Urban Environmental Engineering 6

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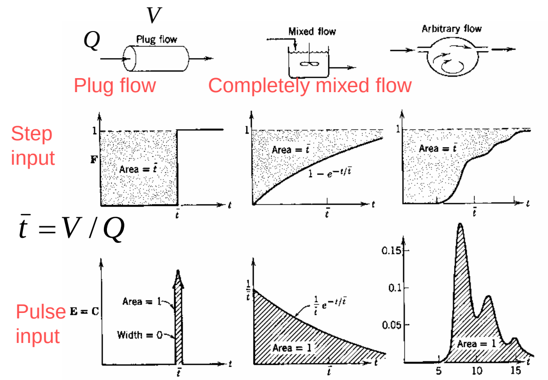
Lecture in the Last time

- Reaction Kinetics
  – First order reaction
  – Zero order reaction
  – Monod Equation
  – Streeter Phelps equation
  – First order reactions in series

Today's Lecture

- Mixing
  – Plug flow and mixed flow
  – Tanks in series model
  – Dispersion model
- Mass Transfer coefficient
  – A melting sphere

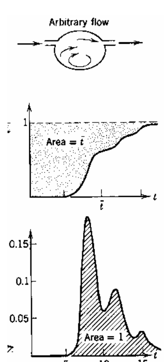
Flow without reaction



$$\bar{t} = V / Q$$

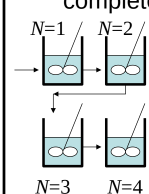
Modeling of Arbitrary Flow

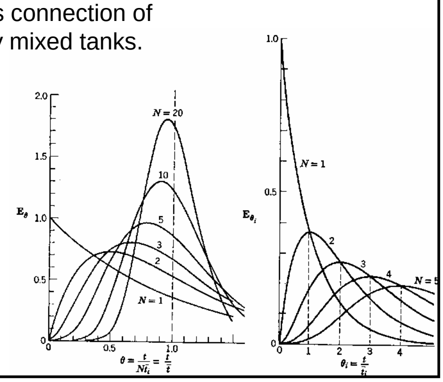
- Tanks in series model
- Dispersion model



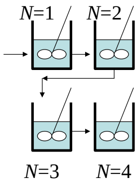
Tanks in series model

- The arbitrary flow was assumed as a series connection of completely mixed tanks.





Tanks in series model (Contd.)



- When N=1, the flow is completely mixed flow
- When N=infinite, the flow is plug flow
- We can characterize arbitrary flow by number of tanks.

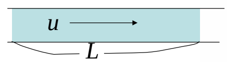
Dispersion model

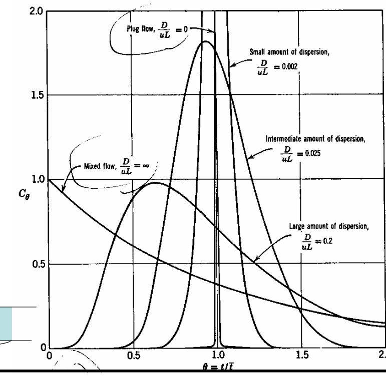
Dispersion equation for 1-D with flow can be written as a dimensionless form:

$$\frac{\partial C}{\partial \theta} = \left(\frac{D}{uL}\right)\frac{\partial^2 C}{\partial z^2} - \frac{\partial C}{\partial z}$$

Where

$$\theta = \frac{tQ}{V} = \frac{t}{\bar{t}}$$





Dispersion model (Contd.)

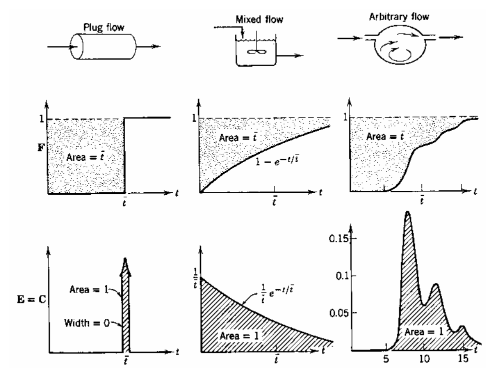
$\frac{D}{uL} \to 0$ Plug flow

$\frac{D}{uL} \to \infty$ Completely mixed flow

We can characterize an arbitrary flow by estimating

$$\frac{D}{uL}$$

Flow without reaction



Flow with zero order reaction

Assuming constant influent concentration, what will be the effluent concentration.

$$r = -\frac{dC}{dt} = k$$

For plug flow reactor $C_{eff} = C_{inf} - k\frac{V}{Q}$

For completely mixed flow reactor,

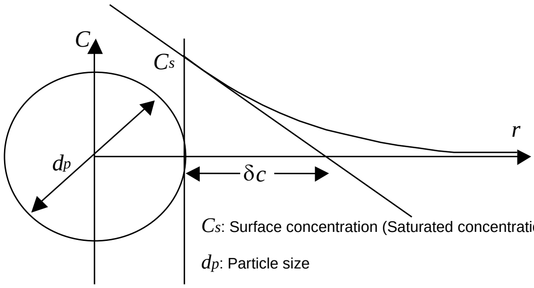
Mass balance equation $C_{inf}Q - C_{eff}Q = rV$

$$C_{eff} = C_{inf} - k\frac{V}{Q}$$

Flow with first order reaction

Homework: In the case of zero order reaction, no difference was found in effluent concentration. But in the case of first order reaction, effluent concentration is affected by the flow. Please calculate the effluent concentration of plug flow reactor and completely mixed flow reactor with first order reaction.

A melting sphere

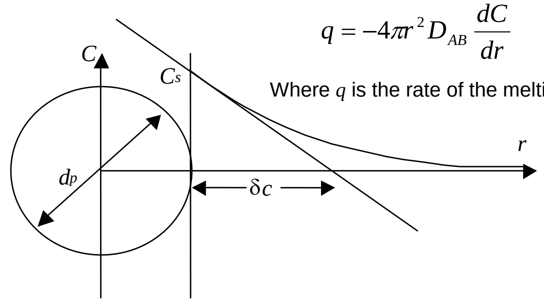


C_s: Surface concentration (Saturated concentration)

d_p: Particle size

δ_c: thickness of concentration polarization layer

A melting sphere



$$q = -4\pi r^2 D_{AB} \frac{dC}{dr}$$

Where q is the rate of the melting

A melting sphere

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Integrating equation above with the condition

C=0 at r = infinite, C=Cs at r = dp/2, we obtain

$$q = 2\pi D_{AB} d_p C_s$$

By defining concentration polarization layer thickness as shown in the figure, we obtain

$$\frac{q}{4\pi\left(\frac{d_p}{2}\right)^2} = D_{AB}\frac{C_s}{\delta_c}$$

A melting sphere

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$$\frac{q}{4\pi\left(\frac{d_p}{2}\right)^2} = D_{AB}\frac{C_s}{\delta_c}$$ This equation is deduced to $\delta_c = \frac{d_p}{2}$

Definition of Sherwood number

Here we define Sherwood number as

$$N_{Sh} = \frac{d_p}{\delta_c}$$

Sherwood number of melting sphere in a fluid without any motion is always 2.

$$N_{Sh} = 2$$

If the fluid is moving, chemical engineers suggest various correlations.

$$N_{Sh} = 2 + 0.6 N_{\mathrm{Re}}^{0.5} N_{Sc}^{0.33}$$ Laminar flow

This kind of correlations are used for the design of the plant because they are dimensionless and are applied without any change even in the case of scaled – up.