2.2 Capacity of RC Section Subjected to Combined Flexural Moment and Axial Force

2.2.1 General Method to Calculate Stresses in RC Section with Axial Force

In principle, the behavior of RC sections in the case of pure flexure, and combined flexure and axial force are almost the same. Hence, the similar assumptions are also applied to calculate the stresses in RC section subjected to combined flexural moment and axial force.

In the case of tensile stresses arising inside the section, the same equations Eqs.(2.3)-(2.13) can be used with a modification of Eqs.(2.12) and (2.13) which include the external axial force *N* into the equilibrium of force and moment.

In the case of no tensile stresses inside the section, the compressive strain of concrete and steel at arbitrary points are expressed by the extreme compressive fiber strains at the top and the bottom ε'_{cc} and ε'_{ct} as follows (Fig. 2.6):

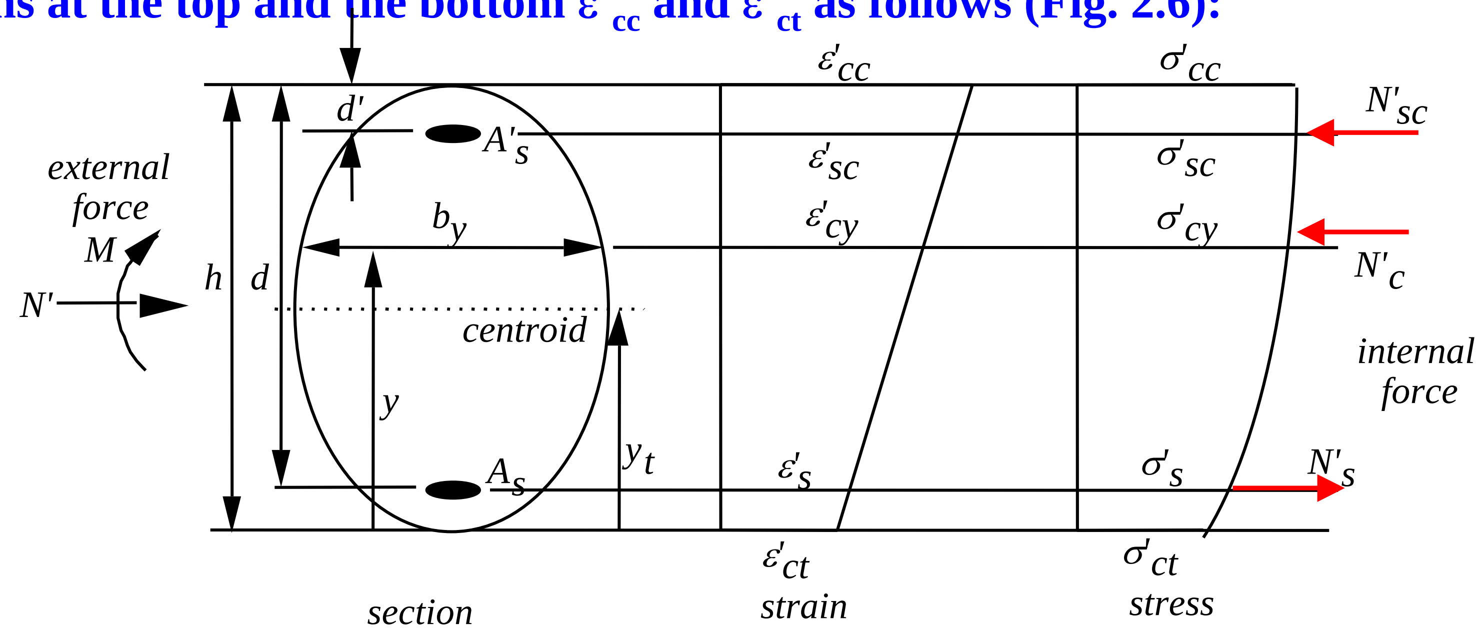

Fig. 2.6 Stress and strain distribution when RC section is subjected to combined moment *M* and axial force *N'*

$$\varepsilon'_{cy} = \varepsilon'_{ct} + \left(\varepsilon'_{cc} - \varepsilon'_{ct}\right)\frac{y}{h} \qquad \textbf{(2.41)}$$

$$\varepsilon'_{s} = \varepsilon'_{ct} + \left(\varepsilon'_{cc} - \varepsilon'_{ct}\right)\frac{h-d}{h} \qquad \textbf{(2.42)}$$

$$\varepsilon'_{sc} = \varepsilon'_{ct} + \left(\varepsilon'_{cc} - \varepsilon'_{ct}\right)\frac{h-d'}{h} \qquad \textbf{(2.43)}$$

The relations between resultant forces and stresses are expressed as

$$N'_{c} = \int_0^h b_y \cdot \sigma'_{cy}\, dy = \frac{h}{\varepsilon'_{cc} - \varepsilon'_{ct}} \int_{\varepsilon'_{ct}}^{\varepsilon'_{cc}} b_y \cdot f_1\left(\varepsilon'_{cy}\right) d\varepsilon'_{cy} \qquad \textbf{(2.44)}$$

$$N'_{s} = A_s\, \sigma'_{s} = A_s\, f_2\left(\varepsilon'_{s}\right) = A_s\, f_4\left(\varepsilon'_{cc}, \varepsilon'_{ct}\right) \qquad \textbf{(2.45)}$$

$$N'_{sc} = A'_s\, \sigma'_{sc} = A'_s\, f_3\left(\varepsilon'_{sc}\right) = A'_s\, f_5\left(\varepsilon'_{cc}, \varepsilon'_{ct}\right) \qquad \textbf{(2.46)}$$

From the equilibrium condition for force and flexural moment,

$$N'_{c} + N'_{sc} + N'_{s} = N' \qquad \textbf{(2.47)}$$

$$\left(\frac{h}{\varepsilon'_{cc} - \varepsilon'_{ct}}\right)^2 \int_{\varepsilon'_{ct}}^{\varepsilon'_{cc}} b_y\, f_1\left(\varepsilon'_{cy}\right)\left(\varepsilon'_{cy} - \varepsilon'_{ct}\right) d\varepsilon'_{cy} + N'_{sc}\left(h - d'\right) + N'_{s}\left(h - d\right) = M + N' y_t \qquad \textbf{(2.48)}$$

There are two equations, i.e., Eqs.(2.47) and (2.48) to solve for four unknowns: M, N', ε'_{cc}, *and* ε'_{ct}. By giving two of them, other two can be calculated.

2.2.2 Case of concentric axial compressive force

For the section subjected to only axial compressive force at the centroid of section, the compressive stress distribution of which is uniform across the section.

The relationship of the axial compressive force N' and compressive strain is shown in Fig. 2.7.

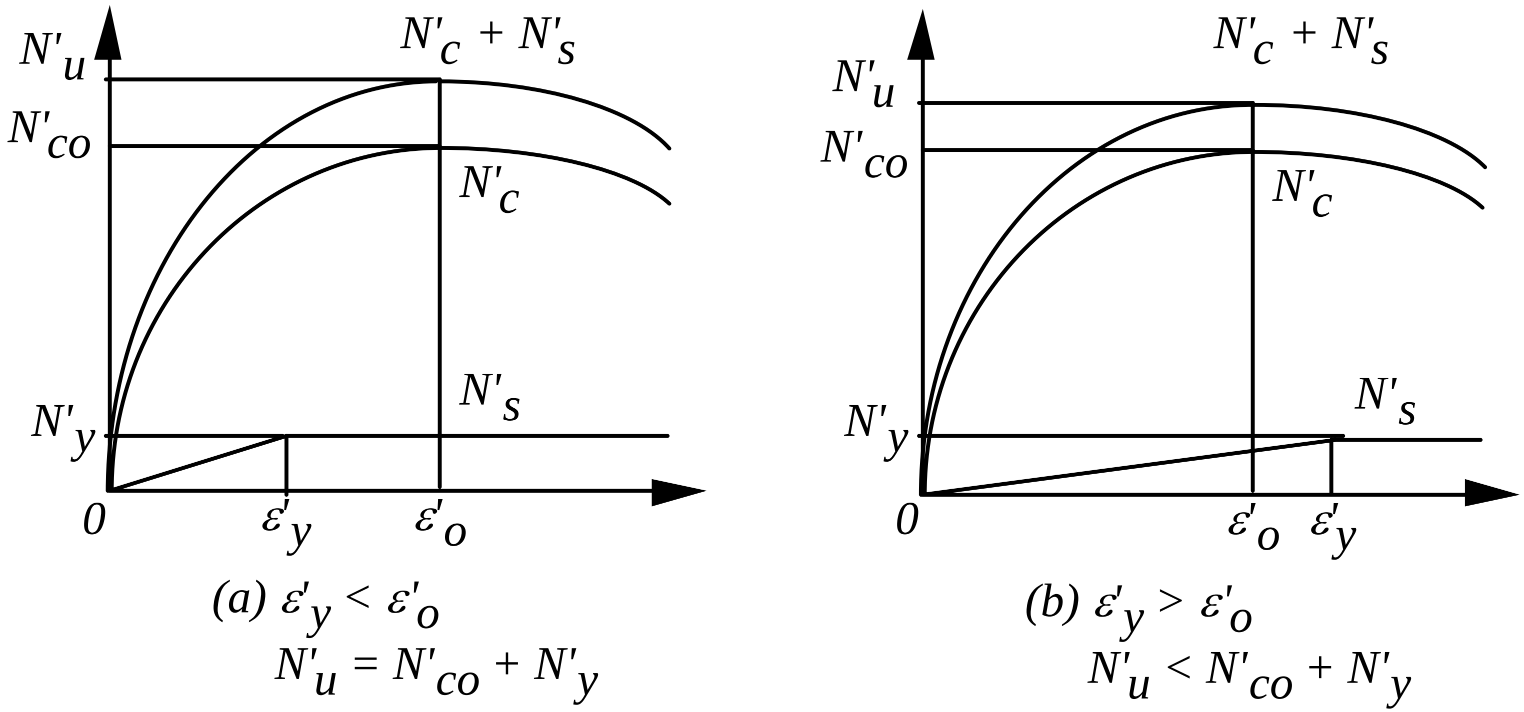

Fig. 2.7 Relationship of concentric axial force N' and axial compressive strain ε'

N' is equal to the sum of compressive force carried by concrete *N'c* and that carried by steel *N's*. The relations of these forces and compressive strain are equivalent to the stress-strain relations. When the yield strain of steel $\varepsilon'y$ is less than the strain of concrete at the peak, $\varepsilon'o$, the axial compressive capacity of RC section N'u is the sum of axial compressive capacity of concrete N'co and the yield force of steel N'y .

$$N'_u = 0.85 A_c f'_c + A_s f_y \qquad (2.49)$$

where, Ac : the area of concrete

As : the area of steel

However, when $\varepsilon'y$ is greater than $\varepsilon'o$, N'u is less than the sum of the capacity of both materials. Except for the case of high strength steel, Eq.(2.49) is applicable for most of the cases.

It is noted that the above discussion is for the case of no buckling of steel. To prevent the buckling, either increasing concrete cover or using tie or spiral reinforcement must be inevitable. The use of tie or spiral reinforcement is required in RC columns due to another reason. At the ultimate state of RC columns, the concrete covering axial reinforcement is suddenly crushed and beyond the peak resistance the load decreases very rapidly. This type of failure is very brittle and is not preferable in the design. The spiral reinforcement confines the lateral strain of concrete due to axial compression, which helps to increase both ductility and capacity of RC columns.

2.2.3 Case of eccentric axial compressive force

In the case that axial compressive force is not applied at the centroid of the section, the stress distribution is non-uniform. Capacity of the section decreases, when the distance between the applying point of force and the centroid which is called "eccentricity, *e*" becomes larger.

When *e* is small, the steel strain in the flexural tension zone ε_s is always in compression. As *e* becomes larger, even when ε_s is compression in the beginning, it becomes tension at the ultimate state. For the larger value of *e*, the steel strain ε_s reaches the yield point at the ultimate state.

In the case of yielding of steel, the deformation of RC section, i.e., the rotation of section, becomes large as the eccentricity increases. The behaviors of a RC section with large eccentricity are similar to those of RC members under pure flexure, i.e., $e/h=\infty$.

2.2.4 Interaction Curve

The capacity of RC section subjected to combined flexure and axial force depends on the ratio of applied moment and axial force. At the ultimate state, the relation between the flexural capacity *Mu* and the axial compressive capacity *N'u* is called as *"interaction curve"*. When a point of combination of flexural moment and axial force is on or outside of this interaction curve, the RC section fails.

For the constant eccentricity, the straight line drawn from the origin expresses the proportional increase of flexural moment and axial force, and the intersection point between this straight line and the interaction curve indicates the capacity of the section (Fig. 2.8).

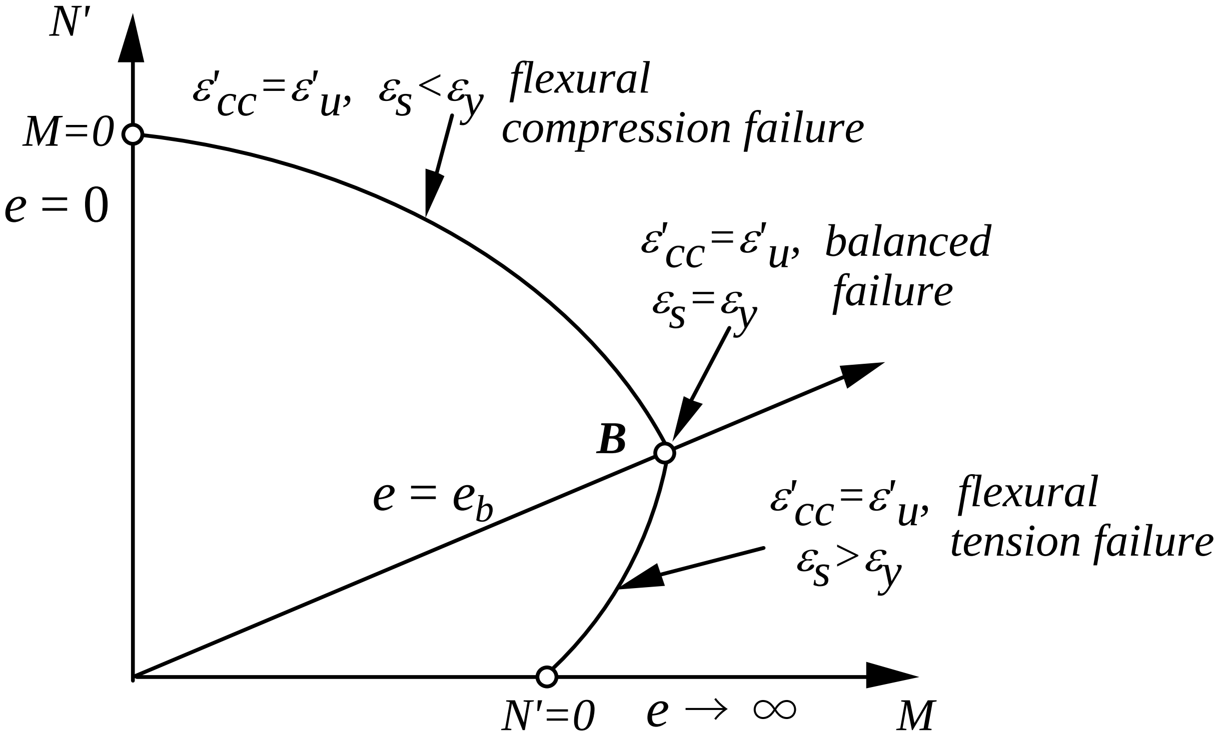

Fig. 2.8 Interaction curve for flexural moment and axial compressive force

At point B in Fig. 2.8, the yield strain of steel and the ultimate compressive strain of concrete occur simultaneously at the ultimate state, which corresponds to the *balanced failure*. The axial force, the flexural moment, and the eccentricity at the point B are called as "balanced axial force N'_b", "balanced moment M_b", and "balanced eccentricity e_b", respectively. When the ratio of applied flexural moment and axial force (M/N') is less than e_b, the failure becomes *flexural compression failure mode* and when M/N' > e_b, the failure becomes *flexural tension failure mode.*

In the case of rectangular section with height h, width b, effective depth d, and symmetric reinforcement (As = A's, h - d = d'), the interaction curve between non-dimensional flexural moment ($M_u/bh^2f'_c$) and non-dimensional axial force (N'_u/bhf'_c) depends on the values of d/h and p f_y/f'_c, where p : reinforcement ratio (=A_s/bd), f_y: yield strength of steel, f'_c: compressive strength of concrete.

2.2.5 Balanced Failure in Case of Combined Flexure and Axial Force

In the case of pure flexure, the balanced failure occurs when the reinforcement ratio is equal to the balanced reinforcement ratio. However, in the case of combined flexure and axial force, depending on the ratio of the combination of flexural moment and axial force, the balanced failure can occur *regardless of reinforcement ratio.*

For rectangular cross section with symmetric reinforcement, the location of the neutral axis from the compression fiber at the balanced failure can be expressed as follows:

$$x = \frac{\varepsilon'_u}{\varepsilon'_u + \varepsilon_y} d \qquad (2.50)$$

Normally it can be assumed that the compression steel has already yielded at the balanced failure. Moreover, if the reinforcement is symmetric, the compressive and tensile forces of reinforcing steel are the same. Therefore, the external compressive force (balanced force) N'_b becomes equal to the resultant compressive force of concrete N'_c.

By using Whitney's equivalent stress block (Fig. 2.9),

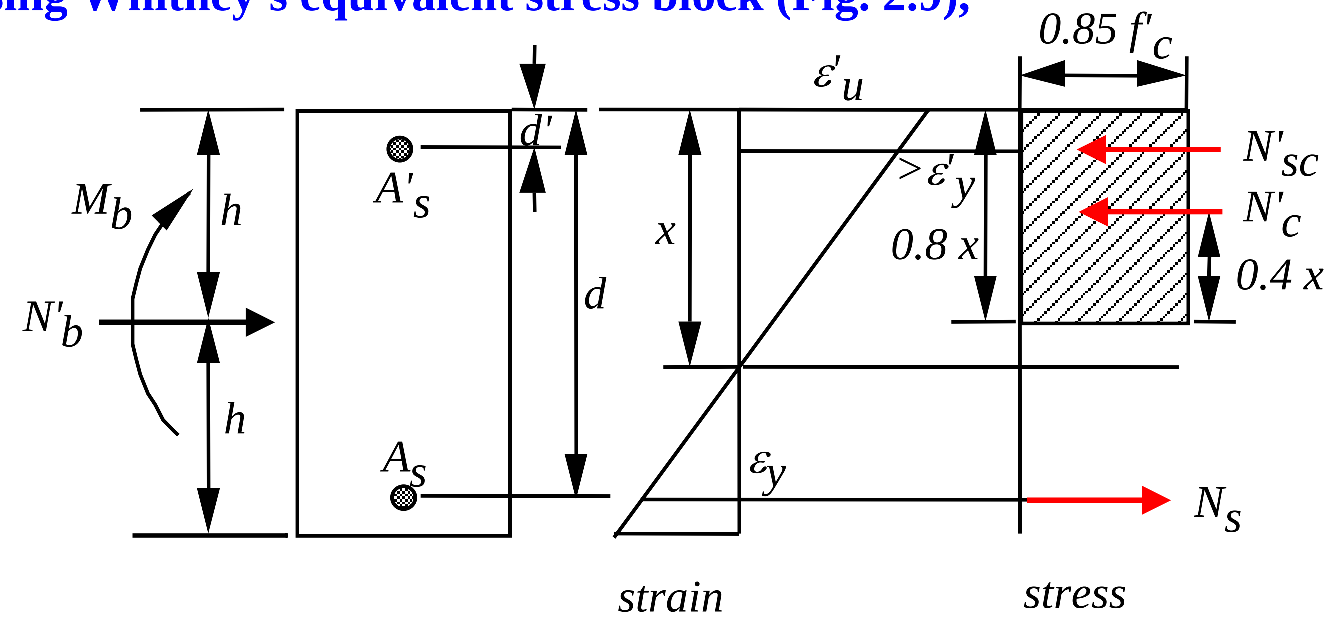

Fig. 2.9 Whitney's equivalent stress block

$$N'_b = N'_c = 0.68\, b\, x\, f'_c \quad \textbf{(2.51)}$$

By substituting Eq.(2.51) into Eq.(2.50), the following relationship can be obtained;

$$\frac{N'_b}{b\, h\, f'_c} = \frac{0.68\, d/h}{1 + \varepsilon_y / \varepsilon'_u} \qquad \textbf{(2.52)}$$

From the equilibrium of moment around the centroid axis,

$$M_b = N'_c\left(\frac{h}{2} - 0.4x\right) + N'_{sc}\left(\frac{h}{2} - d'\right) + N_s\left(d - \frac{h}{2}\right)$$

$$\frac{M_b}{N'_b h} = \frac{e_b}{h} = 0.5 - 0.4\frac{d/h}{1+\varepsilon_y/\varepsilon'_u} + \frac{1+\varepsilon_y/\varepsilon'_u}{0.68}\frac{p f_y}{f'_c}\frac{d-d'}{h} \qquad \textbf{(2.53)}$$

where,

$$p = \frac{A_s}{bd} = \frac{A'_s}{bd}$$

fy : the yield strength of reinforcing steel

In the case of ε_s=0 and $\varepsilon'_{sc}=\varepsilon'_y$, x can be obtained in the same way as Eq.(2.50) and the same procedures can be applied to obtain N'_u and M_u (Fig. 2.10).

2.2.6 Examination for limit states in the case of combined flexure and axial force

Similar to the case of flexure, the design capacity of the section is calculated by using the design stress-strain relationship. The design capacity can be obtained from the design interaction curve by using pf_{yd}/f'_{cd} instead of pf_y/f'_c.

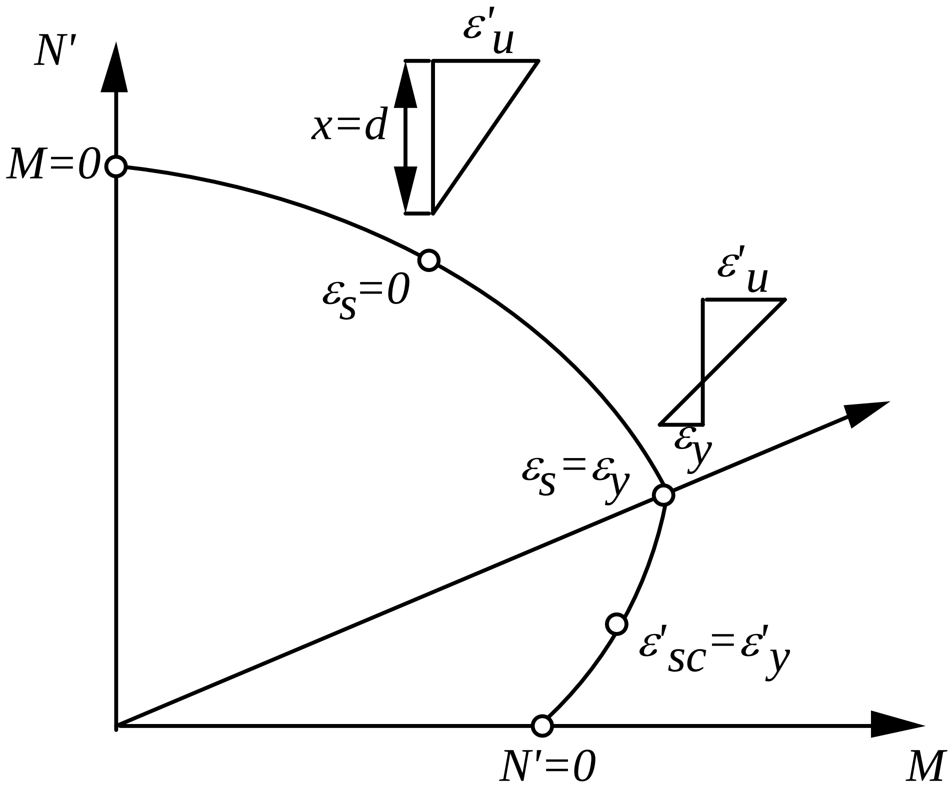

Fig. 2.10 Simple method for drawing interaction curve

Alternative method is to calculate the design flexural capacity M_{ud} based on the constant eccentricity e = M_d/N'_d where M_d : design externally applied flexural moment and N'_d : design externally applied axial force. This will not involve the whole interaction curve but only the related point on the interaction curve.