3 Deterministic identifier

3.1 Adaptive Identifier/Controller

Adaptive Control System is a system that arranges its controller dynamically depends on dynamics of a system to be controlled.

Adaptive Observer is an observer that estimate parameters and state valiables in a system only by inputs and outputs.

Adaptive Identifier is a system that identifies parameters of a system.

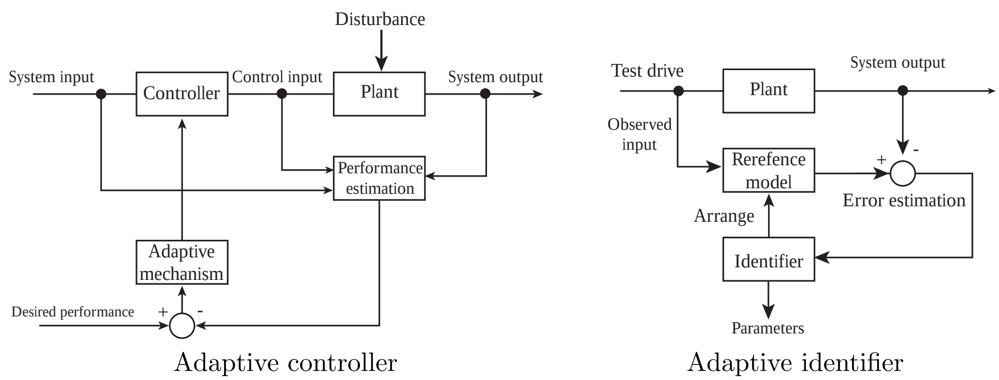

Figure 1: Schematic view of adaptive control system

3.2 Strictly Positive Real

A rational function $f(s)$ is **Positive Real** when

- for real s, $f(s)$ is real, and
- for all $s : \{Re\{s\} > 0\}$, $Re\{f(s)\} \geq 0$.

Additionally, when a real number $\lambda > 0$ that makes $f(s - \lambda)$ positive real exists, $f(s)$ is **Strictly Positive Real (SPR)**.

Examples

- $f(s) = \frac{k}{s}$ $(k > 0)$ is positive real.
- $f(s) = \frac{ks}{a_2 s^2 + a_1 s + 1}$ $(a_1, a_2, k > 0)$ is SPR.

3.3 Positive Real System

Let us consider following system;

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \tag{1}$$

, where $u(t)$ and $y(t)$ is $m \times 1$, $x(t)$ is $n \times 1$.

The transfer function $F(s)$ of the system is $F(s) = C(sI - A)^{-1}B + D$. This system ($F(s)$) is SPR if positive definite simmetric matrices P, Q, W exist that satisfies following conditions.

$$\left\{ \begin{array}{l} A^T P + PA = -Q \\ B^T P = C \\ D + D^T = W^T W \end{array} \right. \tag{2}$$

If Q is semi-positive definit simmetric, $F(s)$ becomes positive real. $D = 0$ also satisfies the conditions. When $D = 0$, the conditions becomes simple as follows.

$$\left\{ \begin{array}{l} A^T P + PA = -Q \\ B^T P = C \end{array} \right. \tag{3}$$

Here, we consider discrete time system denoted as follows.

$$\left\{ \begin{array}{ccccc} x(k+1) & = & Fx(k) & + & Hu(k) \\ y(k) & = & Cx(k) & + & Du(k) \end{array} \right. \tag{4}$$

The transfer function of the system is $G(z) = C(zI - F)^{-1}H + D$. This system is SPR if positive definite simmetric matrices P, Q exist that satisfies following conditions.

$$\left\{ \begin{array}{l} F^T PF - P = -Q \\ H^T PF = C \\ D + D^T = H^T PH \end{array} \right. \tag{5}$$

If Q is semi-positive definit simmetric, $G(z)$ becomes positive real. Please notice that $D = 0$ does not satisfies SPR condition in this case because $H^T PH$ must be positive.

3.4 Deterministic identification; error model

At the beginning, we introduce following error model.

$$\left\{ \begin{array}{ccccc} \dot{e}(t) & = & Ae(t) & + & bf(t) \\ e_1(t) & = & c^T e(t) & + & df(t) \\ f(t) & = & \phi^T(t)\xi(t) & & \end{array} \right. \tag{6}$$

, where $e(t)$: n dimensional state error vector, $f(t)$: scalar control input, $e_1(t)$: observable identification error, $\phi(t)$: parameter error.

The parameter error is represented as

$$\phi(t) = \hat{\theta}(t) - \theta$$

, where θ denotes unknown parameters and $\hat{\theta}(t)$ represents adjustable identifca-tion parameters.

By thedeterministic identification, we want to obtain $\phi(t)(\hat{\theta}(t))$ that realizes $e_1(t) \to 0$ for $t \to \infty$.

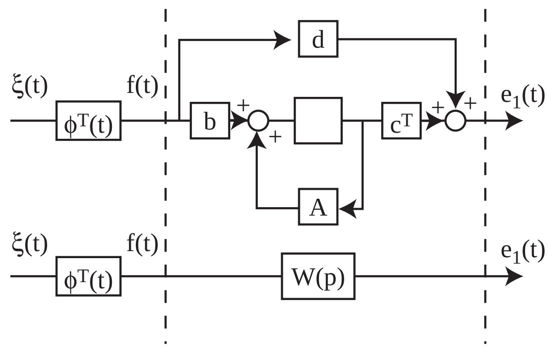

Figure 2: Error model of the deterministic identification.

The transfer function of the error model and observable error are denoted as follows.

$$\begin{cases} W(s) &= c^T(sI-A)^{-1}b+d \\ e_1(t) &= W(p)f(t) = W(p)\{\phi^T(t)\xi(t)\} \end{cases} \tag{7}$$

3.5 Deterministic identifier; continuous time, Narendra's method

For the identification, we apply feedback gain of squared $\xi(t)$ (Narendra's method). In this section, we suppose $d=0$ to simplify the problem.

$$\begin{cases} \dot{e}(t) &= Ae(t)+bf(t) \\ e_1(t) &= c^Te(t) \\ f(t) &= \phi^T(t)\xi(t)-\alpha\xi^T(t)\Lambda\xi(t)e_1(t) \end{cases} \tag{8}$$

, where $\Lambda=\Lambda^T>0$,$\alpha>0$.

The transfer function and output of the feedback system become

$$W(s)=c^T(sI-A)^{-1}b \tag{9}$$

$$e_1(t)=W(p)f(t)=\frac{W(p)\{\phi^T(t)\xi(t)\}}{1+\alpha W(p)\xi^T(t)\Lambda\xi(t)} \tag{10}$$

Here we assume that $W(p)$ is SPR, then

$$\dot{\phi}(t)=\dot{\hat{\theta}}(t)=-\alpha\Gamma\xi(t)e_1(t) \tag{11}$$

realises $e_1(t)\to 0$ for $t\to\infty$. Notice $\Gamma=\Gamma^T>0$.

We can denote (11) in the following form.

$$\hat{\theta}(t)=-\Gamma\int_0^t\xi(\tau)e_1(\tau)d\tau-\Lambda\xi(t)e_1(t) \tag{12}$$

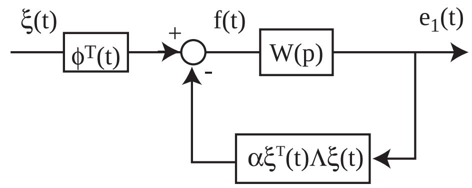

Figure 3: Narendra's adaptive model.

3.6 Deterministic identifier; discrete time

Let us consider identification method in discrete time systems. At first, we introduce Narendra's method in the discrete time. Let us formulate a system as follows.

$$\begin{cases} e(k+1) &= F(e(k)+hf(k) \\ e_1(k) &= c^T e(k)+df(k) \\ f(k) &= \phi^T(k)\xi(k)-\alpha\xi^T(k)\Lambda\xi(k)e_1(k) \end{cases} \tag{13}$$

where $\Lambda = \Lambda^T > 0$,$\alpha > 0$.

The transfer function and observable output of the system are

$$\begin{aligned} W(z) &= c^T(zI-F)^{-1}h+d \\ e_1(k) &= W(q)f(k) \end{aligned} \tag{14}$$

Here we assume $W(q)$ is SPR, then (15) realizes deterministic identification.

$$\hat{\theta}(k+1) = \hat{\theta}(k) - \alpha\Lambda\xi(k)e_1(k) \tag{15}$$