1 Concept of adaptive control

1.1 Overview of adaptive control

Key words: Adaptive controller, Adaptive observer, Adaptive identifier

Let us consider a system represented by a state space model (1) and square evaluation function(2).

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), x(0) = x_0 \\ y &= Cx(t) \\ & \quad x(t) : n \times 1, u(t) : r \times 1 \end{aligned} \tag{1}$$

$$J = \int_0^\infty \left\{ y^T(t)Qy(t) + u^T(t)Ru(t) \right\} dt \tag{2}$$

We can obatin u(t) that minimize J by (3),

$$u(t) = -Kx(t), \tag{3}$$

where Q and R are weight matrix, and K is feedback matrix $(r \times n)$. K is a solution of Riccati equation, $K = f(A, B, C, Q, R)$. We call K as SAS (Stability Argumentation System) gain.

If we can estimate matrix A and B, above K realizes SAS. Of course, we have to know all sate valiables exactly. However, actual system always contains model error. Additionally, we cannot always observe state valiables. In such case, we usually apply observer and estimate system status, $\hat{x}(t)$. To construct a feedback system $u(t) = -K\hat{x}(t)$, we have to know exact dynamics of a system in advance.

Adaptive Control System is a system that arranges its controller dynamically depends on dynamics of a system to be controlled.

Adaptive Observer is an observer that estimate parameters and state valiables in a system only by inputs and outputs.

Adaptive Identifier is a system that identifies parameters of a system.

1.2 Formally known system

1.2.1 High Gain System

Here, we assume $y(s) = y_n(s) + y_u(s)$, where

$$\left\{ \begin{array}{lcll} y_n(s) &=& \frac{G_P(s)}{1+G_C(s)H(s)G_P(s)} n(s) & u_m(s) = 0 \\ y_u(s) &=& \frac{G_C(s)G_P(s)G_M(s)}{1+G_C(s)H(s)G_P(s)} u_m(s) & n(s) = 0 \end{array} \right. .$$

When we choose $G_C(s)$ and $H(s)$ so that $|G_C(s)H(s)| >> 1$,

$$\left\{ \begin{array}{lcl} \frac{y_n(s)}{n(s)} &\simeq& \frac{1}{G_C(s)H(s)} \to 0 \\ \frac{y_u(s)}{u_m(s)} &\simeq& \frac{G_M(s)}{H(s)} \end{array} \right. ,$$

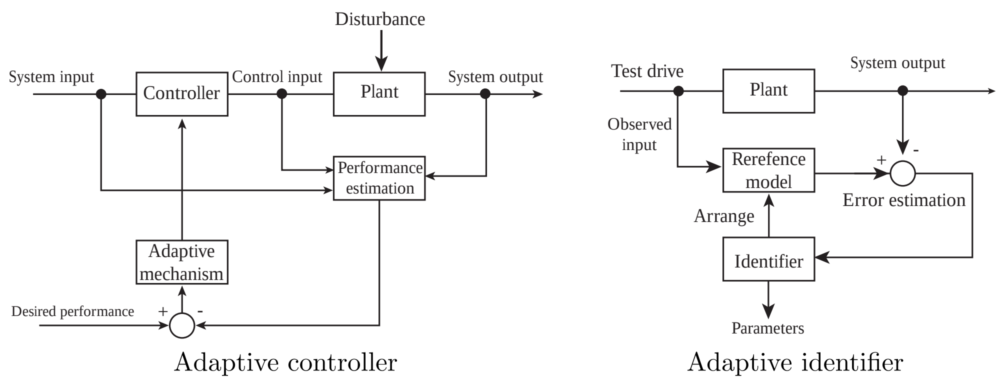

Figure 1: Schematic view of adaptive control system

we can reduce effects from noise. Additionally, we we can set $H(s) = 1$, $y_u(s) \to G_M(s)u_m(s)$. Output of the plant becomes equal to that of reference model.

So, we should set large $G_C(s)$. This system is called "High-gain method". This system has some problem on stability and actual control limits.

1.2.2 Gain Scheduled Method

Gain scheduled method is not classified into "adaptive control", but it is often used. In the gain scheduled method, an user has to investigate feasible parameters of the controller according to the environment conditions in advance. The controller changes its parameters based on the pre-fix conditions.

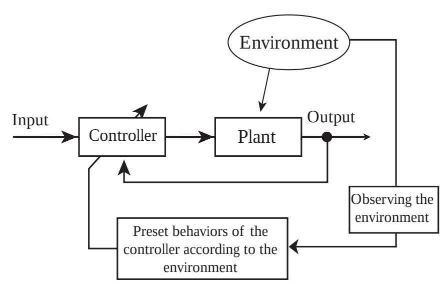

Figure 2: Gain Scheduled Method; GSM

1.3 Basic structure of adaptive controller

Key words: High gain system, MRACS, STR, Gain Scheduled Method

Let us assume following system (Fig. 3).

Reference Model	Transfer function $G_M(s)$.
Unknown plant (system)	$G_P(s)$: Its parameters are unknown.
Disturbance	$n(s)$
System output	$y(s)$
System input	$u(s)$

Our goal is obtaining $G_C(s)$ and $H(s)$ so that $y(s)$ follows output of the reference model even if any disturbance exists.

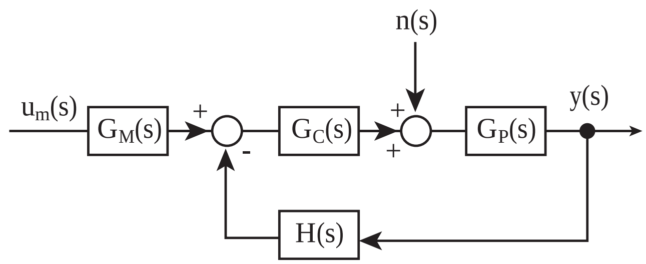

Figure 3: Assumed system

Here, we introduce typical methods. We will discuss them precisely in following sections.

1.3.1 Model Reference Adaptive Control System (MRACS)

A MRACS system arranging controller (Fig. 4) so that output of combined system (controller and unknown plant) matches that of refenrece model. Monopoli et al. have proposed a method that calculates feasible control parameters only by input and output signals of the plant.

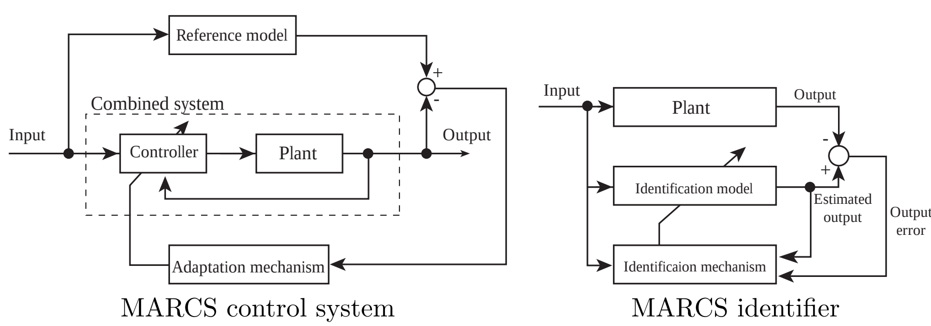

Figure 4: Model Reference Adaptive Control System; MRACS

1.3.2 Self Tuning Regulator (STR)

STR is online-tuning feedback controller. At first, STR assume parameters of unknown plant arbitrarily. Then it construct feedback controller based on given evaluation function. Next, it identifies parameters according to control inputs and outputs of the plant. After that, it repeats tuning procedure.

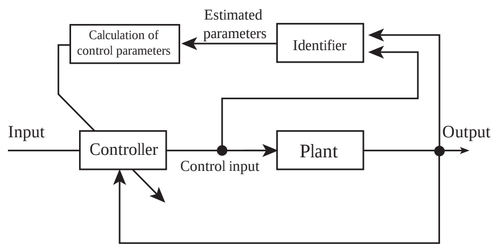

Figure 5: Self Tuning Regulator; STR

1.4 Expression of unknown plants

Level of "unknown".

White box Structure: known (linear differential equation), parameters and state valiables: known and observable.

Gray box partially known, partially unknown.

Black box Only inputs and outputs are observable.

Expressions of systems

- Lumped Parameter System
  - Linear, time-invaliant $\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases}$
  - Linear, time-valiant $\begin{cases} \dot{x}(t) = A(t)x(t) + B(t)u(t) \\ y(t) = C(t)x(t) + D(t)u(t) \end{cases}$
  - Non-linear, time-invaliant $\begin{cases} \dot{x}(t) = f(x(t), u(t)) \\ y(t) = g(x(t), u(t)) \end{cases}$
  - Non-linear, time-valiant $\begin{cases} \dot{x}(t) = f(x(t), u(t), t) \\ y(t) = g(x(t), u(t), t) \end{cases}$
- Distributed parameter system
- Lag-time system