In Table 3.2, "C" is concrete cover (mm). For the case of C > 100 mm, C = 100 mm. In Table 3.2, the permissible crack widths for prestressing steel in corrosive environment and severely corrosive environment are not determined. It is considered that prestressed concrete can be designed prohibiting the formation of flexural cracks by using prestressing, and that it is necessary to examine the corrosion of prestressing steel much more carefully. It is advisable to design the prestressed concrete member prohibiting the formation of cracks in such environmental conditions.

3.1.3 Prediction of Flexural Crack Width

(1) Derivation of equation for predicting flexural crack width

When the flexural moment is applied to RC beams, a flexural crack due to flexural tensile stress occurs at the extreme tension fiber. Then, as the moment is increased, the number of cracks increases and the *crack spacing* gets smaller. This will cause the tensile stress in concrete between the adjacent cracks. After the initiation of number of flexural cracks, new cracks are hardly formed because of a relatively short development length for bond stress. Hence, *the stable state of cracking* is obtained. At this stage, cracked portion around the tensile reinforcement in a flexural member can be considered to be equivalent to a concrete member having a single reinforcement subjected to pull-out force at both ends (Fig. 3.1).

Denote l as the crack spacing. From the equilibrium of forces at the mid section between cracks, the following relationship can be derived:

$$\int_0^{l/2} \tau_b(x)\,U\,dx = \overline{\tau_b}\,U\,\frac{l}{2} = \overline{\sigma_{ct}}\,A_e \qquad (3.1)$$

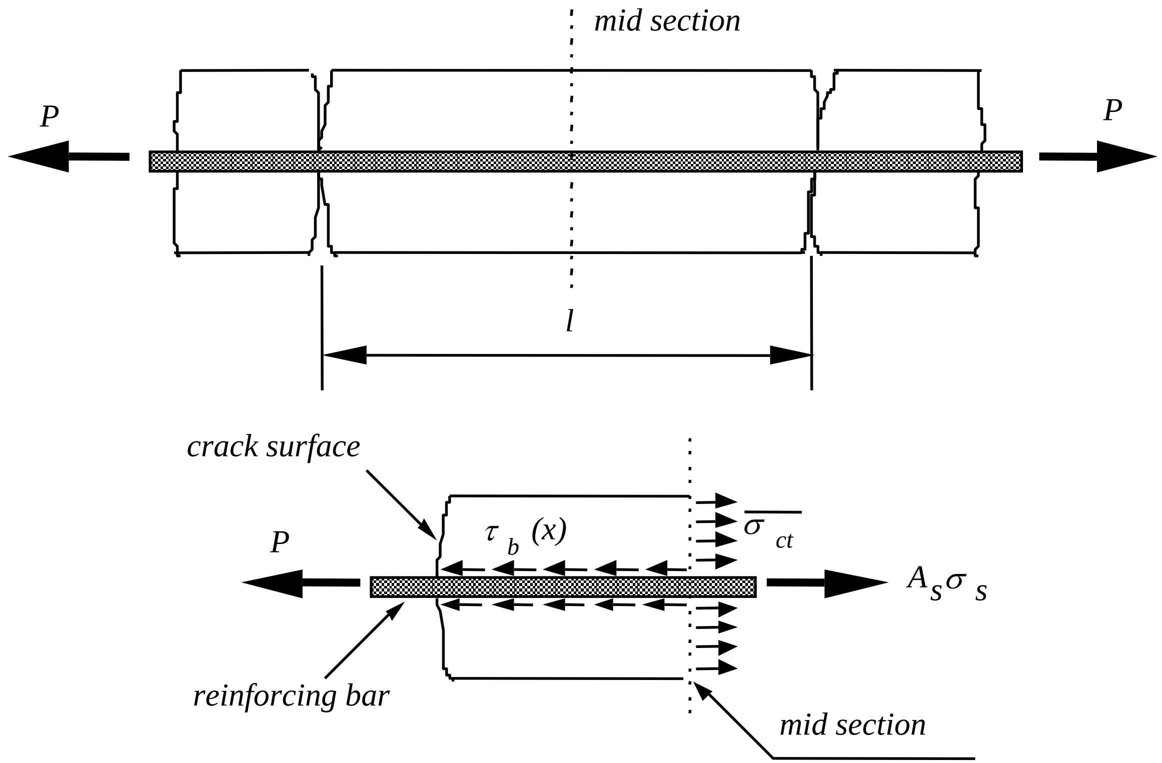

Fig. 3.1 Idealized conditions of bond between steel and concrete

where, $\overline{\tau}_b$ **is the average bond stress between the two adjacent cracks**

l is a spacing of cracks.

U is a perimeter length of reinforcement.

A_e is effective area of cover concrete.

$\overline{\sigma}_{ct}$ **is the average tensile stress of concrete at the mid section.**

The relation between $\overline{\tau}_b$ and the tensile force P of steel shows that the maximum value of $\overline{\tau}_b$ exists. At this point $\overline{\tau}_b = \overline{\tau}_{b.max}$, the crack spacing *l* becomes stable (Fig. 3.2).

When $\overline{\sigma}_{ct}$ reaches a certain limit value, i.e., k f_t (where, k : coefficient representing the stress distribution over the mid-section and f_t : tensile strength of concrete), the new crack between adjacent two cracks occurs. Hence, the maximum crack spacing is defined as follows:

$$l_{max} = \frac{2 k f_t A_e}{\overline{\tau}_{b.max} U} \qquad \textbf{(3.2)}$$

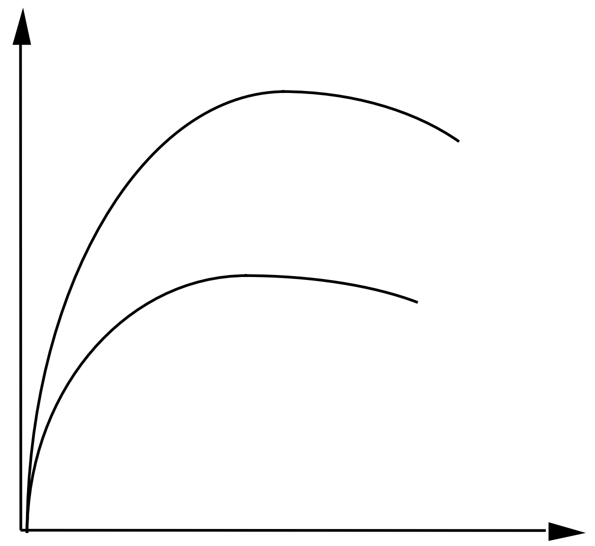

Fig. 3.2 Relationship between pull-out force and average bond stress

It is noted that in the case of more than two reinforcing bars, A_e is defined by the centroid of total reinforcing bars (Fig. 3.3). For simplicity, if the maximum average bond stress is assumed to be proportional to the tensile strength of concrete, f_t, the following relationship can be obtained.

$$\overline{\tau}_{b.max} = k_1 f_t \qquad \textbf{(3.3)}$$

where, k_1 : coefficient representing bond characteristics

Substituting Eq. (3.3) into Eq. (3.2) and defining the ratio of reinforcement area to the effective area of cover concrete A_e as an effective reinforcement ratio p_e will lead to the following equation.

$$l_{max} = k_2 \frac{\phi}{p_e} \qquad \textbf{(3.4)}$$

where, k_2 : coefficient representing bond characteristics, ϕ: diameter of reinforcement

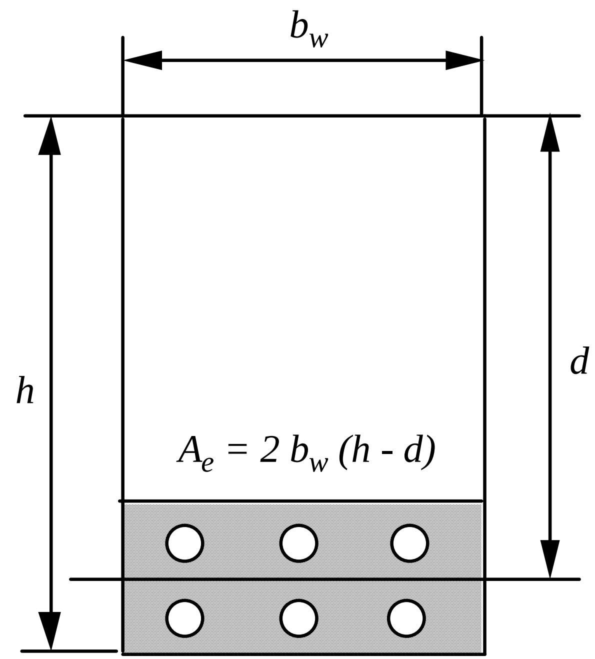

Fig. 3.3 Definition of effective concrete area A_e

This equation proposed by Saliger in 1938. However, it was found later that this equation cannot be applied for RC members in flexure. Kakuta in Hokkaido University proposed that the maximum average bond stress $\overline{\tau_{b.max}}$ is affected by not only the tensile strength of concrete f_t but also the diameter of reinforcement ϕ, the effective area of concrete A_e, and concrete cover c. The empirical formula to obtain $\overline{\tau_{b.max}}$ is expressed as follows:

$$\overline{\tau_{b.max}} = k_3 f_t \frac{A_e}{c\phi} \qquad \textbf{(3.5)}$$

Substituting Eq. (3.5) into Eq. (3.2) yields,

$$l_{max} = k_4 c \qquad \textbf{(3.6)}$$

where, k_4 : coefficient representing bond characteristics. k_4 approximates as 5.4 for D16~32 (f = 16~ 32 mm) deformed bars.

In the case of more than two reinforcing bars, a clear spacing between reinforcing bars e_s has to be considered, and Eq. (3.6) is modified as follows:

$$l_{max} = \frac{k_4 c}{1.45}\left(1 + 0.18\frac{e_s}{c}\right) \qquad \textbf{(3.7)}$$

To determine the crack width *w*, the following relationship is adopted (Fig. 3.4).

$$w = \left(\overline{\varepsilon_s} - \overline{\varepsilon_c}\right) \cdot l \qquad \textbf{(3.8)}$$

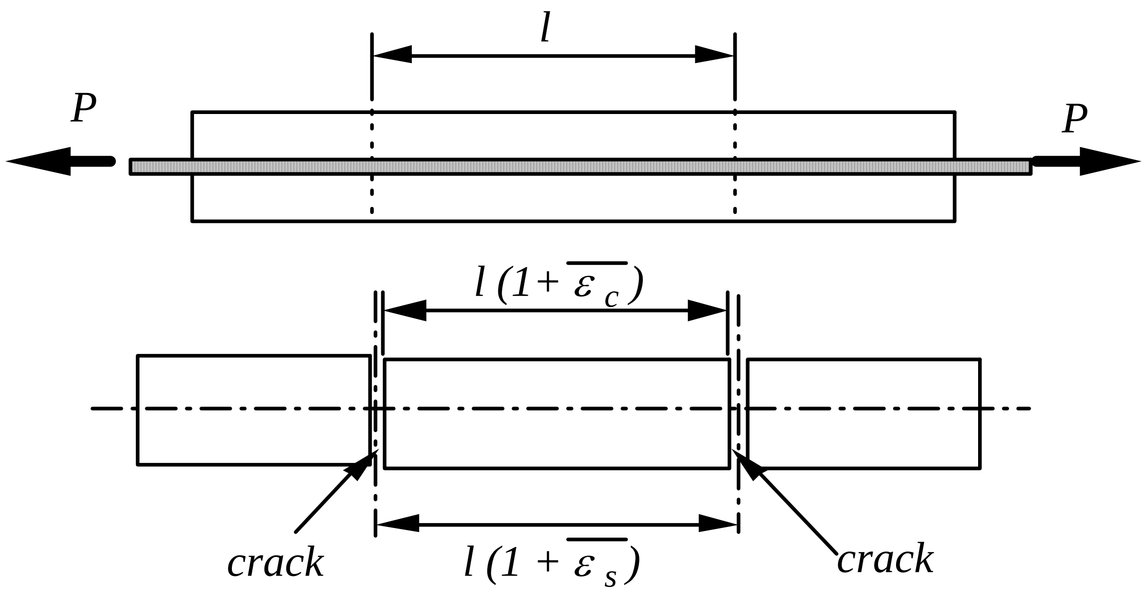

Fig. 3.4 Crack width

where, $\overline{\varepsilon_s}$: average tensile strain of reinforcement between cracks

$\overline{\varepsilon_c}$: average tensile strain in concrete between cracks

l : crack spacing (crack interval)

The average tensile strain of concrete $\overline{\varepsilon_c}$ is divided into two components as:

$$\overline{\varepsilon_c} = \overline{\varepsilon_{ce}} - \varepsilon'_{cs} \quad (3.9)$$

where, $\overline{\varepsilon_{ce}}$: average tensile strain due to external tensile force

ε'_{cs} : compressive strain due to drying shrinkage and creep in concrete

On the other hand, the average tensile strain of reinforcement $\overline{\varepsilon_s}$ is expressed as follows:

$$\overline{\varepsilon_s} = \frac{\sigma_s}{E_s} - \frac{\overline{\sigma_t}}{E_s p_e} \qquad (3.10)$$

where, σ_s : tensile stress of reinforcement at the crack position

E_s : Young's modulus of steel

$\overline{\sigma_t}$: average tensile stress of concrete between cracks

This $\overline{\sigma_t}$ can be considered as the effect of tension stiffening due to bond. In general, ε_{ce} in Eq. (3.9) is negligibly small as compared with other components. Thus, by using Eqs. (3.7) to (3.10), the following equation to predict the maximum crack width w_{max} was proposed by Kakuta.

$$w_{max} = \left(\overline{\varepsilon_s} - \overline{\varepsilon_c}\right) l = \frac{5.4c}{1.45}\left(1 + 0.18\frac{e_s}{c}\right)\left(\frac{\sigma_s}{E_s} - \frac{\overline{\sigma_t}}{E_s p_e} + \varepsilon'_{cs}\right) \qquad (3.11)$$

(2) Design equation for prediction of flexural crack width

Based on Eq. (3.11), the design equation for prediction of flexural crack width is stipulated in the JSCE Specification.

$$w_{max} = k\,\{4c + 0.7(c_s - \phi)\}\left(\frac{\sigma_{se}}{E_s} + \varepsilon'_{cs}\right) \qquad \textbf{(3.12)}$$

where, **k : constant to take into account the influence of bond characteristics of reinforcing bar, 1.0 for deformed bars, 1.3 for round bars.**

c : concrete cover

c_s : center-to-center distance of reinforcement

ϕ : diameter of reinforcement

ε'_{cs} : compressive strain due to drying shrinkage and creep in concrete = 150×10^{-6} in general

σ_{se} : tensile stress in reinforcement due to external loads (the external loads to be considered are the combination of permanent and variable loads in the service condition.)

E_s : Young's modulus of reinforcement

(3) Stress calculation under service loads

For the examination of serviceability limit states, stresses need to be computed, such as σ_{se} in Eq. (3.12). Under service loads, the stresses in concrete and reinforcement are small, and hence concrete and reinforcement can be assumed to be elastic with the modulus of elasticity, E_c and E_s, respectively. As in the case of stress calculation for serviceability limit state, the following assumptions are used:

(i) linear strain distribution along the cross section
(ii) perfect bond
(iii) neglecting tensile stress in concrete
(iv) concrete in compression and reinforcement are elastic

3.2 Deflection of RC Beam

3.2.1 Limit State on Deflection

In order to preserve the functions and serviceability of structures, the control of displacement and deformation of structures is also required. For RC beams especially in bridge structures, the excess in deflection is highly related to the safety and comfort of vehicle traveling on the bridge.

There are two types of deformations. One is short-term deformation that occurs immediately after the application of load. The other is additional deformation caused by creep and drying shrinkage of concrete due to permanent load. The sum of these two deformations is called long-term deformation.
(long-term deformation = short-term deformation + creep & shrinkage deformation)

Similar to the limit state on crack width, the deflection of RC beams is examined to be less than or equal to the permissible value. The permissible deflection is determined considering the type and purpose of structures and the type of loads. For example, for the ordinary concrete bridge, the permissible deflection is set to be approximately 1/600 of the span length.

3.2.2 Prediction of Short-term Deflection

There are many previous research works related to the prediction of short-term deflection of RC beams. However, in the JSCE Specification the method based on an effective moment of inertia empirically taking into account of flexural cracking is stipulated to determine the short-term deflection. When the concrete does not have any flexural cracks, the gross moment of inertia, i.e., all concrete and steel, is effective in use. The following equation to determine an effective moment of inertia after flexural cracking which was originally proposed by Branson is adopted in the JSCE Specification.

$$I_e = \left[\left(\frac{M_{cr}}{M_{max}} \right)^3 I_g + \left\{ 1 - \left(\frac{M_{cr}}{M_{max}} \right)^3 \right\} I_{cr} \right] \le I_g \qquad (3.13)$$

where, I_e : effective moment of inertia considering flexural cracks

M_{cr} : cracking moment, which is equivalent to the moment of the cross section causing the tension fiber stress to be equal to the flexural strength of concrete.

$$M_{cr} = \frac{f_b \cdot I_g}{y_t} \qquad (3.14)$$

where, y_t : the distance from the neutral axis to the extreme tension fiber

f_b : the flexural strength of concrete

I_g : the gross moment of inertia

M_{max} : the maximum moment in a member concerned

I_{cr} : moment of inertia for cracked cross section neglecting the concrete in tension

Eq. (3.13) is an approximated evaluation for a simply-supported beam subjected to uniformly distributed load. However, even for RC beams with other loading and boundary conditions, this equation can be adopted to predict a deflection within an acceptable error.