Fuzzy theory

1. Overview
2. Fuzzy sets
3. Operations for fuzzy sets
4. Fuzzy I/O
5. Fuzzy functions
6. Fuzzy rules
7. Fuzzy control

1 Overview

- Fuzzy theory began with a paper on “fuzzy sets”, written by Prof. L.A. Zadeh in 1965.
- Fuzzy sets are those sets whose boundary is not clear. Fuzzy logics are calculation procedures on fuzzy sets.
- A technology in which the whole system can be roughly defined, that is “fuzzy theory” was proposed.

ATTENTION

- Fuzzy system is deterministic. Neither stochastic nor ambiguous.

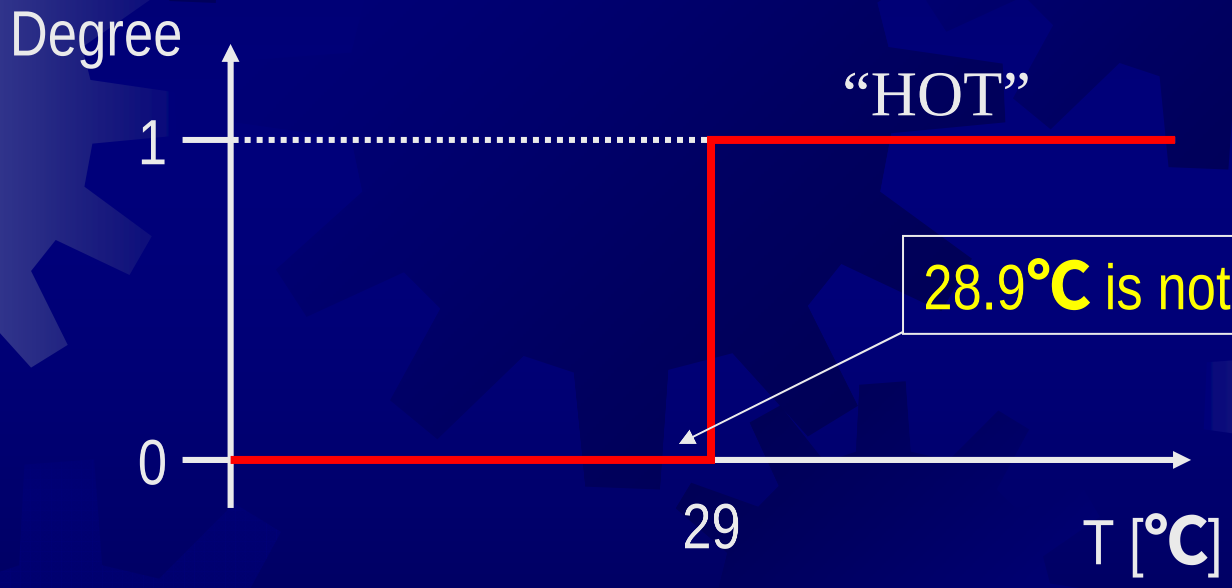
2 Fuzzy sets

- Ordinal set = Crisp set

- Fuzzy set

Crisp set: feeling "HOT"

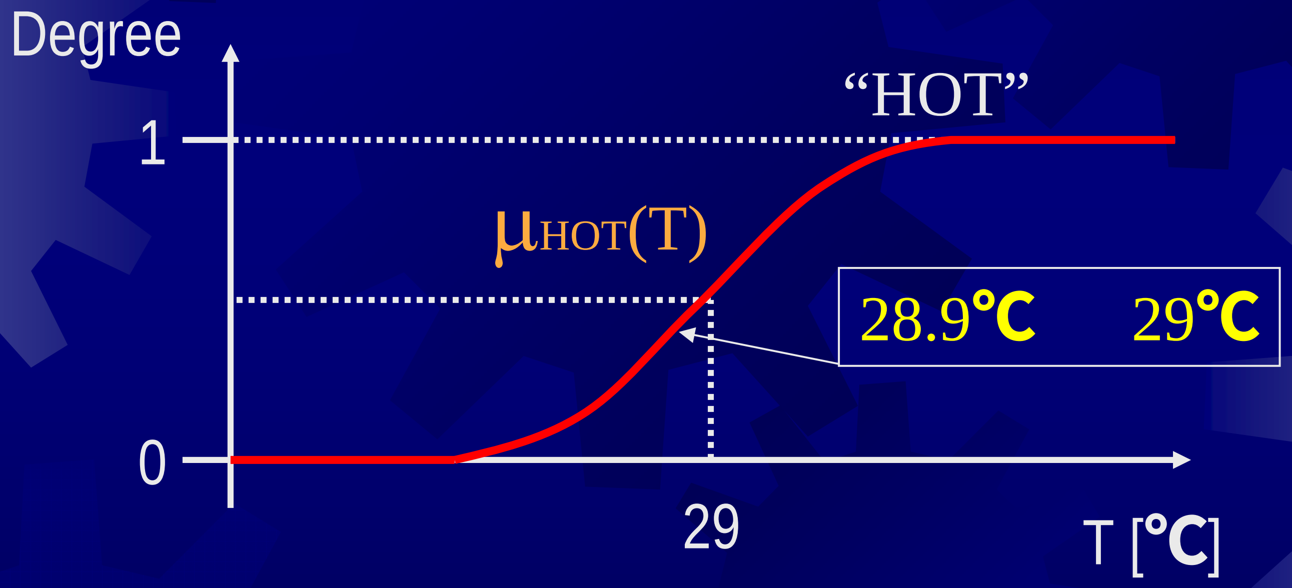
$$X_{HOT}(T) = \begin{cases} 1 & T \geqq 29 \\ 0 & T < 29 \end{cases}$$



HOT : {T | T ≧ 29°C }

Only Yes (degree=1) or NO (degree=0)

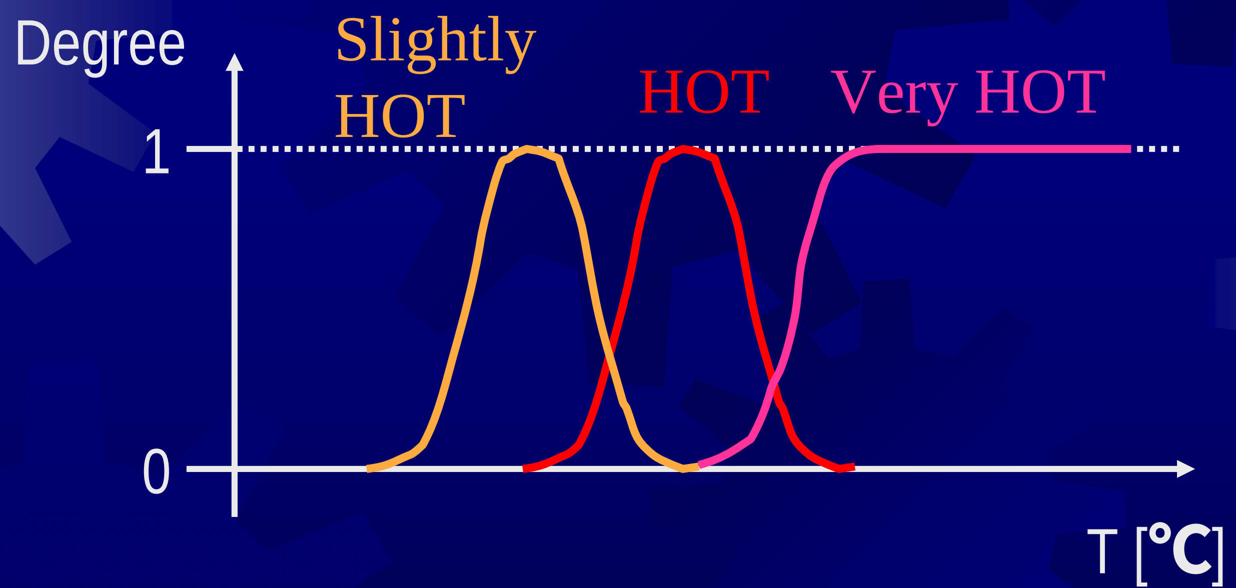
Fuzzy set: feeling "HOT"



Membership function $\mu_{HOT}(T)$ defines a fuzzy set

Definition of many fuzzy sets

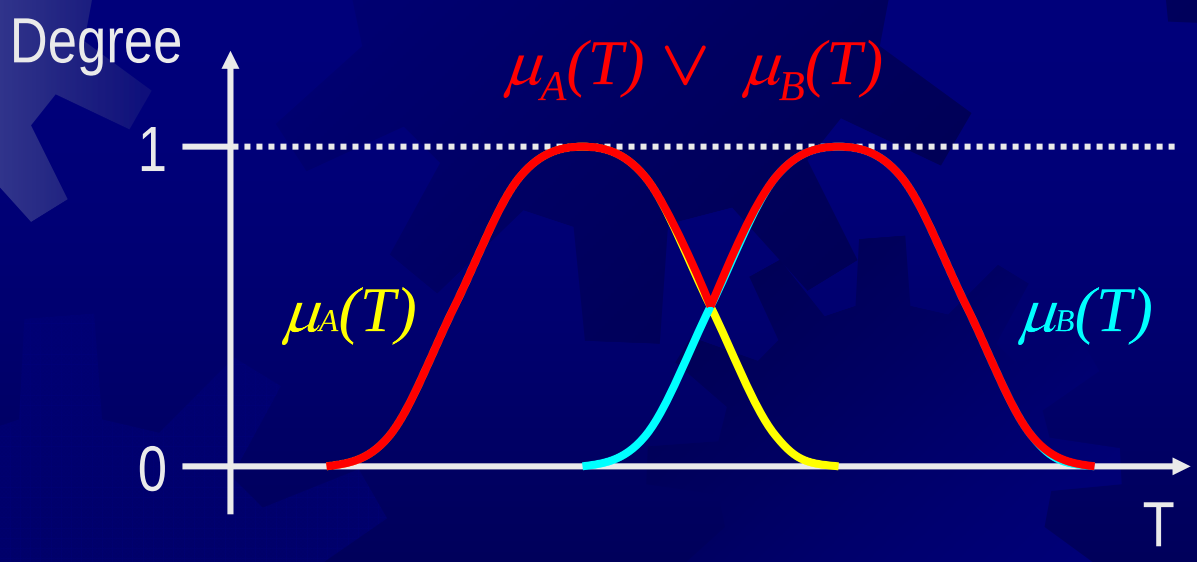
- Only by defining each membership function, we can express many status.



3 Operations for fuzzy sets

$$\mu_A(T) \vee \mu_B(T) = \max\{\mu_A(T), \mu_B(T)\}$$

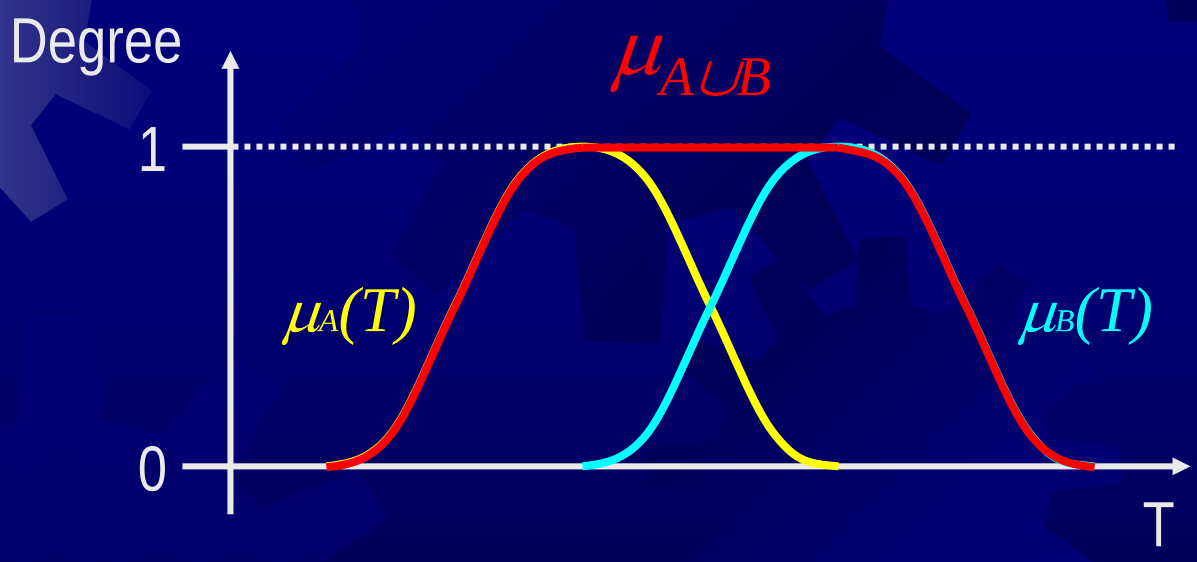
Similar to “OR” logic.



Set of maximum values in $\{\mu_A(T), \mu_B(T)\}$

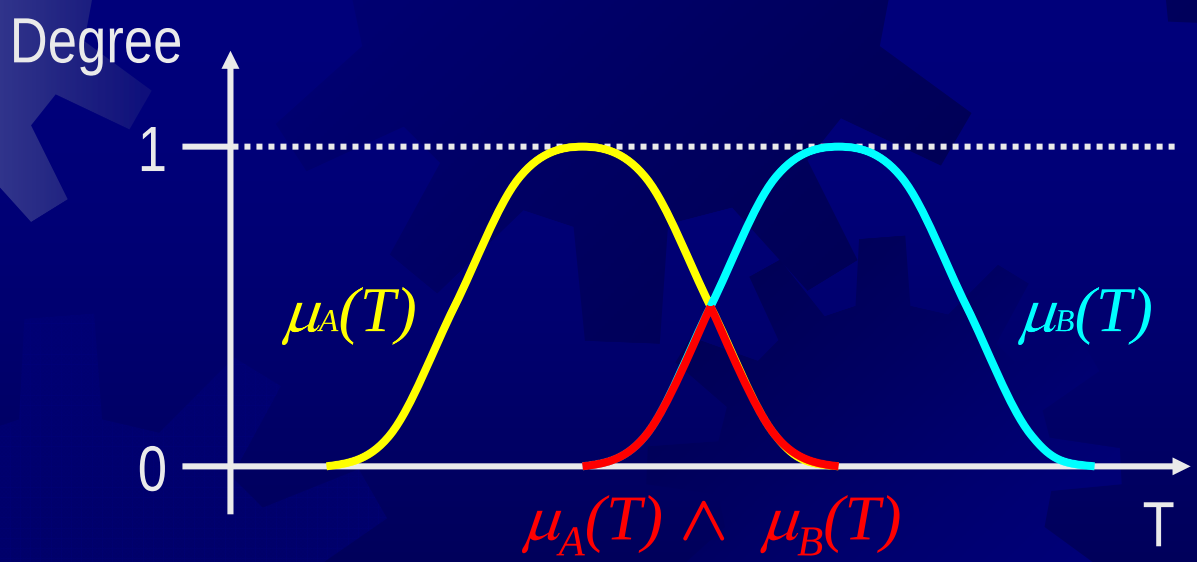
$$\mu_{A \cup B} = \min\{1, \mu_A(T) + \mu_B(T)\}$$

Saturated sum of μ_A and μ_B



$$\mu_A(T) \wedge \mu_B(T) = \min\{\mu_A(T), \mu_B(T)\}$$

Similar to “AND” logic.



4 Fuzzy Input and Output

Let us input a fuzzy set into a crisp function

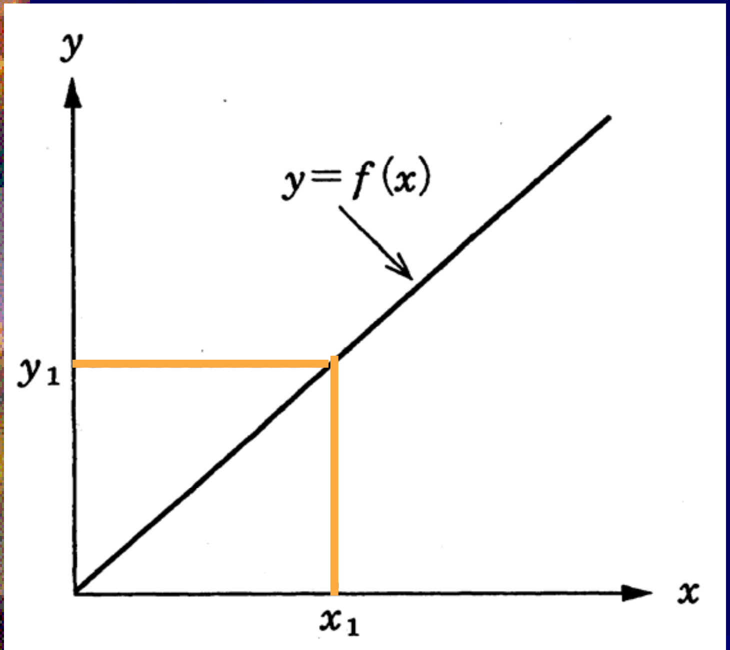
A crisp function $y = f(x)$ e.g. $y = 2x+1$

Let us input a fuzzy set A $y=f(x),\ x \in A$

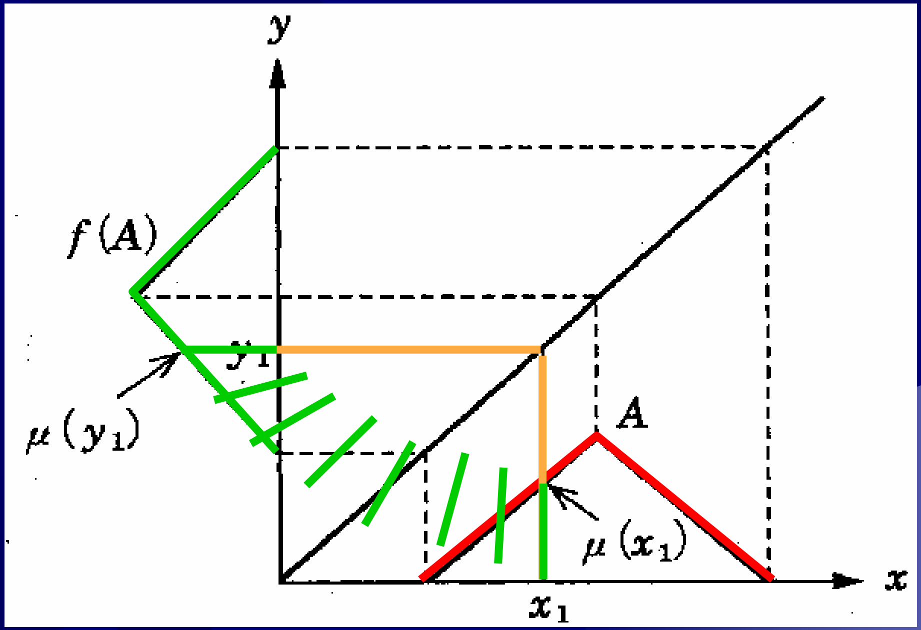
Output of the function becomes a membership function.

Examples will appear on the next slide.

Examples (1)



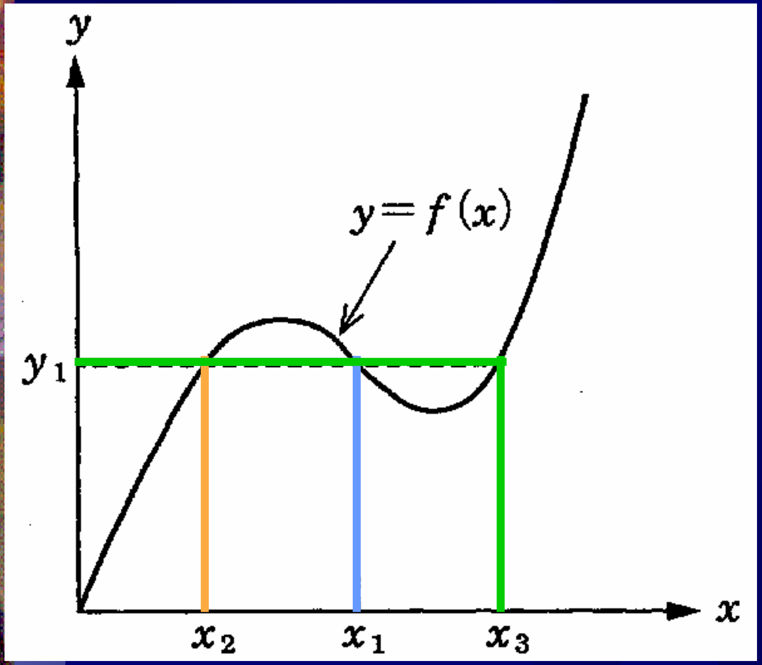
Crisp function   y=x



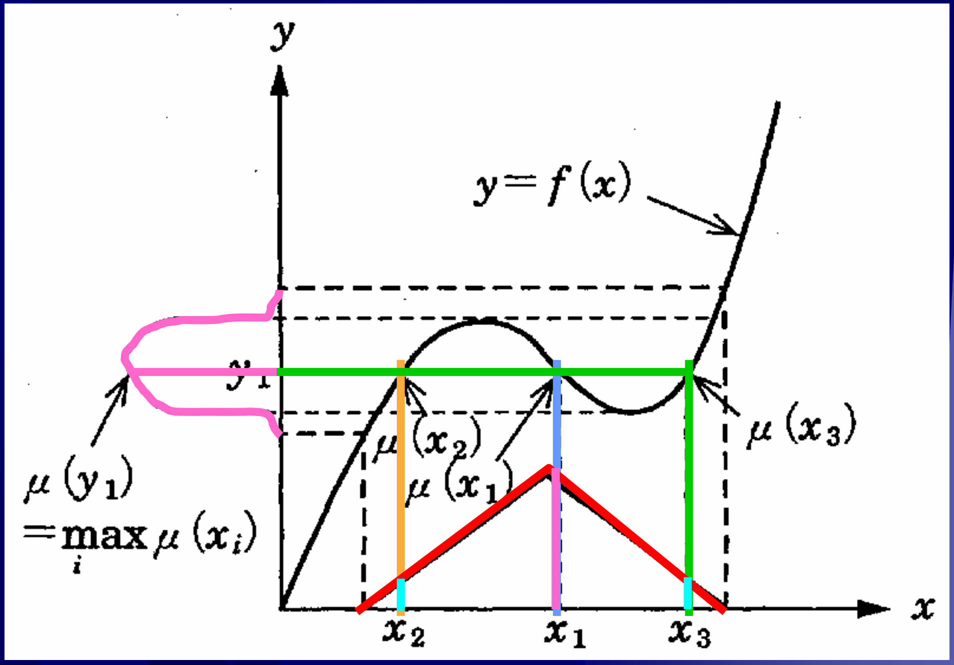
Input a fuzzy set A

Membership function (output)

Examples（2）



Crisp inputs

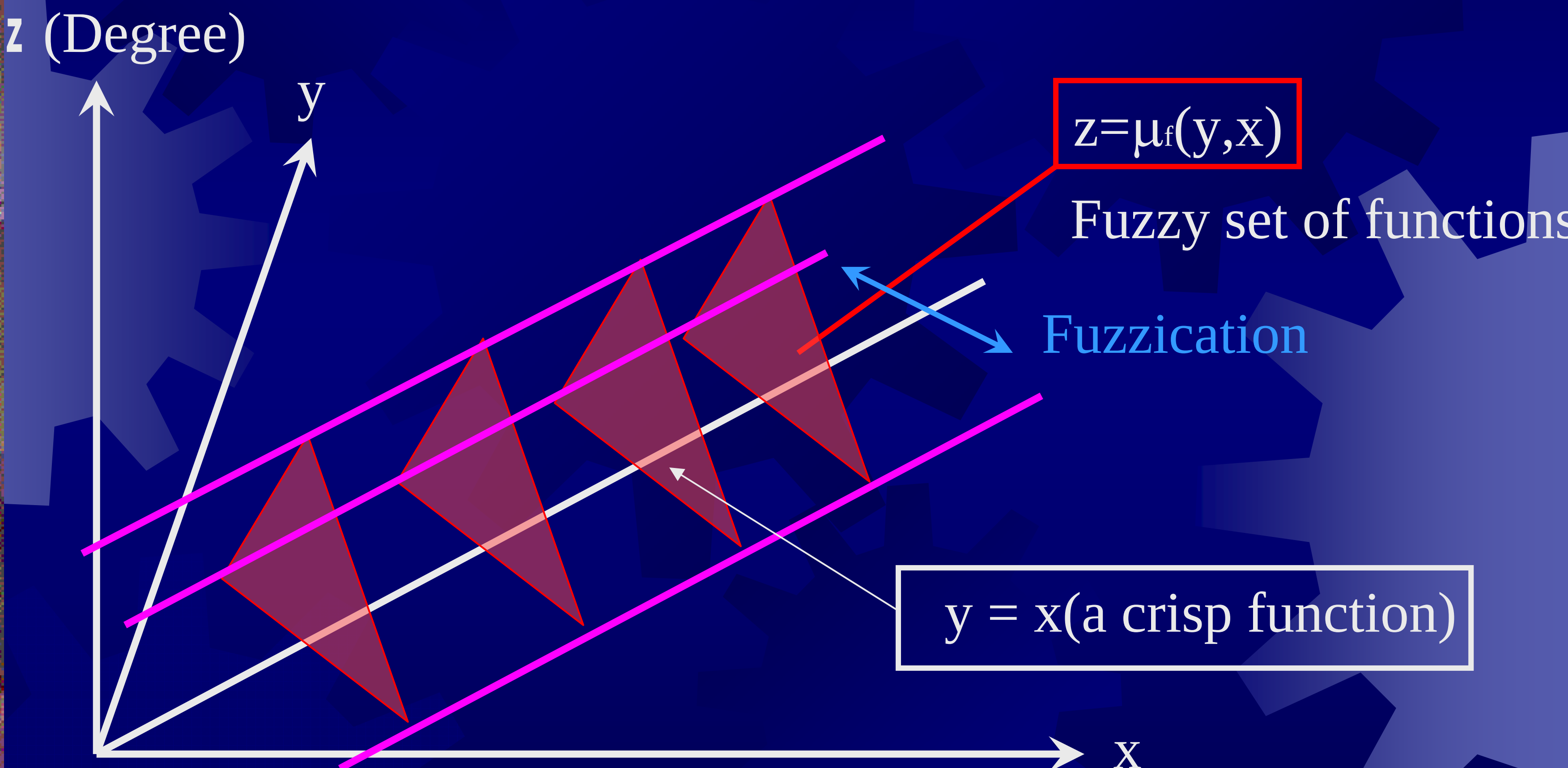


Input a fuzzy set

$\mu(y_1) = \max\{\ \mu(x_1), \mu(x_2), \mu(x_3)\ \}$

, where $y_1 = f(x_1) = f(x_2) = f(x_3)$

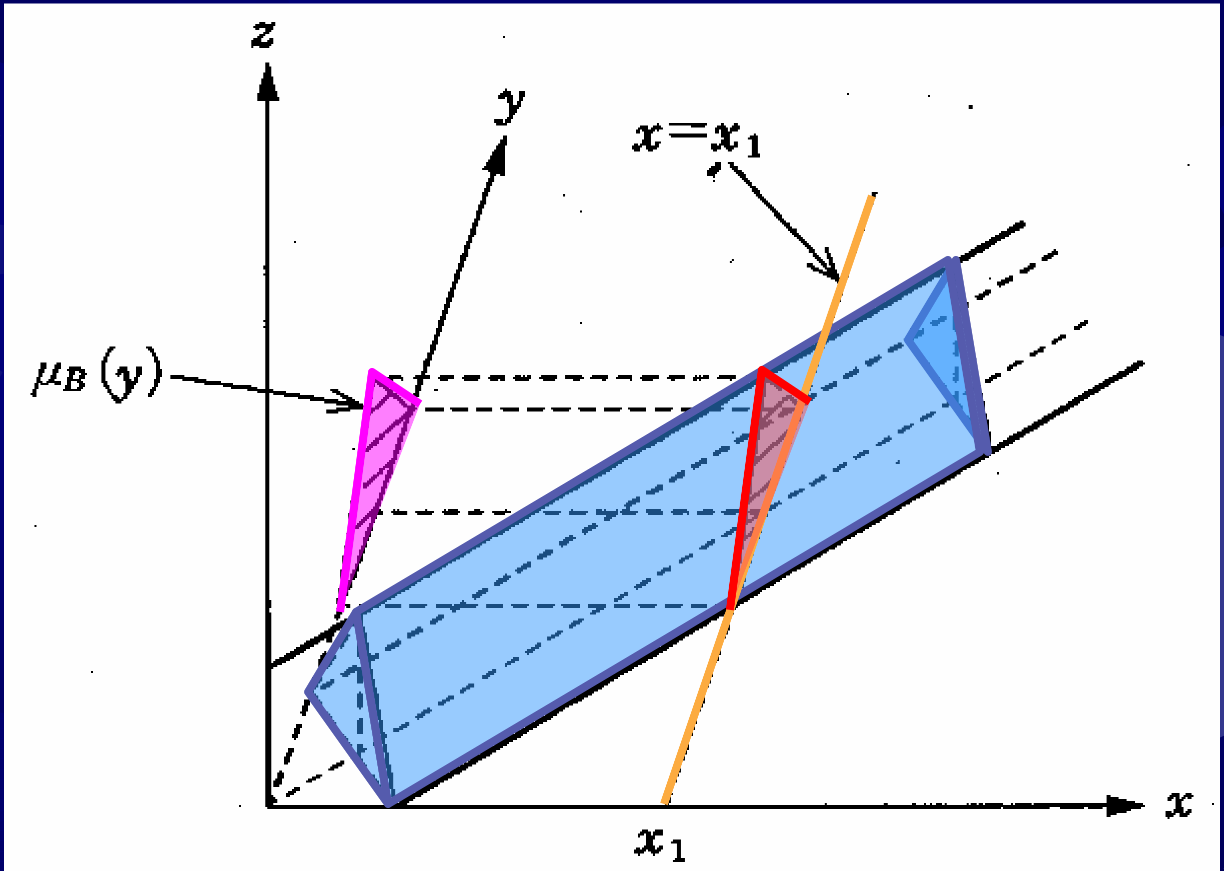
5 Fuzzy functions



A fuzzy function becomes a “cylinder”.

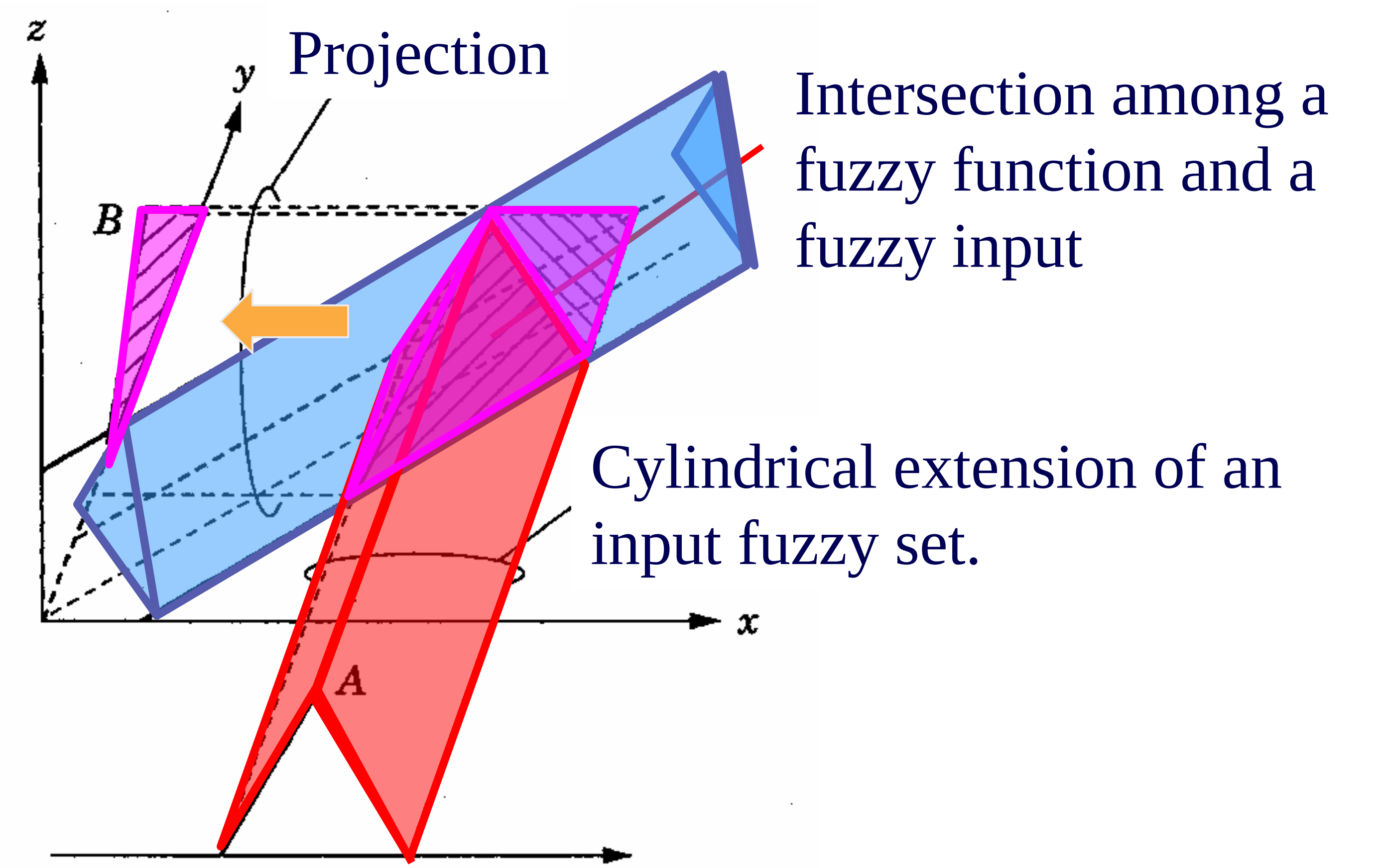
A crisp input to a fuzzy function

$$\boxed{\mu_B(y) = \mu_f(y, x_1)}$$



Output is obtained by an intersection among a fuzzy function and an input.

Input a fuzzy set to a fuzzy function



$$\mu_B(y) = \vee_{x \in X}\{\mu_f(y, x) \wedge \mu_A(x)\}$$

6 Fuzzy rules

Rules between fuzzy sets

IF x is $\underline{R}$ then y is $\underline{C}$

Crisp sets

IF x $\in$ $\underline{\mu_A}$ then y $\in$ $\underline{\mu_B}$

Fuzzy sets

Illustration of fuzzy rules

Projection

Intersection

A fuzzy rule: an intersection among cylindrical extensions of input and output fuzzy sets

Input A’ to the fuzzy rule

Input a fuzzy set to many fuzzy rules

Projections

Intersections

Two fuzzy rules: A1 and A2

Input a fuzzy set A'

7 Fuzzy control

- Control system of a “cooler”
- Input：temperature T [℃]
- Output：driving voltage V

- Designing fuzzy sets
- Defining fuzzy rules
- Defuzzication

Procedures of fuzzy control

Input T (crisp input)

↓

Output of fuzzy rules (fuzzy)

↓

Evaluate fitness

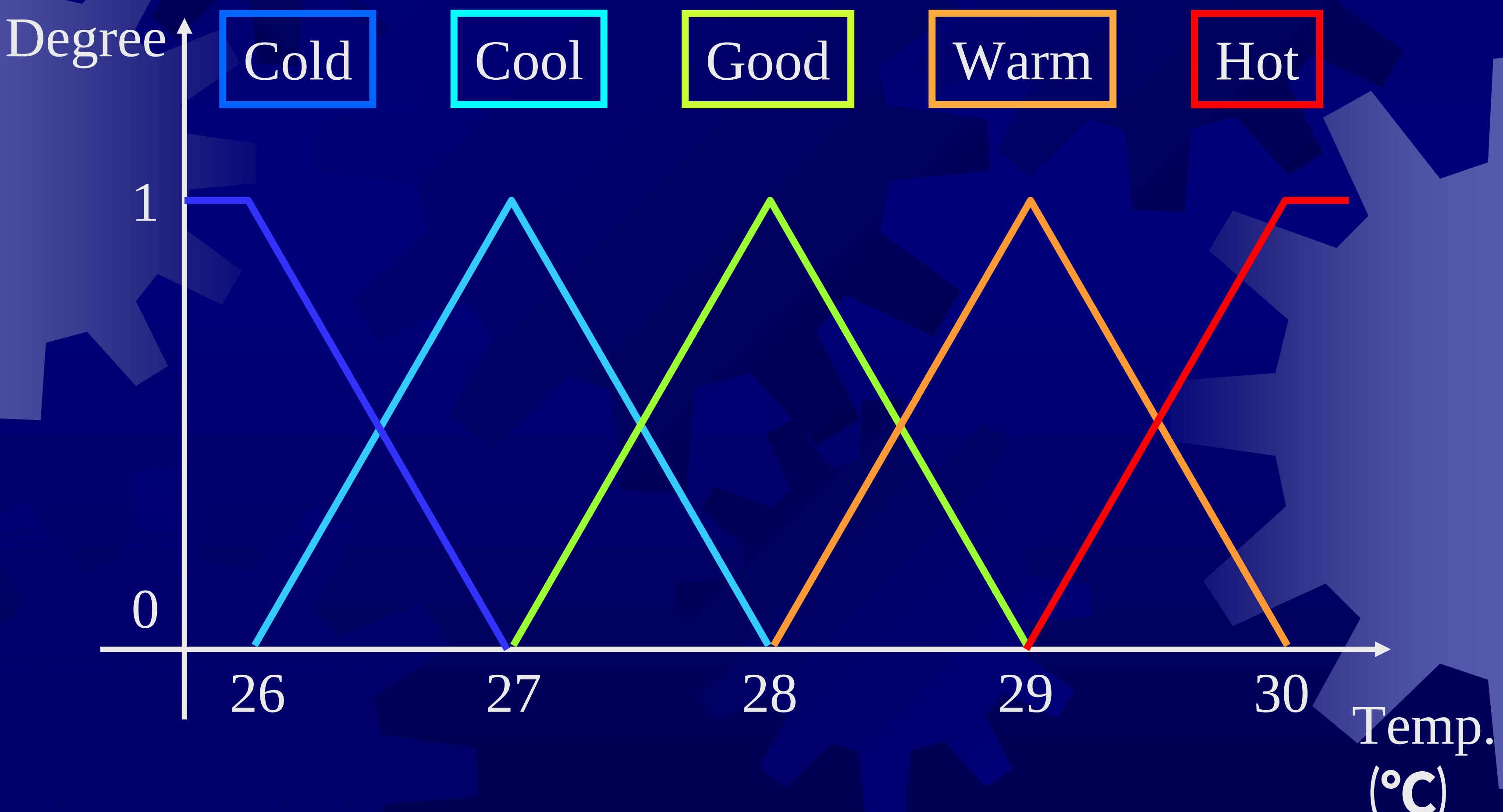
↓

Defuzzication (crisp output)

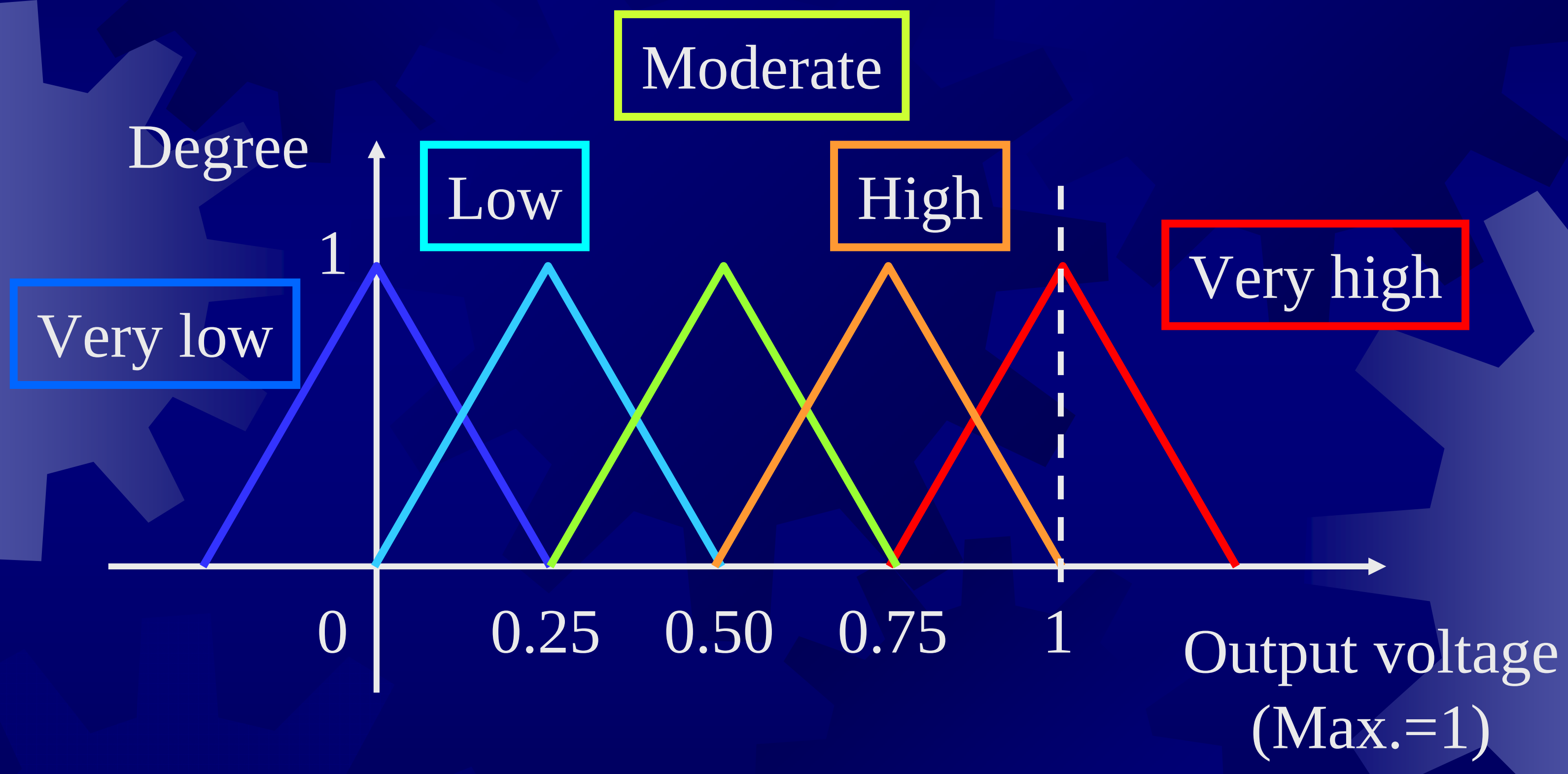
↓

Driving voltage (crisp)

Fuzzy sets for feelings



Fuzzy sets for outputs



Fuzzy rules

Defining relationship between inputs and outputs

Input (feeling)		*Output (voltage)*
Cold	➡	**Very low**
Cool	➡	**Low**
Good	➡	**Moderate**
Warm	➡	**High**
Hot	➡	**Very high**

Operation of fuzzy rules

If x_i then y_i

$$\Downarrow$$

if $x_i(\mu_{xi}(t))$ then $y_i(\mu_{yi}(v))$

"t" temp., "v" voltage

Calculation of output

For all rules (relationship between x_i and y_i)

$$\boxed{\mu_{\text{output}}(v) = \bigvee_{\text{i}}(\mu_{\text{yi}}(v) \wedge \mu_{\text{xi}}(t))}$$

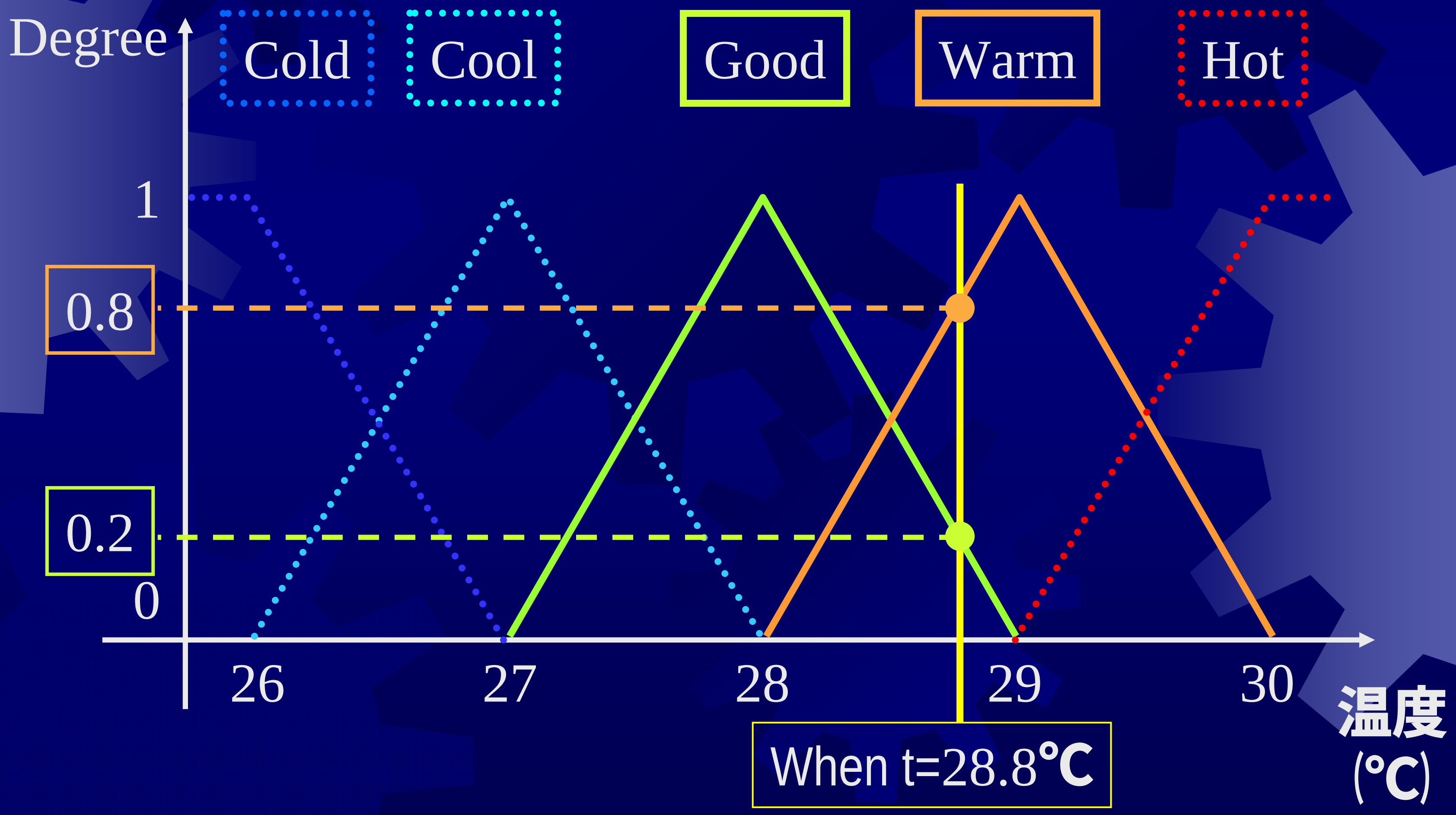
in other words,

$$\mu_{\text{output}}(v) = \max_{\text{i}} \{ \min\{ \mu_{\text{yi}}(v), \mu_{\text{xi}}(t)\} \}$$

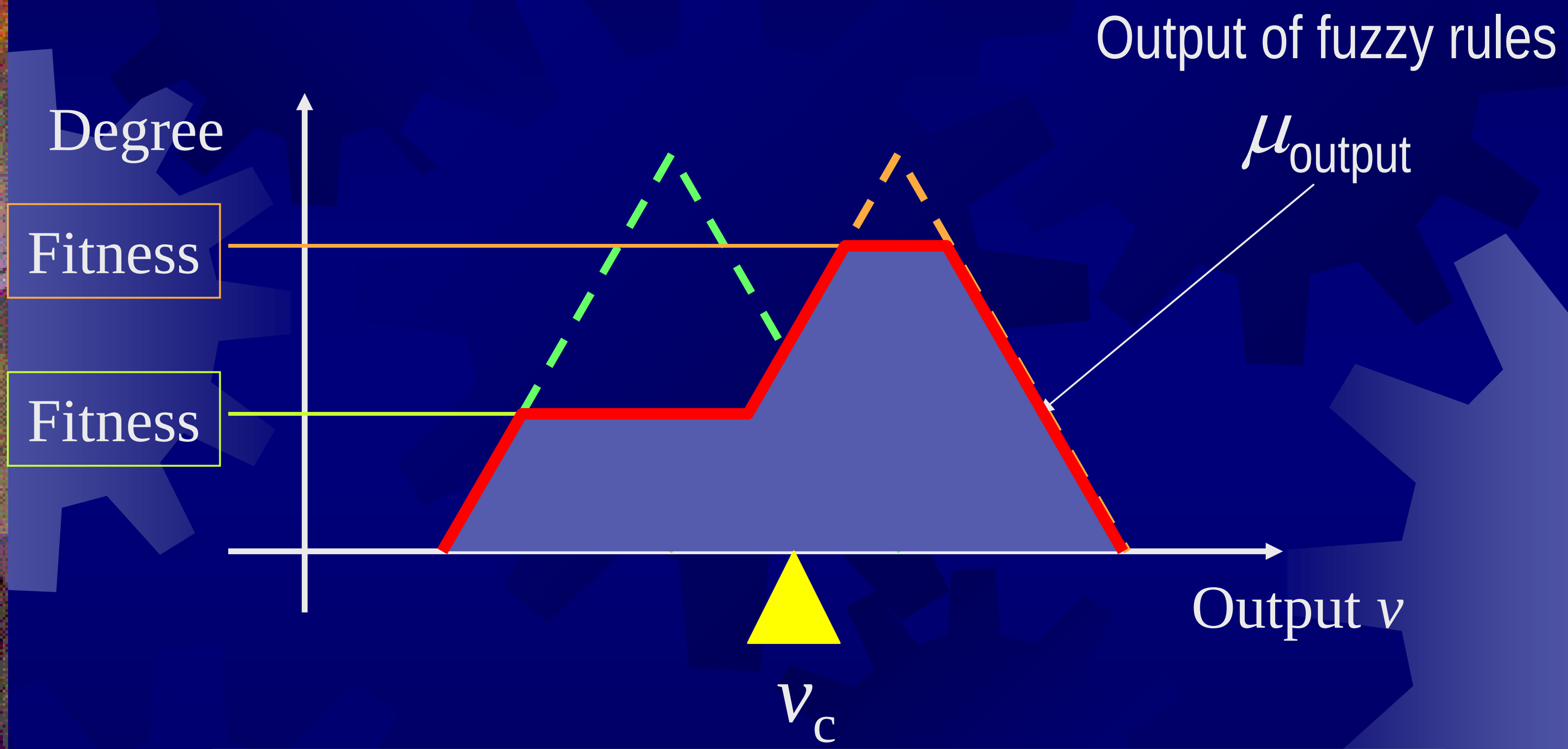
When we input ***t (crisp),*** we obtain a membership function of ***v.***

Fitness of a fuzzy rule

Degree of a membership function of input (feeling)



Defuzzication to obtain crisp output



Choosing the center of the gravity v_c

Control output for input T

Driving voltage

- Only setting several fuzzy sets, a fuzzy controller generates continuous output function of input T.

Changing membership functions for inputs

Degree

Cold

Cool

Good

Warm

Hot

... then, a shape of a control function is changed.

Driving voltage

Conclusion

- "Fuzzy" is a tool for designers. It does not contribute to the performance of controller.
- We can handle "feelings" by fuzzy sets without finding "crisp" boundaries.
- Only by changing membership functions, we can arrange a control function.