\varepsilon_1)\varepsilon_2 \sigma T_2^4}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2}, \quad W_2 = \frac{\varepsilon_2 \sigma T_2^4 + (1-\varepsilon_2)\varepsilon_1 \sigma T_1^4}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2}$$

$$q_{1\rightarrow} = W_1 - W_2 = \frac{\varepsilon_1 \varepsilon_2 \sigma \left(T_1^4 - T_2^4\right)}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2} = \frac{\sigma \left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

Boiling, Condensation Heat Transfer

Boiling Heat Transfer



Boiling Curve

AB: free convection
BC: nucleate boiling
CD: transition boiling
DE: film boiling
C: burnout point
D: Leidenfrost point

Condensation Heat Transfer

Film Condensation

$$\rho_L\, g\,\mathrm{d}x\,\mathrm{d}y + \left\{\left(\tau + \frac{\mathrm{d}\tau}{\mathrm{d}y}\mathrm{d}y\right) - \tau\right\}\mathrm{d}x = 0, \quad \rho_L\, g\,\mathrm{d}x\,\mathrm{d}y + \mu\frac{\mathrm{d}u}{dy}dx = 0, \quad \rho_L\, g + \mu\frac{\mathrm{d}^2u}{\mathrm{d}y^2} = 0$$

$$B.C.\ \frac{\mathrm{d}u}{dy} = 0 \text{ at } y = \delta,\ u = 0 \text{ at } y = 0 \Rightarrow u = \frac{\rho_L\, g\,\delta^2}{\mu}\left[\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right]$$

$$u_0 = \frac{\rho_L\, g\,\delta^2}{2\mu} \Rightarrow \frac{u}{u_0} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

$$u_m = \frac{1}{\delta}\int_0^{\delta} u\,\mathrm{d}y = \frac{\rho_L\, g\,\delta^2}{3\mu} \Rightarrow \quad G = u_m\,\delta\,\rho_L = \frac{\rho_L{}^2\, g\,\delta^3}{3\mu}$$

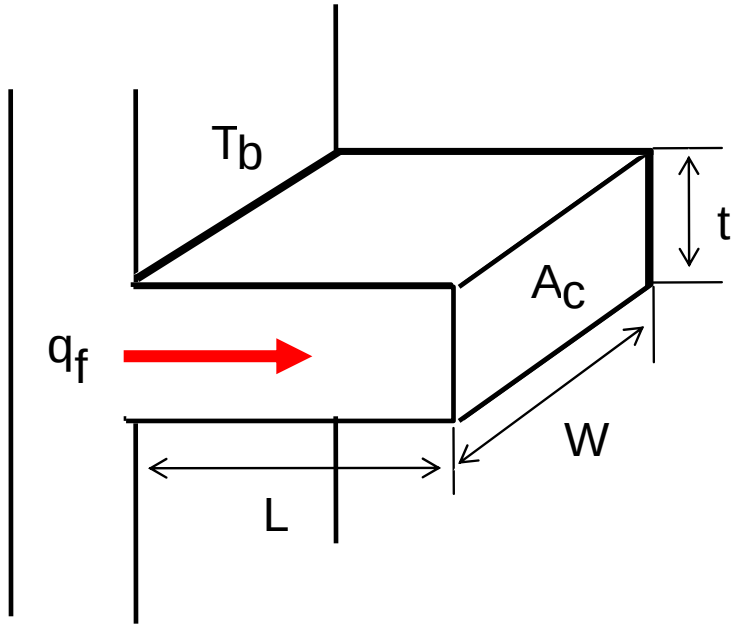
$$L\,\mathrm{d}G = L\frac{\rho_L{}^2\, g}{3\mu}\mathrm{d}(\delta^3) = h(T_{sat} - T_w)\mathrm{d}x \Rightarrow \int_0^{x}\frac{k}{L}(T_{sat} - T_w)\mathrm{d}x = \int_0^{\delta}\frac{\rho_L{}^2\, g}{3\mu}3\delta^3 d\delta$$

$$\therefore \quad \delta = \left[\frac{4\mu k(T_{sat} - T_w)x}{\rho^2\, g\, L}\right]^{1/4}$$

$$Nu_x = \frac{h\,x}{k} = \frac{x}{\delta} = \left[\frac{\rho^2\, g\, L\, x^3}{4\mu k(T_{sat} - T_w)}\right]^{1/4} = 0.707\left(\frac{L}{Cp(T_{sat} - T_w)}\right)^{1/4} Gr^{1/4}\, Pr^{1/4}$$

$$\overline{Nu} = \frac{4}{3}Nu_x = 0.943\left(\frac{L}{Cp(T_{sat} - T_w)}\right)^{1/4} Gr^{1/4}\, Pr^{1/4}$$

Heat Conduction – *Fin Performance*



$$\frac{d^2T}{dx_2} - \frac{hP}{kA_c}(T - T_\infty) = 0$$

$$\theta = T(x) - T_\infty$$

$$\frac{d^2\theta}{dx_2} - m^2\theta = 0, \quad m^2 = \frac{hP}{kA_c}$$

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

Boundary Conditions

$$\frac{d^2T}{dx_2} - \frac{hP}{kA_c}(T - T_\infty) = 0$$

$$\theta = T(x) - T_\infty$$

$$x = 0; \ \theta(0) = T_b - T_\infty = \theta_b$$

$$x = L; \ \text{four types}$$

1. infinite fin

$$\theta(\infty) = 0, \quad \frac{\theta}{\theta_b} = e^{-mx}, \quad q_f = \sqrt{hPkA_c}\,\theta_b$$

2. convection heat transfer

$$hA_c\,\theta(L) = -kA_c \left.\frac{d\theta}{dx}\right|_{x=L}$$

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$$

$$q_f = \sqrt{hPkA_c}\,\theta_b \frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$$

3. Adiabatic

$$\left.\frac{d\theta}{dx}\right|_{x=L} = 0$$

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x)}{\cosh mL}$$

$$q_f = \sqrt{hPkA_c}\,\theta_b \tanh mL$$

4. prescribed temperature

$$\theta(L) = \theta_L$$

$$\frac{\theta}{\theta_b} = \frac{\left(\frac{\theta_L}{\theta_b}\right)\sinh mx + \sinh x(L-x)}{\sinh mL}$$

$$q_f = \sqrt{hPkA_c}\,\theta_b \frac{\cosh mL - \left(\frac{\theta_L}{\theta_b}\right)}{\sinh mL}$$