5 MRACS

5.1 Assumptions and system expressions

In this section, we introduce Model Reference Adaptive Control System (MRACS). To simplify the problem, we only consider a target plant that has following proverties.

- Time-invaliant linear system.
- Without noise (only unknown parameters).
- Orders of a transfer function are known.
- We know b_m is positive or not.
- An invert system is stable.
- Control input is unlimited.

Let us consider a system represented by (1).

$$y(t) = \frac{B(p)}{A(p)}u(t) \tag{1}$$

$$where \left\{ \begin{array}{lcl} A(p) & = & p^n a_{n-1} p^{n-1} \cdots a_1 p + a_0 \\ B(p) & = & b_m p^m b_{m-1} p^{m-1} \cdots b_1 p + b_0 \end{array} \right.$$

5.2 MRACS in continuous time

As mentioned in section 1, MRACS has an architecture denoted in Fig. 1.

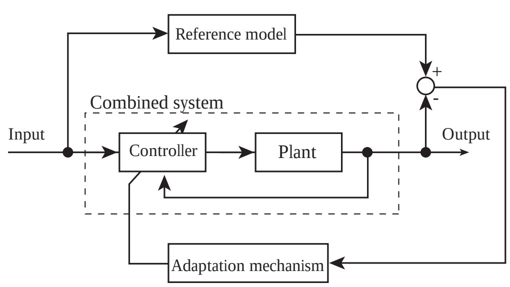

Figure 1: Model Reference Adaptive Control System; MRACS

Our goal is realizing control system so that a combined system works as the same as a reference mode. For simplify the problem, we assume single input single output system denoted in (2),

$$\left\{ \begin{array}{l} \dot{x}(t) = A_P x(t) + b_P u(t) \\ y(t) = c_P^T x(t) \end{array} \right. \tag{2}$$

and we set a model system and tracking error as (3) and (4), respectively. In (3), $A_M(p)$ and $B_M(p)$ are order p and q, respectively.

$$\begin{aligned} y_m(t) &= \frac{B_M(p)}{A_M(p)} u_m(t) & (3) \\ e_1(t) &= y_m(t) - y(t) & (4) \end{aligned}$$

We have to notice following condition;

$$p - q \geq n^* \quad , where \ \ n* = n - m.$$

In order to realize an adaptive system without differential signals (= future values of a system), we have to satisfy the condition.

We are going to construct MRACS according to following procedure.

1. Construct non-minimal realization of an unknown plant.
2. Design model-matching controller to zero the output error.
3. Calculate control input.

5.3 non-minimal realization by Diophantine equation

We again consdier Diophantine euqation. For proper $A(p)$ and $B(p)$, by introducing monic stability polinominals $Q(p)$ (order n) and $D(p)$ (order $n-m$), we can find polinominals $R(p)$ and $H(p)$ that satisfy (5).

$$R(p)A(p) + H(p)B(p) = Q(p)(b_m A(p) - D(p)B(p)) \tag{5}$$

$$, where \left\{ \begin{array}{lcl} R(p) & = & r_{n-1}p^{n-1} + r_{n-2}p^{n-2} + \cdots + r_1 p + r_0 \\ H(p) & = & h_{n-1}p^{n-1} + h_{n-2}p^{n-2} + \cdots + h_1 p + h_0 \end{array} \right.$$

Equation (6), obtained from (5), represents non-minimal realization of the system.

$$y(t) = \frac{1}{D(p)} \left(b_m u(t) - \frac{R(p)}{Q(p)} u(t) - \frac{H(p)}{Q(p)} y(t) \right) \tag{6}$$

Next, we consider control input that converges the tracking error to zero. Based on (4),

$$\begin{aligned} D(p)e_1(t) &= D(p)y_m(t) - D(p)y(t) & (7) \\ &= D(p)y_m(t) - b_m u(t) + \frac{R(p)}{Q(p)} u(t) + \frac{H(p)}{Q(p)} y(t) \end{aligned}$$

We can $D(p)e_1(t) = 0$ by following $u(t)$.

$$u(t) = \frac{1}{b_m} \left(D(p)y_m(t) + \frac{R(p)}{Q(p)} u(t) + \frac{H(p)}{Q(p)} y(t) \right) \tag{8}$$

5.4 Design of adaptive system

As (8), we want to know b_m,$R(p)$, and $H(p)$ in order to zero the tracking error. So, we denote estimated parameters as $\hat{b}_m(t)$,$\hat{R}(p,t)$, and $\hat{H}(p,t)$. Then,

$$u(t) = \frac{1}{\hat{b}_m(t)} \left(y_c(t) + \frac{\hat{R}(p,t)}{Q(p)} u(t) + \frac{\hat{H}(p,t)}{Q(p)} y(t) \right) \tag{9}$$

, where $y_c(t) = D(p)\frac{B_M(p)}{A_M(p)} u_m(t)$ (model output).

Next, we define $w(t)$ called state valiable filer such as

$$w^T(t) = \left[\frac{1}{Q(p)} u(t), \frac{p}{Q(p)} u(t), \cdots, \frac{p^{n-1}}{Q(p)} u(t), \frac{1}{Q(p)} y(t), \frac{p}{Q(p)} y(t), \cdots, \frac{p^{n-1}}{Q(p)} u(t) \right] \tag{10}$$

, and unknown parameter vector θ such as

$$\theta^T = [r_0, r_1, \cdots, r_{n-1}, h_0, h_1, \cdots, h_{n-1}] . \tag{11}$$

Then, we can describe $u(t)$ as

$$u(t) = \frac{1}{b_m} \left(y_c(t) + \theta^T w(t) \right) . \tag{12}$$

We can represent the non-minimal realization of (12) and the tracking error by (13) and (14) respectively.

$$y(t) = \frac{1}{D(p)} \left(b_m u(t) - \theta^T w(t) \right) \tag{13}$$

$$e_1(t) = \frac{1}{D(p)} \left(-b_m u(t) + \theta^T w(t) + y_c(t) \right) \tag{14}$$

By introducing estimated parameters into (12),

$$\begin{aligned} u(t) &= \frac{1}{\hat{b}_m(t)} \left(y_c(t) + \hat{\theta}^T(t) w(t) \right) & (15) \\ \hat{e}_1(t) &= \frac{1}{D(p)} \left(-\hat{b}_m(t) u(t) + \hat{\theta}^T(t) w(t) + y_c(t) \right) & (16) \end{aligned}$$

5.5 Adjustment of control parameters

Here, we define argumented error $\epsilon_1(t) = e_1(t) - \hat{e}_1(t)$, then

$$\left\{ \begin{array}{lcl} \epsilon_1(t) &=& W(p) \left(\phi^T(t) \xi(t) \right) , \\ W(p) &=& \frac{1}{D(p)}, \\ \phi^T(t) &=& [\hat{b}_m(t) - b_m, -(\hat{\theta}^T(t) - \theta^T)], \\ \xi^T(t) &=& [u(t), w^T(t)]. \end{array} \right. \tag{17}$$

Now, we can apply algorithms of adaptive identifiers.

For example (we assume $W(p)$ is SPR),

$$\begin{cases} \dot{\hat{b}}_m(t) &= -\gamma_0 u(t)\epsilon_1(t) \quad (\gamma_0 > 0) \\ \dot{\hat{\theta}}(t) &= \Gamma w(t)\epsilon_1(t) \quad (\Gamma = \Gamma^T > 0) \end{cases}$$

When we apply this method to actual systems, we have to consider b_m because $u(t) \to \infty$ for $b_m \to 0$.

5.6 Example

Let us construct MRACS for following plant and model.

$$Plant \begin{cases} \dot{x}(t) &= -a_0 x(t) + b_0 u(t) \\ y(t) &= x(t) \end{cases} \tag{18}$$

$$Model \begin{cases} \dot{x}_m(t) &= -a_{m0} x_m(t) + b_{m0} u_m(t) \quad (a_{m0} > 0) \\ y_m(t) &= x_m(t) \end{cases} \tag{19}$$

We can transform (18) as

$$y(t) = \frac{b_0}{p + q_0} u(t).$$

Then, we obtain

$$\begin{cases} A(p) &= p + a_0 \quad (n = 1) \\ B(p) &= b_0 \quad (m = 0) \end{cases} , and$$

$$\begin{cases} A_M(p) &= p + a_{m0} \\ B_M(p) &= b_{m0} \end{cases} .$$

Next, we calculate control input $u(t)$.

$$u(t) \frac{1}{\hat{b}_{m0}(t)} \left(y_c(t) + \hat{\theta}^T(t) w(t) \right) \tag{20}$$

, where $y_c(t) = D(p) \frac{b_{m0}}{p + a_{m0}} u_m(t)$.

For non-minimal realization, we introduce $Q(p) = p + q_0$ and $D(p) = p + d_1$, and denotes estimated $\hat{R}(p, t) = \hat{r_0}(t)$, $\hat{H}(p, t) = \hat{h_0}(t)$. Then the state variable filter becomes

$$w^T(t) = \left[\frac{1}{p + q_0} u(t), \frac{1}{p + q_0} y(t) \right].$$

Here, we set unknown vector $\hat{\theta}(t)$ as

$$\hat{\theta}^T(t) = [\hat{r_0}(t), \hat{h_0}(t)].$$

Instead of $\epsilon_1(t)$, we simply introduce $\phi(t)$

$$\phi^T(t) = [\hat{b}_{m0}(t) - b_{m0}, r_0 - \hat{r}_0(t), h_0 - \hat{h}_0(t)].$$

Because obviously $W(p) = \frac{1}{D(p)} = \frac{1}{p + d_1}$ is SPR, we can apply algorithms of adaptive identifiers. By setting $\xi^T(t) = [u(t), w(t)]$,$\dot{\phi}(t) = -\Gamma \xi(t) e_1(t) (\Gamma = \Gamma^T > 0)$ realizes $e_1(t) \to 0$.