6 MRACS on discrete time systems

6.1 Formulation

In this section, we apply MRACS to discrete time systems. At first, we formulate plants and models by (1) and (2).

$$
Plant \quad : \quad A(q^{-1})y(k) = q^{-d}B(q^{-1})u(k) \tag{1}
$$
$$
\begin{cases} d \geq 1 & : \quad Timedelay(q^{-d}y(k) = y(k-d)) \\ A(q^{-1}) & = \quad 1 + a_1 q^{-1} + \cdots + a_n q^{-n} \\ B(q^{-1}) & = \quad b_0 + b_1 q^{-1} + \cdots + b_m q^{-m} \end{cases}
$$

$$
Model \quad : \quad A_M(q^{-1})y_m(k) = q^{-d}B_M(q^{-1})u_m(k) \tag{2}
$$
$$
\begin{cases} A_M(q^{-1}) & = \quad 1 + a_{m1} q^{-1} + \cdots + a_{mn} q^{-n} \\ B_M(q^{-1}) & = \quad b_{m0} + b_{m1} q^{-1} + \cdots + b_{mm} q^{-m} \end{cases}
$$

, where $b_{m0} > 0,\ A_M(q^{-1})$ is stability polynominal.

We want $e_1(k) = y_m(k) - y(k) \to 0$ for $k \to \infty$.

6.2 Diophantine equation

Next, we introduce Diophantine equation (3) and non-minimal realization.

$$
D(q^{-1}) = A(q^{-1})R(q^{-1}) + q^{-d}H(q^{-1}) \tag{3}
$$

, where $D(q^{-1}) = 1 + d_1 q^{-1} + \cdots + d_n q^{-n}$ is n order monic stability polynominal that we can set arbitrary. Then $R(q^{-1})$ and $H(q^{-1})$ always exist.

$$
\begin{aligned}
R(q^{-1}) &= 1 + r_1 q^{-1} + \cdots + r_{d-1} q^{-(d-1)} \\
H(q^{-1}) &= h_0 + h_1 q^{-1} + \cdots + h_{n-1} q^{-(n-1)}
\end{aligned}
$$

By multiplying (3) by $y(k)$,

$$
\begin{aligned}
D(q^{-1})y(k) &= A(q^{-1})R(q^{-1})y(k) + q^{-d}H(q^{-1})y(k) \\
&= q^{-d}B(q^{-1})R(q^{-1})u(k) + H(q^{-1})y(k-d) \\
&= B(q^{-1})R(q^{-1})u(k-d) + H(q^{-1})y(k-d).
\end{aligned}
$$

Here, we introduce θ, set of unknown parameters, and ξ, input and output data

$$
\begin{aligned}
\theta^T &= [b_0,\ b_0 r_1 + b_1,\ b_0 r_2 + b_1 r_1 + b_2,\ \cdots,\ b_m r_{d-1},\ h_0,\ h_1,\ \cdots,\ h_{n-1}] \\
\xi^T &= [u(k),\ u(k-1),\ \cdots,\ u(k-(m+d-1)),\ y(k),\ y(k-1),\ \cdots,\ y(k-(n-1))]
\end{aligned}
$$

, then we obtain (4) that denotes non-minimal realization.

$$
D(q^{-1})y(k) = \theta^T \xi(k) \tag{4}
$$

By the way, let us assme that order $n = 2$, $d = 2$. The Diohpantine equation (3) becomes

$$1 + d_1 q^{-1} + d_2 q^{-2} = (1 + a_1 q^{-1} + a_2 q^{-2})(1 + r_1 q^{-1}) + q^{-2}(h_0 + h_1 q^{-1}).$$

By comparing coeficients, we can derive following equations.

$$\begin{cases} r_1 &= d_1 - a_1 \\ h_0 &= d_2 - a_2 - a_2 d_1 + a_1^2 \\ h_1 &= a_2(a_1 - d_1) \end{cases}$$

These equations mean that coeficients r_i and h_j depend on $A(q^{-1})$ and $D(q^{-1})$.

Let us get back to the system error $e_1(k)$. By multiplying $e_1(k)$ by $D(q^{-1})q^{-d}$,

$$\begin{aligned} D(q^{-1})e_1(k+d) &= D(q^{-1})y_m(k+d) - B(q^{-1})R(q^{-1})u(k) - H(q^{-1})y(k) \\ &= D(q^{-1})y_m(k+d) - \theta^T \xi(k) \end{aligned} \tag{5}$$

, and we separate b_0 and $u(k)$ as

$$\begin{cases} \theta^T &= [b_0, \bar{\theta}^T] \\ \xi^T(k) &= [u(k), \bar{\xi}^T(k)] \end{cases} .$$

Then,

$$D(q^{-1})e_1(k+d) = D(q^{-1})y_m(k+d) - b_0 u(k) - \bar{\theta}^T \xi(k) \tag{6}$$

For (6) $\rightarrow 0$,

$$u(k) = \frac{1}{b_0}\left\{D(q^{-1})y_m(k+d) - \bar{\theta}^T \xi(k)\right\}. \tag{7}$$

6.3 Direct control

Unknown parameters in (6) are $\hat{\theta}(\hat{b}_0(k)\ and\ \hat{\bar{\theta}}(k)$. We denotes $\hat{y}(k)$ that is calculated by means of expected values. Then, we define $\epsilon_1(k)$:

$$\begin{aligned} \epsilon_1(k) &= D(q^{-1})(\hat{y}(k) - y(k)) \\ &= \phi^T(k)\xi(k-d) \end{aligned} \tag{8}$$

where $\phi^T(k) = \hat{\theta}(k) - \theta$. This equation represents a case of deterministic identifier that has $W(p) = 1$. So, we can apply algorithms of deterministic identifiers.

$$\hat{\theta}(k) = \hat{\theta}(k-1) - \Pi(k-1)\xi(k-d)\epsilon_1(k) \tag{9}$$

$$\Pi(k) = \frac{1}{\lambda_1(k)}\left\{\Pi(k-1) - \frac{\lambda_2(k)\Pi(k-1)\xi(k-d)\xi^T(k-d)\Pi(k-1)}{\lambda_1(k) + \lambda_2(k)\xi_T(k-d)\Pi(k-1)\xi(k-d)}\right\} \tag{10}$$

, where $0 < \lambda_1(k) \leq 1$, $0 \leq \lambda_2(k) \leq \lambda$, $\Pi(0) = \Pi(0)^T > 0$. By removing $\hat{\phi}(k)$ $(\hat{\theta}(k))$ from $\epsilon_1(k)$,

$$\epsilon_1(k) = \frac{-D(q^{-1})y(k) + \hat{\theta}^T(k-1)\xi(k-d)}{1 + \xi^T(k-d)\Pi(k-1)\xi(k-d)}. \tag{11}$$

$\epsilon_1(k)$ will goes to zero for $k \rightarrow \infty$) by (9)-(11).

7 STR: Self-tuning regulator

7.1 Formulation

In this section, we introduce STR that considers stochastic noize added to plants. At first, we formulate a plant by (12).

$$Plant \quad : \quad A(q^{-1})y(k) = q^{-d}B(q^{-1})u(k) + C(q^{-1})w(k) \tag{12}$$

$$\begin{cases} A(q^{-1}) &= 1 + a_1 q^{-1} + \cdots + a_n q^{-n} \\ B(q^{-1}) &= b_0 + b_1 q^{-1} + \cdots + b_m q^{-m} \\ C(q^{-1}) &= 1 + c_1 q^{-1} + \cdots + c_n q^{-n} \end{cases}$$

where $w(k)$ denotes white noize (average=0, distribution=σ^2). Known parameters are m, n, d, and unknown parameters are a_i, b_j, c_k.We assume that $B(q^{-1})$ and $C(q^{-1})$ are stability polynominals.

In this section, our goal is minimizing $J = E[(y_m(k) - y(k))^2]$ (minimizing distribution).

7.2 Diophantine equation

Next, we consider Diophantine equation and non-minimal realization. Diophantine equation is represented by (13).

$$C(q^{-1}) = A(q^{-1})R(q^{-1}) + q^{-d}H(q^{-1}) \tag{13}$$

$$\begin{aligned} R(q^{-1}) &= 1 + r_1 q^{-1} + \cdots r_{d-1} q^{-(d-1)} \\ H(q^{-1}) &= h_0 + h_1 q^{-1} + \cdots h_{n-1} q^{-(n-1)} \end{aligned}$$

By mutiplying (13) by $y(k)$,

$$\begin{aligned} C(q^{-1})y(k) &= A(q^{-1})R(q^{-1})y(k) + q^{-d}H(q^{-1})y(k) \\ &= B(q^{-1})R(q^{-1})u(k-d) + H(q^{-1})y(k-d) + C(q^{-1})R(q^{-1})w(k) \end{aligned} \tag{14}$$

Here, we define system error $e_1(k) = y_m(k) - y(k)$,

$$\begin{aligned} C(q^{-1})e_1(k) &= C(q^{-1})y_m(k) - C(q^{-1})y(k) \\ &= C(q^{-1})y_m(k) - B(q^{-1})R(q^{-1})u(k-d) - H(q^{-1})y(k-d) - C(q^{-1})R(q^{-1})w(k) \end{aligned} \tag{15}$$

. For (15)$\to 0$,

$$u(k) = \frac{1}{b_0}\left\{C(q^{-1})y_m(k+d) - H(q^{-1})y(k) - B_R(q^{-1})u(k)\right\} \tag{16}$$

where $B_R(q^{-1}) = B(q^{-1})R(q^{-1}) - b_0$. Then, we obtain

$$e_1(k) = -R(q^{-1})w(k)$$

7.3 Control

Same as the case of MRACS, we define θ and ξ as follows.

$$\begin{aligned}
\theta^T &= [b_0,\ b_0 r_1 + b_1,\ b_0 r_2 + b_1 r_1 + b_2,\ \cdots,\ b_m r_{d-1},\ h_0,\ h_1,\ \cdots,\ h_{n-1},\ c_1,\ c_2,\ \cdots,\ c_n] \\
\xi^T &= [u(k),\ u(k-1),\ \cdots,\ u(k-(m+d-1)),\ y(k),\ y(k-1),\ \cdots,\ y(k-(n-1)), \\
&\quad -y_m(k+d-1),\ \cdots,\ -y_m(k+d-n)]
\end{aligned}$$

Then,

$$u(k) = \frac{1}{b_0}\left\{y_m(k+d) - \bar{\theta}^T \bar{\xi}(k)\right\} \tag{17}$$

$$y_m(k+d) = \bar{\theta}^T \bar{\xi}(k) \tag{18}$$

where, $\theta^T = [b_0, \bar{\theta}^T]$, $\xi^T(k) = [u(k), \bar{\xi}^T(k)]$.

By $\phi^T(k) = \hat{\theta}^T - \theta^T$,

$$\epsilon_1(k) = \phi^T(k)\xi(k-d) = y_m(k) - y(k)$$

Then, we can apply algorithms of identifiers.