The distribution of shear stress $\tau(y)$ is of parabolic curve within the flexural compressive zone of the cross section, i.e., between the extreme compression fiber and the neutral axis (y=0). At y=0, $\tau(0)$ becomes the maximum and equal to ***V/(b jd)***. Since the tensile resistance of concrete is neglected, the shear stress below the neutral axis becomes constant. Here, the flexural stress is assumed to be zero, and hence the principal tensile stress is equal to the shear stress. The diagonal cracking strength of RC beams is expressed by this maximum shear stress (***V/(b jd)***). The value of ***j*** in Eq.(2.54) is approximately equal to 7/8. However, this maximum shear stress is only ***an index*** of the principal tensile stress, and often ***j*** is set to be 1 for the sake of simplicity. ***V/(b d)*** is called ***"nominal shear stress".***

Nominal shear stress: $$\tau = \frac{V}{b_w d}$$

where, V **: shear force**

b_w **: the width of the web of cross-section (Fig. 2.14)**

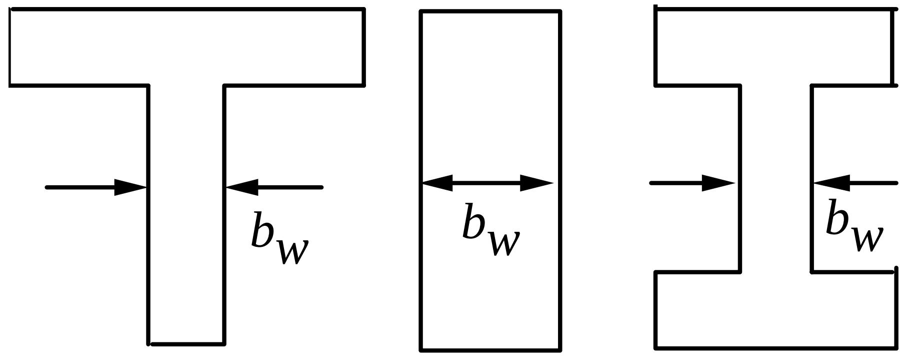

Fig. 2.14 Definition of web width for various cross sections

2.3.2 Beams without Shear Reinforcement

<u>**(1) Shear stress at the formation of diagonal crack**</u>

In a RC beam without shear reinforcement under shear force, once diagonal crack is formed, the beam will fail very suddenly. However, the nominal shear stress at the formation of diagonal crack cannot be obtained by the elastic theory, because it involves many factors such as concrete strength, shear span-effective depth ratio (a/d), longitudinal reinforcement ratio, effective depth, etc.

After a flexural crack occurs, the shear stress along the crack plane is considered to be resisted by the following effects (Fig. 2.15).

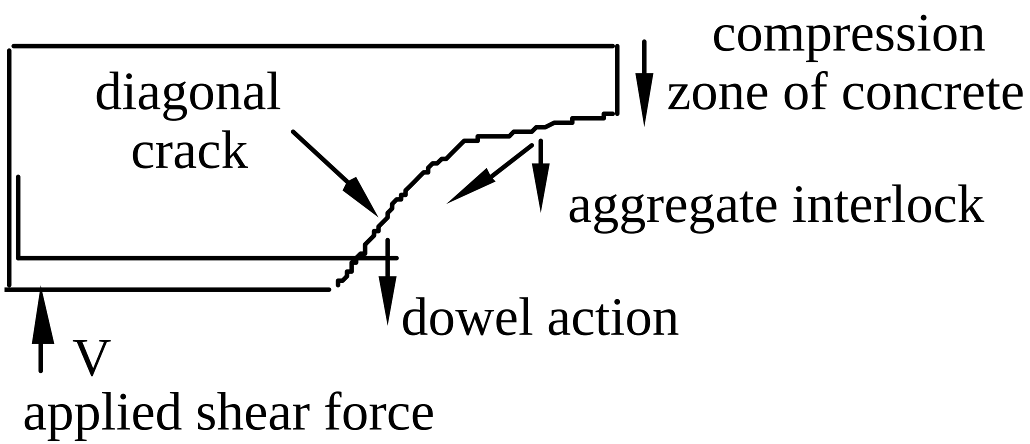

Fig. 2.15 Internal forces to resist applied shear force

- *direct shear resistance* **in the flexural compression zone**
- *aggregate interlocking* **along the crack surface**
- *dowel action* **of longitudinal steel**

Up to now, the amount of shear force due to each of these effects has not been formulated yet.

(a) Aggregate Interlock along Crack Plane

Along the diagonal crack of concrete, ***the shear transfer due to the effect of aggregate interlock*** **can be expected. This effect is especially large, when the crack width is small and concrete strength is high. Since the crack width is proportional to the stress in steel which depends on the longitudinal reinforcement ratio, as the reinforcement ratio increases, the effect of aggregate interlock becomes larger.**

The above discussion is based on the assumption of the same ratio of the dimension of section to the maximum size of coarse aggregates. The effect of ***aggregate interlock*** **depends on the relation between the sectional dimension and the aggregate size, and for the same aggregate size, the effect of aggregate interlock on the small RC section is more pronounced than that in the large section. Since the maximum size of coarse aggregates in ordinary RC beams practically does not change even when the dimension of the section is increased, the nominal shear strength of large beams tends to decrease. This is a** ***classical explanation*** **for the** ***size effect*** **in the shear strength.**

(b) Dowel Action of Longitudinal Reinforcement

A part of the shear force can be transferred by the *dowel action* of longitudinal reinforcement. The main factors influencing this action are flexural rigidity of longitudinal reinforcement and flexural rigidity of surrounding concrete. Actually, there are additional factors involving this effect such as the number and arrangement of longitudinal reinforcement, spacing of flexural cracks, etc. However, the contribution of each factor has not been formulated so far. At present, the dowel action is represented by using the reinforcement ratio and concrete compressive strength.

(c) Flexural Compressive Zone of Concrete

A part of shear carried by *uncracked flexural compression zone* of concrete is closely related to the area of compression zone. Since the position of the neutral axis after flexural cracking depends largely on the elastic modulus of concrete and reinforcement ratio of longitudinal steel, this effect can be represented by the reinforcement ratio and the strength of concrete which is also the function of the elastic modulus.

(d) Main Factors Defining the Shear Strength

According to the above *qualitative* consideration, the shear stress corresponding to the diagonal cracking depends on the following main factors:

- concrete strength f'_c
- reinforcement ratio of longitudinal steel p_w
- effective depth d

Besides these factors, the axial force and *a/d* ratio have also the strong effect.

(2) Shear capacity of RC beam without shear reinforcement

(a) Diagonal Cracking Capacity

As described earlier, the diagonal tension failure occurs immediately after the diagonal crack is formed. Therefore, the shear stress at the diagonal cracking *can be assumed* to be the ultimate shear strength in the case of the diagonal tension failure.

From numerous experimental data on the shear strength of RC beams *without shear reinforcement*, the empirical equation was proposed by Okamura and Higai (*Okamura & Higai's equation*) in 1980. Based on this equation, the modification has been made to incorporate *the size effect* directly in 1986. This modification was proposed by Niwa and Okamura. This revised equation has been adopted into JSCE Shear Design Specification.

$$V_c = 0.20 f'^{1/3}_c p_w^{1/3} d^{-1/4} \left[0.75 + \frac{1.4}{a/d} \right] b_w d \quad (2.55)$$

where, f'_c : compressive strength of concrete (N/mm^2)

p_w : longitudinal reinforcement ratio (%) $= \dfrac{100A_s}{b_w\, d}$

d : d [m]

Eq.(2.55) is an *empirical* equation. However, the accuracy is quite high. Predicted size effect is almost similar to the numerical result by the nonlinear fracture mechanics.

2.3.3 Beams with Shear Reinforcement

(1) Classical Truss Analogy

Shear reinforcement is called as the web reinforcement according to its function. This shear reinforcement can be classified as *stirrup or bent up bar* in the case of beams, or *tie or spiral reinforcement* in the case of columns (Fig. 2.16).

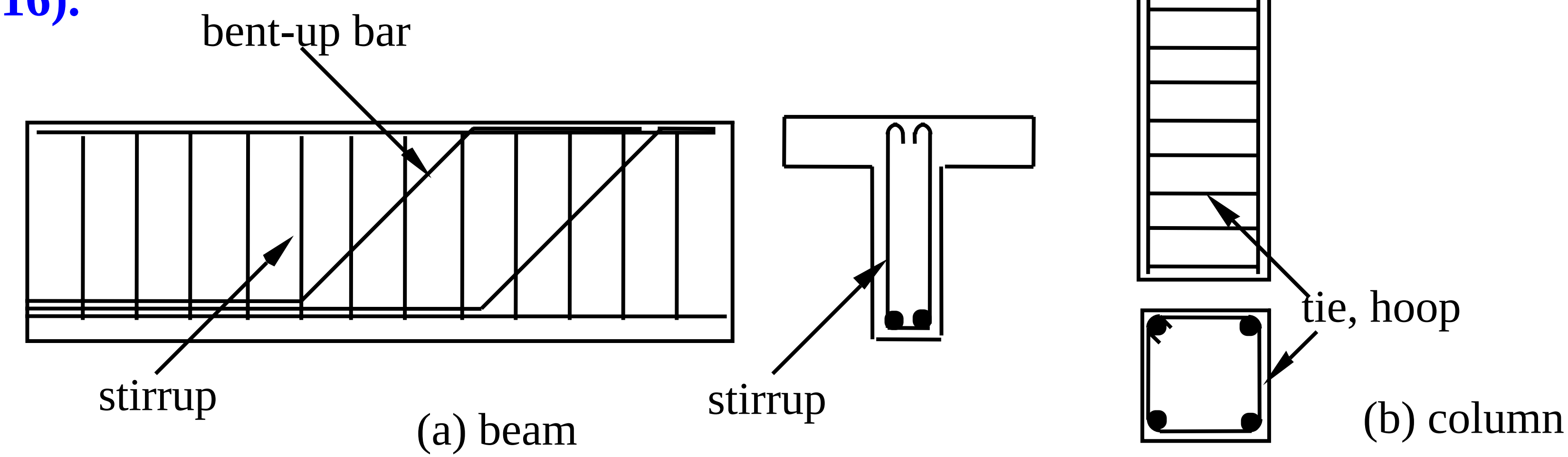

Fig. 2.16 Stirrup, bent-up bar, and tie bar (various types of shear reinforcement)

To determine the effect of the shear reinforcement, *the truss analogy* was proposed by Ritter-Moersh. This theory assumes that a RC beam behaves as a truss. After diagonal cracks propagate, according to the flow of internal stresses, the truss can be composed of the following members: (Fig. 2.17)

- ***horizontal compression member*** **: flexural compression zone of concrete**
- ***horizontal tension member*** **: longitudinal tension steel**
- ***diagonal compression member*** **: compression zone of web concrete**
- ***inclined tension member*** **: shear reinforcement**

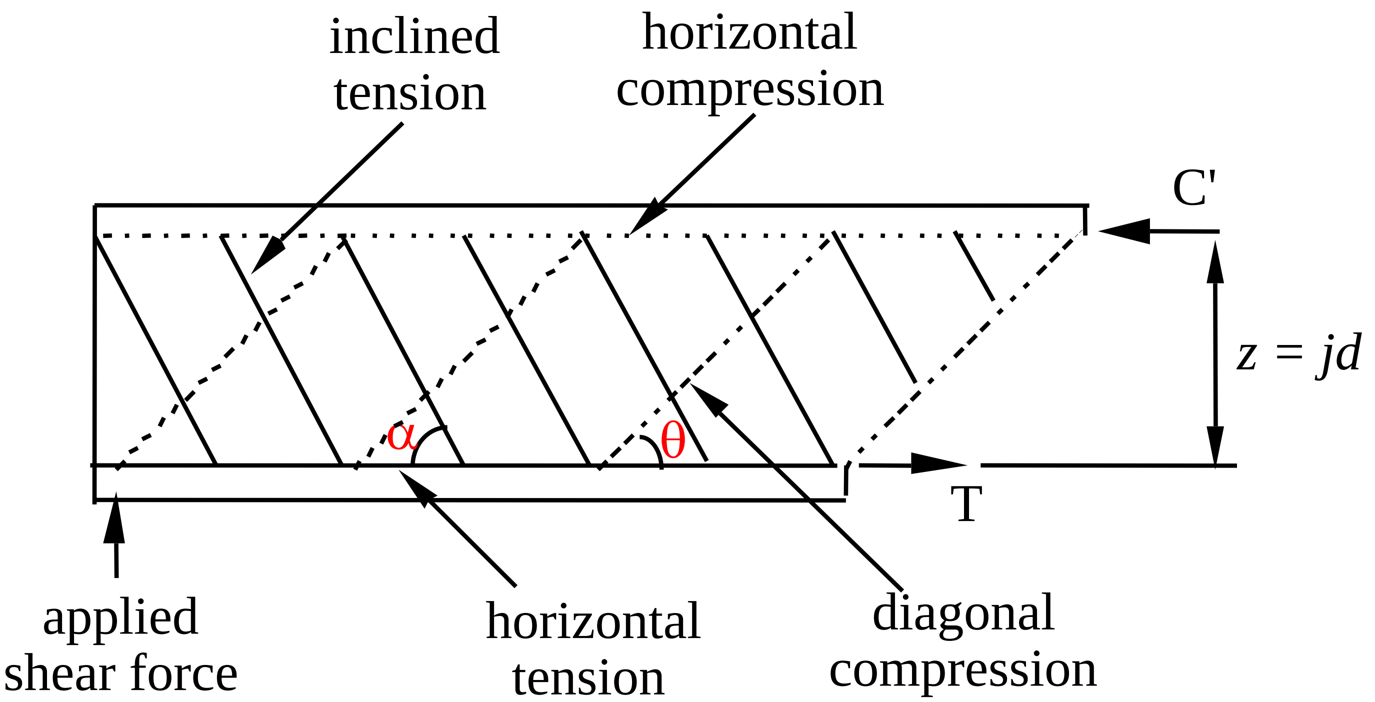

Fig. 2.17 Concept of truss analogy

In the classical truss analogy, the angle between the diagonal compression member and the horizontal line, θ is usually assumed to be 45 degrees. Therefore, the relation of shear force *V* and the tensile stress of shear reinforcement σ_w can be expressed as:

$$V = A_w \, \sigma_w \left(\sin\alpha + \cos\alpha\right)\frac{z}{s} \qquad \textbf{(2.56)}$$

where,		
where,	A_w	**: the area of one set of shear reinforcement with spacing *s***
	s	**: the spacing of shear reinforcement**
	z	**: distance between the horizontal compression and tension members (= jd)**
	α	**: the angle between shear reinforcement and the member axis**

This classical truss analogy is very simple and *objective*. However, according to the experimental observation, it is admitted that it cannot predict the actual shear behavior of RC beams accurately. To fix this problem, *the modified truss analogy* has been proposed and used in various design codes for the shear design of RC beams.

(2) Modified Truss Analogy

The relationship of applied shear force and tensile stress in stirrups shows that the tensile stress in stirrups is much smaller than that predicted by the classical truss analogy. In other words, after the diagonal cracking, it can be considered that the shear force is divided into two components, such as

(a) shear force carried by the shear reinforcement

(b) shear force carried by another mechanism than shear reinforcement (contribution of concrete)

$$V = V_c + V_s \qquad \textbf{(2.57)}$$

where, V **: applied shear force**

V_s **: the contribution of shear reinforcement**

V_c **: the contribution of concrete. This is approximated to be the shear capacity at the diagonal cracking from Eq. (2.55).**

At the ultimate stage, if we assume the yielding of shear reinforcement, V_s **can be obtained from Eq. (2.56) as follows:**

$$V_s = A_w f_{wy} (\sin\alpha + \cos\alpha)\frac{z}{s} \qquad \textbf{(2.58)}$$