$$q = \tau t = \frac{T}{2A_o} \qquad \textbf{(2.61)}$$

**Eq. (2.61) is called as Bredt's formula.**

### *2.4.3 Torsion Capacity of Reinforced Concrete Member Based on Space Truss Analogy*

**In the *space truss analogy* proposed by Raush, the original solid cross section is converted into an imaginary thin-walled tubular cross section. The following basic assumptions are made to calculate the ultimate torsion capacity.**

**(1) After torsion cracks initiate and propagate, the core part of concrete in a solid cross section can be neglected and an original solid cross section can be assumed to be equivalent to the imaginary thin-walled tubular cross section. The shear stress due to torsion becomes zero at the center of the cross section and the magnitude of the shear stress will increase when it approaches to the perimeter of a cross section. Moreover, resisting torsion moment is proportionally increasing with the distance from the center of the cross section. Thus, the assumption to neglect the effect of core part of concrete can be accepted.**

**(2) From Bredt's theory, applied torsion can be converted into an uniform shear flow. If the thickness of the imaginary thin-walled tubular cross section is assumed to be uniform, the shear stress also becomes uniform.**

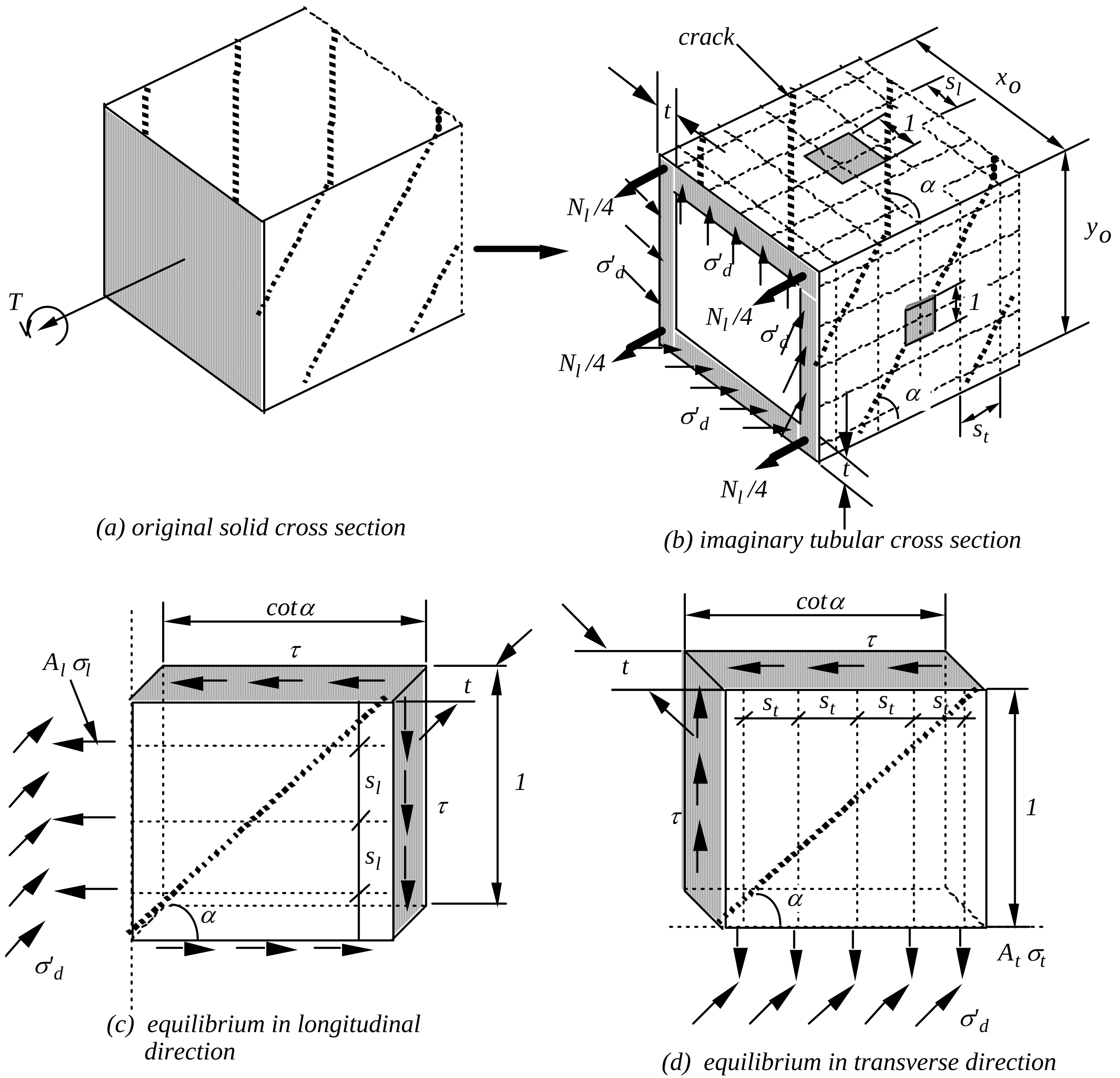
**(3) From Fig. 2.22, the shear stress is resisted by concrete and reinforcement. For simplicity, the contribution of concrete for tension after cracking is completely neglected (the tension stiffening including the tension softening is not taken into account). According to this assumption, concrete can be assumed to be in uniaxial compressive stress state. The direction of this compressive stress of concrete is parallel to the direction of torsion cracks. Reinforcing bars can resist only axial forces (the dowel action is neglected).**

**According to these assumptions, the following relationships can be obtained. From the equilibrium condition in the horizontal and vertical surfaces, Eqs. (2.62) and (2.63) are derived.**

$$A_l \sigma_l / s_l - \sigma'_d t \cos^2 \alpha = 0 \qquad \textbf{(2.62)}$$

$$A_t \sigma_t / s_t - \sigma'_d t \sin^2 \alpha = 0 \qquad \textbf{(2.63)}$$

**where, $A_l$, $\sigma_l$ : area and stress of one longitudinal torsional reinforcing bar. $A_t$, $\sigma_t$ : area and stress of one transverse torsional reinforcing bar. $s_l$, $s_t$ : the spacing of longitudinal and transverse reinforcing bars, respectively. $\sigma'_d$, $t$ : compressive stress of concrete and an imaginary wall thickness. $\alpha$ is an angle between the direction of diagonal compressive stress of concrete and the longitudinal direction of a member. $\alpha$ is assumed to be uniform throughout the wall surface.**




(a) original solid cross section

(b) imaginary tubular cross section

(c) equilibrium in longitudinal direction

(d) equilibrium in transverse direction

**Fig. 2.22 Torsional resisting model**

**If we consider the extended element having the length 2 ($x_o$+$y_o$), Eq. (2.62) can be rewritten as follows:**

$$A_l \sigma_l \cdot 2(x_o + y_o)/s_l - \sigma'_d t\, 2(x_o + y_o)\cos^2\alpha = 0 \qquad \textbf{(2.64)}$$

**$A_l$ 2($x_o$+$y_o$)/$s_l$ means the total area of longitudinal reinforcement included in this cross section. Thus, Eq.(2.64) can be written as follows:**

$$\sum A_l \sigma_l - \sigma'_d t \cdot 2(x_o + y_o)\cos^2\alpha = 0 \qquad \textbf{(2.65)}$$

**If applied shear stress is designated as $\tau$ , the relationship between $\tau$ and $\sigma'_d$ can be written as follows:**

$$\tau = \sigma'_d \sin\alpha\cos\alpha \qquad \textbf{(2.66)}$$

**For applied torsion T and shear stress $\tau$ , the following relationship can be obtained.**

$$T = \tau \cdot t \cdot (y_o / 2 \times x_o \times 2 + x_o / 2 \times y_o \times 2) = 2 x_o y_o \tau t \qquad \textbf{(2.67)}$$

**From the relationships, $x_o$ $y_o$=$A_o$ and 2($x_o$+$y_o$) = $p_o$, and Eqs. (2.62) to (2.67), the stress of reinforcement in both directions can be determined as follows:**

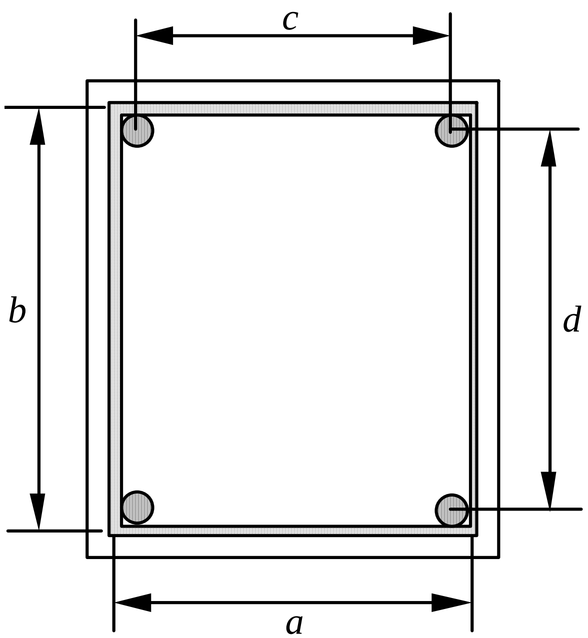
$$\sum A_l \sigma_l = T p_o \cot\alpha / \left(2 A_o\right) \qquad \textbf{(2.68)}$$

$$A_t \sigma_t = T s_t \tan\alpha / \left(2 A_o\right) \qquad \textbf{(2.69)}$$

**At the ultimate stage, we can assume torsional reinforcement in both directions has already yielded. From Eqs. (2.68) and (2.69), we can delete $\alpha$ and we can use $f_{ly}$ and $f_{ty}$ instead of steel stresses. Finally, we can get the torsional capacity based on the yielding of both torsional reinforcements as follows:**

$$T_{max} = 2 A_o \sqrt{\frac{A_t f_{ty}}{s_t} \frac{\sum A_l f_{ly}}{p_o}} \qquad \textbf{(2.70)}$$

**To calculate $T_{max}$, we need to determine $x_o$ and $y_o$ in advance. Two ideas are proposed to determine $x_o$ and $y_o$ (Fig. 2.23). One is the idea to determine $x_o$ and $y_o$ according to the center line connecting longitudinal reinforcement. Another idea is to trace the location of the transverse reinforcement. The former will give an conservative estimation.**




**center line of transverse reinforcement:**

$x_o = a,\ y_o = b$

**center line of longitudinal reinforcement:**

$x_o = c,\ y_o = d$

**Fig. 2.23**
**$x_o$ and $y_o$ (Effective area for torsion)**

**Equation (2.70) can be applied when both torsional reinforcements have already yielded. In an actual reinforced concrete member, once the reinforcement in one direction yields, the torsional resisting mechanism will be changed and it will cause the change in the angle $\alpha$. For example, when the transverse torsional reinforcement has yielded due to applied torsion, the angle $\alpha$ will be decreased and the resisting force in longitudinal reinforcement will be increased. Finally, both reinforcements will yield. However, if the reinforcement ratio is too unbalanced, Eq. (2.70) will overestimate the ultimate torsional capacity.**