2.3 Shear Capacity of RC Beam

2.3.1 Formation of Diagonal Cracks due to Shear

The so-called shear failure is one of the failure modes of RC structures of which the mechanism is much different from flexural failure. In actual RC structures, there is a combination of forces such as shear and flexural moment, axial force, torsional moment, and their failure modes are very complicated.

The shear failure follows a formation of diagonal cracks. It is brittle failure compared with flexural tension failure. Therefore, in the case of design involving the ductility of structures such as seismic design, this type of failure has to be avoided by assigning the safety factor greater than that for flexural failure.

(1) Principal tensile stress in an elastic beam

In the case of simply supported beam subjected to two-points loading, the moment and shear distribution is such that the moment is constant in the mid span and in two side spans, shear force is constant. These two side spans are called "shear span". For the elastic beam, the flexure stress σ, shear stress τ, and the principal tensile stress σ_1 are determined according to the beam theory. Since concrete material is weak in tension, the magnitude and direction of principal tensile stresses are important. At the location of zero shear stress, i.e., the extreme tension fiber, the principal tensile stress takes the horizontal direction. At the point of zero normal stress, i.e., the neutral axis, the principal tensile stress is equal to shear stress, and its direction is 45 degrees with respect to the member axis (Fig. 2.11).

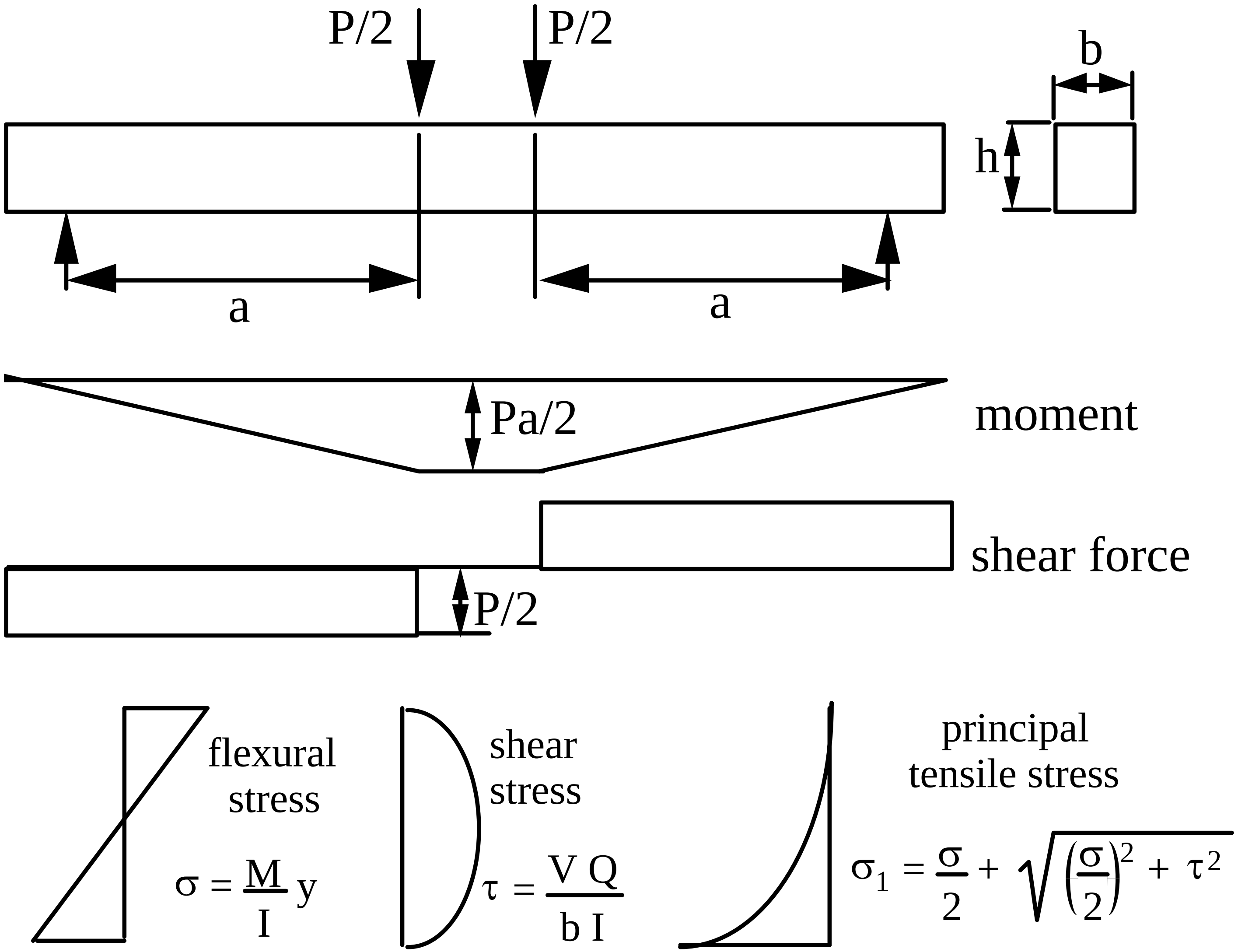

Fig. 2.11 Stress conditions in an elastic beam subjected to shear and moment

(2) Principal tensile stress in RC beam

Before cracking, RC beam can be considered as an elastic body. Hence, the maximum principal tensile stress occurs at the extreme tension fiber within the mid span, and its direction is parallel to the member axis. As this principal tensile stress increases and exceeds the tensile strength of concrete, crack occurs in the direction perpendicular to the direction of principal tensile stress. This crack is called *flexural crack*. After the flexural crack is formed, a RC beam is no longer considered to be an elastic body. However, since the tensile force is carried by longitudinal reinforcement, the state of stress even after flexural cracking is still similar to that of the principal stress of an elastic beam. When applied load is increased, the flexural crack propagates to the compression zone of the cross section. Also in both of side spans, the formulation of cracks occurs with an inclination with respect to the member axis. This crack is called *"diagonal crack"*.

When this diagonal crack occurs, the tensile force carried by concrete is released, and if reinforcement effective in the direction of principal tensile stress is not provided, the RC beam fails suddenly under the so-called *"diagonal tension failure"* mode.

Another type of failure following the diagonal crack is *"shear compression failure"*. A RC beam can resist increasing loads after the diagonal crack. The stress state becomes like a compression arch formed by diagonal cracks. In this case the beam fails when this arch crushes under diagonal compression. These two types of failure modes depend largely on the shear span-effective depth ratio (a/d).

If the shear span-effective depth ratio is large, diagonal tension failure occurs, but when small, shear compression failure occurs. For the case of so-called *deep beam*, i.e., the shear span-effective depth ratio is very small (a/d < 1.0), the tied-arch shear resisting mechanism is formed as a compression strut joining the loading and support points, and this failure mode is called *deep beam failure* (Fig. 2.12). Since all of these failures are preceded by diagonal cracking of web concrete, so they are called as *"shear failure"*.

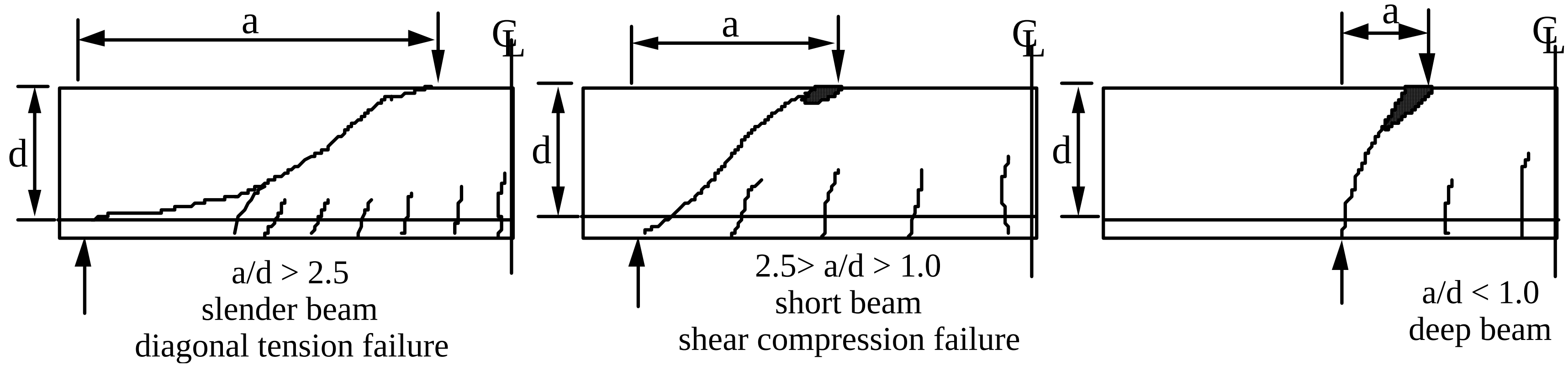

Fig. 2.12 Typical shear failure modes of RC beams

(3) Diagonal tensile stress and nominal shear stress

When flexural cracks and diagonal cracks are combined, the stress state of RC beams becomes very complicated and it is very difficult to evaluate the stress state exactly. The so-called *nominal shear stress* is commonly used to be an index of measuring principal tensile stresses.

This nominal shear stress is obtained based on the elastic theory of RC beam and the assumption of neglecting tensile stress of concrete. In this case, the concept of transformed section is used, in which the area of steel is transformed to be that of concrete by multiplying a factor n (= E_s/E_c, E_c, and E_s are elastic modulus of concrete and steel, respectively). An effective transformed section consists of compression zone of concrete and transformed area of steel. From the theory of elastic beam, the following relationships can be obtained (Fig. 2.13).

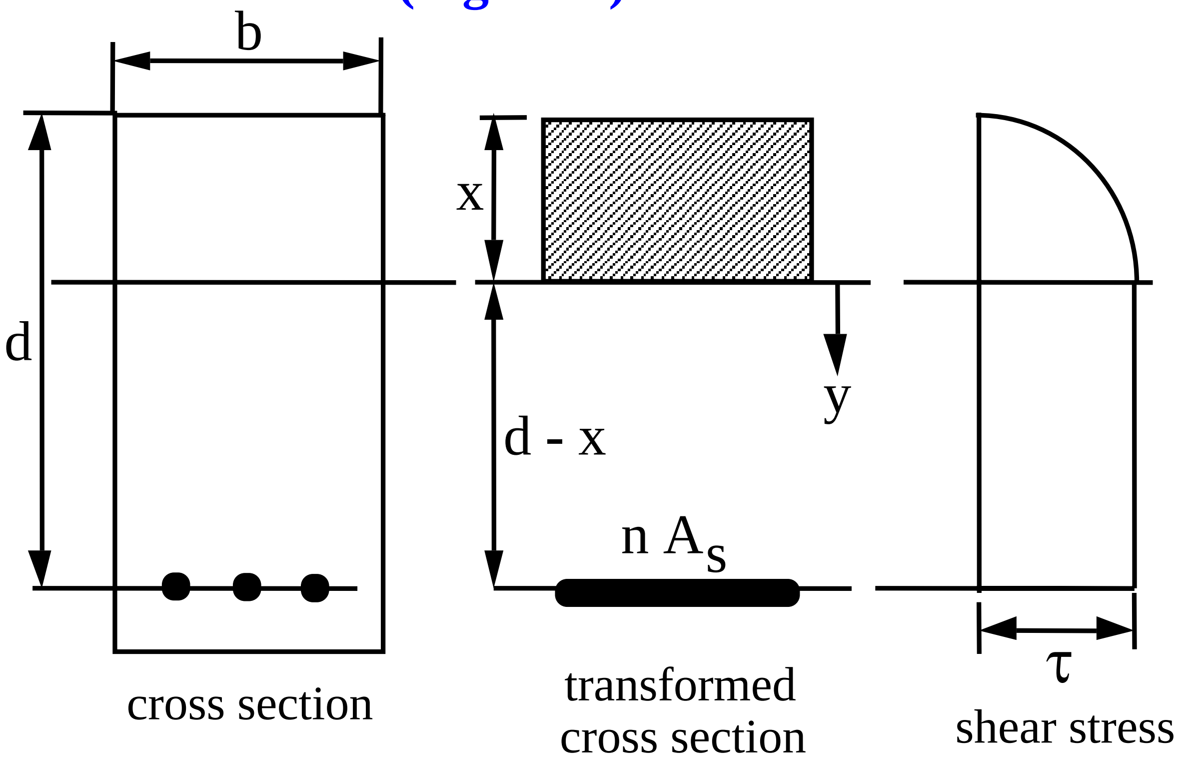

Fig. 2.13 Shear stress based on the elastic theory
(tensile stress in concrete is neglected)

$$\text{For} -x \le y \le 0,\ \ Q(y) = \int_y^{-x} b\,y\,dy = \frac{b}{2}\left(x^2 - y^2\right)$$

$$\text{For}\ 0 < y \le d - x,\ \ Q(y) = \int_0^{-x} b\,y\,dy = \frac{b\,x^2}{2}$$

$$I = I_{cr} = \frac{b\,x^3}{12} + b\,x\left(\frac{x}{2}\right)^2 + n\,As\left(d - x\right)^2 = \frac{b\,x^3}{3} + \frac{b\,x^2}{2}(d - x) = \frac{b\,x^2}{2}\left(d - \frac{x}{3}\right) = \frac{b\,x^2}{2}\cdot jd$$

$$\text{For}\ -x \le y \le 0,\ \ \tau(y) = \frac{V\,Q(y)}{b\,I_{cr}} = \frac{V\frac{b}{2}\left(x^2 - y^2\right)}{b\frac{b\,x^2}{2}jd} = \frac{V}{b\,jd}\left[1 - \left(\frac{y}{x}\right)^2\right]$$

$$\text{For}\ 0 < y \le d - x,\ \ \tau(y) = \frac{V\,Q(y)}{b\,I_{cr}} = \frac{V\frac{b\,x^2}{2}}{b\frac{b\,x^2}{2}jd} = \frac{V}{b\,jd} \qquad \textbf{(2.54)}$$

where,

$$jd = d - \frac{x}{3},\quad x = \frac{-n\,A_s + \sqrt{\left(n\,A_s\right)^2 + 2\,n\,A_s\,b\,d}}{b}$$