ASTRONOMY ACROSS CULTURES

The History of Non-Western Astronomy

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SPRINGER-SCIENCE+BUSINESS MEDIA, B.V.

A catalogue record for this book is available from the Library of Congress.

ISBN 978-94-010-5820-9 ISBN 978-94-011-4179-6 (eBook)
DOI 10.1007/978-94-011-4179-6

Printed on acid-free paper

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REMARKS ON THE ORIGIN OF INDO-TIBETAN ASTRONOMY

In studying the history of science, we may divide all natural sciences into two groups, modern science and traditional sciences. Modern science should not be called 'Western', because it incorporated several traditional sciences from other parts of the world, directly or indirectly, and has grown up to become a global science. There are three main lines of traditional sciences in the Old World, East Asian (predominantly Chinese), South Asian (predominantly Indian), and Ancient Mediterranean-Islamic-European science. Of course, there are also several small lines and branch lines of traditional sciences.

Tibetan traditional science may be considered one small line of traditional science which was influenced by both Indian and Chinese science, although it is closer to Indian. Two major branches of Tibetan traditional science are astronomy and medicine.

There are four branches of Tibetan astronomy:

1. *sKar-rtsis* (star calculation) – mathematical astronomy based on the Kālacakra astronomy of India,
2. *dByańs-'char* (appearance of voice) – divination based on Indian divination called *svarodaya*,
3. *Nag-rtsis* (black calculation) – astrology based on Chinese astrology and natural philosophy, and
4. *rGya-rtsis* (Chinese calculation) – mathematical astronomy based on the Shixian calendar of China.

Among these branches, the sKar-rtsis is the basis of traditional calendars in Tibet, Mongolia, and Bhutan. In this essay, I will discuss its origin. The sKar-rtsis is based on the astronomical section of the *Kālacakratantra*, originally written in Sanskrit and translated into Tibetan and Mongolian, which is the fundamental text of the last stage of Indian Esoteric Buddhism. We will call the system of astronomy in the *Kālacakratantra* 'Kālacakra astronomy'. As it was originally written in Sanskrit, it is clear that it is closely connected with Indian astronomy.

Before discussing the origin of Kālacakra astronomy, let us briefly review

H. Selin (ed.), Astronomy Across Cultures: The History of Non-Western Astronomy, 341–369.

the history of Indian astronomy. The origin of Indian astronomy itself is controversial, and what I am going to present below is my own view.[1] [Editor's note: Subhash Kak presents another view in his article on the development of Indian astronomy in this volume.]

The Indus valley civilization (ca. 2500 BC–ca. 1700 BC) is famous for its excellent town planning and agriculture, so we can easily suppose that it had some astronomical knowledge. However, we do not have enough material to estimate its development.

The Indo-Aryans were originally pastoral people, and entered northwest India in about 1600 BC or a little earlier. There they composed the *Ṛgveda*, one of the basic books of Indian thought. After entering India, they gradually developed agricultural activity and acquired astronomical knowledge. At this stage, they were already using a luni-solar calendar and knew some asterisms. We call this period, from ca. 1500 BC–ca. 1000 BC, the Ṛgvedic period.

In the Later Vedic period (ca. 1000 BC–ca. 500 BC), Indo-Aryans moved to the basin of the River Gaṅgā and became essentially agricultural people. At the same time, their astronomical knowledge developed, because it was necessary to determine seasons for agricultural activity. One year was divided into six seasons, the complete system of *nakṣatras* (lunar mansions) was already known, and one day was divided into 30 parts called *muhūrtas*. (At this time, day and night were divided into 15 muhūrtas each. Later, from the Vedāṅga astronomy period onwards, one whole day was divided into 30 equal muhūrtas.) The sun's seasonal northward and southward movement was noticed. In this period, the regular calendar and agricultural activities were symbolized in Brahmanic rituals such as new and full moon offerings and seasonal (four monthly) offerings.

The period sometime between the 6th and the 4th centuries BC marks the formation of Vedāṅga astronomy. The Vedāṅga (limb of the Veda) is a class of works which is regarded as auxiliary to the Veda. It consists of six divisions, one of which is astronomy (*jyotiṣa*), which was necessary to determine the schedule of rituals.

The fundamental Sanskrit text of Vedāṅga astronomy is the *Vedāṅgajyotiṣa*, of which two recensions, Ṛgvedic and Yajurvedic, are extant.[2]

The main structure of Vedāṅga astronomy is as follows.

1 *yuga* = 5 years,
= 60 solar months (one solar month is 1/12 of a year),
= 61 sāvana months (one sāvana month is 30 civil days),
= 1830 sāvana days (civil days),
= 62 synodic months,
= 1860 *tithis* (one *tithi* is 1/30 of a synodic month),
= 67 sidereal months,
= 1835 sidereal days.

The Vedāṅga calendar is a luni-solar calendar, and there are two intercalary months in a yuga (five years). One sāvana day (civil day) is from sunrise to sunrise.

David Pingree (1973) argued that Vedāṅga astronomy was formed under Mesopotamian influence during the Achaemenid occupation of the Indus valley. However, Pingree's argument is refuted by my own research. I shall show that Vedāṅga astronomy is based on actual astronomical observations in North India (Ōhashi, 1993). As this is very important for the history of Indian astronomy and necessary for the later discussion, I shall explain it in detail.

First, let us examine the length of a year. The Yajurvedic recension of the *Vedāṅgajyotiṣa* states that one yuga consists of 61 sāvana months (= 1830 sāvana days) and that the number of sidereal days in a yuga is the number of sāvana days plus five. This means that one year consists of 366 civil days or 367 sidereal days. The Ṛgvedic recension does not mention this explicitly. Pingree argued that one year in the Ṛgvedic recension was 366 sidereal days and not 366 civil days, and that the statement in the Yajurvedic recension was wrong because of the misunderstanding of its compiler. He concluded that one year of the original Vedāṅga astronomy was 365 civil days (one day less than the number of sidereal days), and that it was introduced into India through Persia, because the Egyptian-Persian year was also 365 days. Pingree's argument is, however, not borne out by the evidence. I shall show that one year of the Vedāṅga astronomy was definitely 366 civil days. According to the *Vedāṅgajyotiṣa* itself, the purpose of Vedāṅga astronomy was to determine the proper time for sacrifices. As mentioned above, there were some Brahmanic rituals which symbolized the division of time. Actual observations at the time of the new and full moon offerings fairly accurately determined the dates of the new and full moons. This is clear from the fact that the *Śāṅkhāyanaśrautasūtra* (I.3.5), which is one of the ritualistic works of Vedāṅga literature, states that the two days of the full moon are the day on which the moon appears full about the setting of the sun and its succeeding day (Caland, 1953: 5). By this method, the day of the full moon can be determined quite accurately, because the time of moonrise changes by about 49 minutes on the average per day and this difference can easily be observed by naked eye observations. However, the change of season cannot be determined so accurately. Therefore we can suppose that Vedāṅga astronomy could predict the date of the new and full moons for at least five years accurately, even if it could not predict the seasons with the same accuracy. Now, the modern exact value of 62 synodic months is 1830.90 days, and that of 67 sidereal months is 1830.55 days. Then, one yuga of Vedāṅga astronomy could not be different from 1830 days or so. If Pingree's argument is true, one yuga becomes 1825 days, and it produces nearly 6 days' error of the new and full moon days, which would cause a panic at the time of the new and full moon offerings. One year in Vedāṅga astronomy was 366 civil days. Since there is no similar calendar anywhere in ancient West Asia, Vedāṅga astronomy must be the original Indian astronomy.

Second, let us examine the seasonal change of the length of day and night. The *Vedāṅgajyotiṣa* states that the length of day is given by the following zigzag function.

The length of day = $(12 + \frac{2}{62}n)$ muhūrtas, where n is the number of days after or before the winter solstice. One muhūrta is $\frac{1}{30}$ of a day. According to this formula, the period of one solar month produces one muhūrta's difference of daytime, and the proportion of day to night at the solstice becomes 2 : 3. This proportion is observed at the latitude 35°N or so, which is around Kashmir and far north of the basin of the River Gaṅgā, which was the central area in the Vedāṅga period. This proportion is very famous among historians of Indian science, and some people conjectured that Vedāṅga astronomy was produced around Kashmir. Pingree also regarded this proportion as important, and argued that this value was borrowed from Mesopotamia, where the central area is at the same latitude. I believe this argument also has flaws. I shall show that the formula above is based on actual observations in North India.

The seasonal movement of the sun was well observed by Vedic people. For example, the *Kauṣītakibrāhmaṇa* (XIX.3), one of the later books of Vedic literature, states that the sun goes north for six months and stands still, being about to turn southwards, and then goes south for six months and stands still, being about to turn northwards (Keith, 1920: 242). This statement probably refers to the change of the position of sunrise or sunset. They change much around the equinox but not around the solstice. So, the sun looks as if it is standing still around the solstice. This must have produced the idea that the seasonal change of certain phenomena should be obtained from observations around the equinox and not from around the solstice. So, formula must have been obtained by extrapolation from the observation of the change of the length of daytime around the equinox and not by interpolation from the observation around the solstice. Practically, there are two possibilities for the extrapolation. If we assume that the formula was extrapolated from one muhūrta's difference of the length of day during one solar month after the equinox, the most suitable latitude for this observation becomes 27°N. If we assume that the formula was extrapolated from two muhūrtas' difference during two solar months after the equinox, the most suitable latitude becomes 29°N. In any case, it is clear that the formula is based on observations in North India. The actual length of daytime at 35°N, 29°N, and 27°N, and the formula above are graphed together in Figure 1.

The five-year cycle of Vedāṅga astronomy was used in the *Arthaśāstra*, a political work attributed to Kautilya, a minister of Candragupta Maurya, enthroned in 321 BC, although the actual date of composition is controversial (Kangle, 1965–72). It was also used in the *Śārdūlakarṇāvadāna*, a Buddhist work (Vaidya, 1959: 314–425); and the *Sūriyapannatti*, a Jaina work (Kohl, 1937). The *Paitāmahasiddhānta* (quoted in chapter XII of the *Pañcasiddhāntikā* of Varāhamihira (6th century AD)) is also a text of Vedāṅga astronomy. (Thibaut and Dvivedī, 1889; Neugebauer and Pingree, 1970–71; Sastry, 1993).

The epoch of the *Paitāmahasiddhānta* is AD 80. This demonstrates that Vedāṅga astronomy must have been in use in the 1st century AD. The *Śārdūlakarṇāvadāna* was translated into Chinese as the *Madengqie jing* in the 3rd century AD. The *Sūriyapannatti* is included in the canon of the Śvetāmbara sect of Jainism which is said to have been edited in the 5th century AD. It may be that Vedāṅga astronomy was still used in their time. The *Śārdūlakarṇāvadāna*

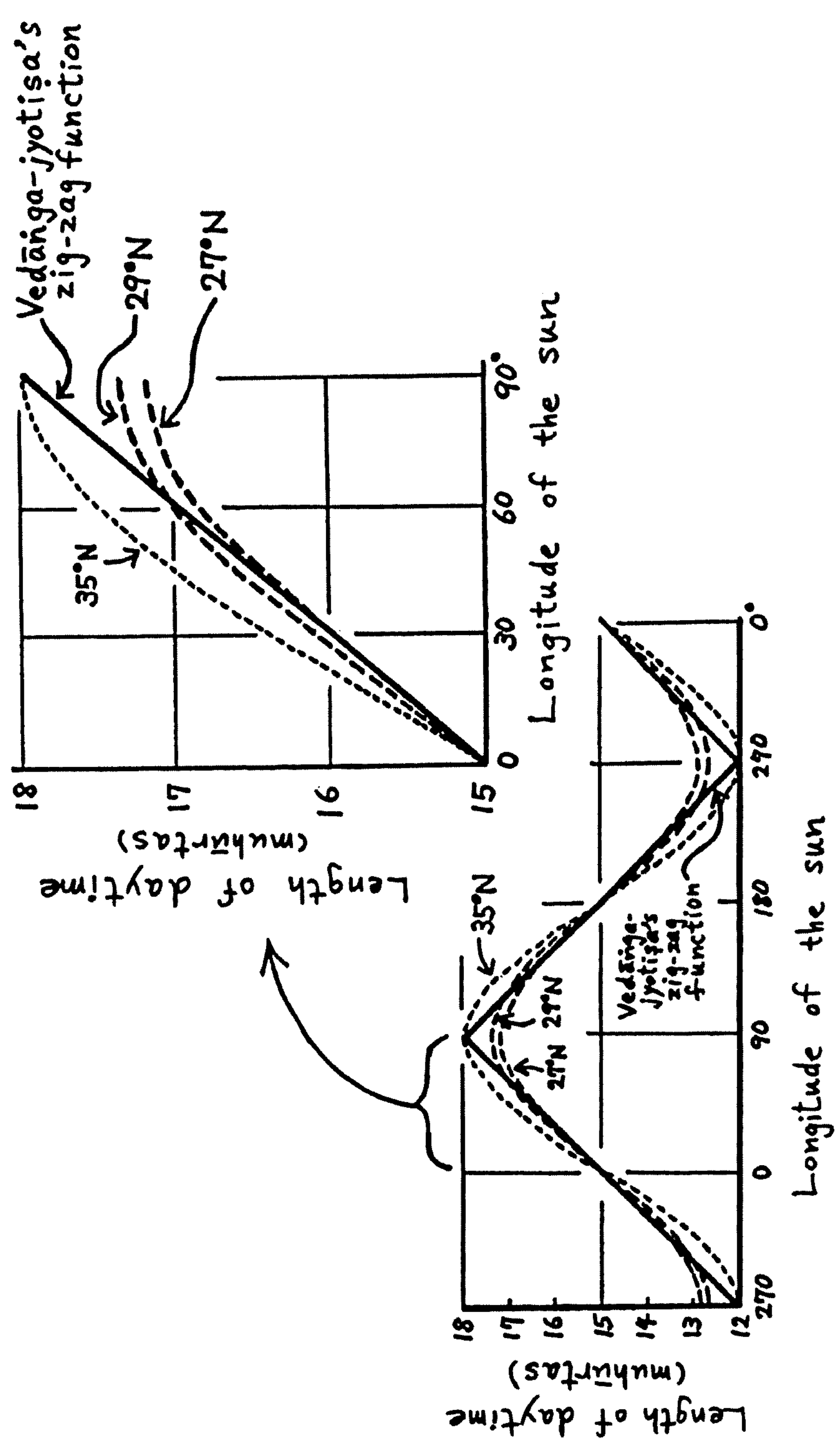

Figure 1 *Vedāṅgajyotiṣa*'s annual variation of the length of daytime

was also translated into Tibetan, and is included in the *bKa'-'gyur*, the Tibetan translation of the Buddhist canon.[3] What is interesting is that the annual variation and diurnal variation of the gnomon shadow are mentioned in some of those texts, and they show that they are also based on observations in North India.

The *Arthaśāstra* (II.20.41–42) gives the annual variation of the gnomon shadow. It is graphed in Figure 2, together with the actual variation at 27°N and 21°N. From this figure, it is clear that the data in the *Arthaśāstra* is based on observation in North India. Similar data are found in the *Śārdūlakarṇāvadāna*, the *Sūriyapannatti*, and other works.

The *Arthaśāstra* (II.20.39–40) gives the diurnal variation of the gnomon shadow. As George Abraham (1981) has pointed out, it follows the following formula:

$$\frac{d}{2t} = \frac{s}{g} + 1,$$

where d/t is the fraction of daytime which has elapsed since sunrise or is remaining until sunset, and s is the length of the gnomon of length g. It is graphed in Figure 3, along with the actual variation at the summer solstice at the Tropic of Cancer (23.7°N in ca. 300 BC). This again shows that this formula was based on observation in North India. The *Sūriyapannatti* provides similar data.

The above discussion shows clearly that Vedāṅga astronomy was produced in North India without explicit foreign influence.

Indian knowledge of natural phenomena and astrology, probably systematized in this period, was later developed into the *saṁhitā* branch of Hindu astronomical science, *jyotiḥśāstra*. This kind of knowledge is found in the *Śārdūlakarṇāvadāna*. There are several recensions of the *Gargasaṁhitā*, which is another group of astrological works of this kind. A Tibetan translation of

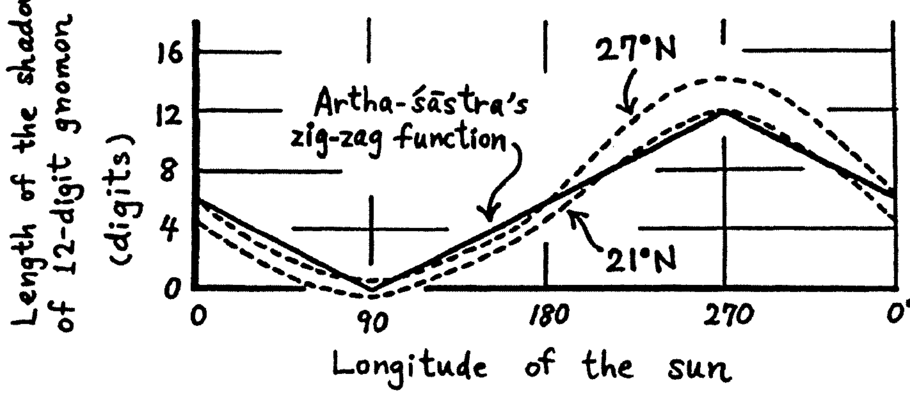

Figure 2 *Arthaśāstra*'s annual variation of the midday shadow

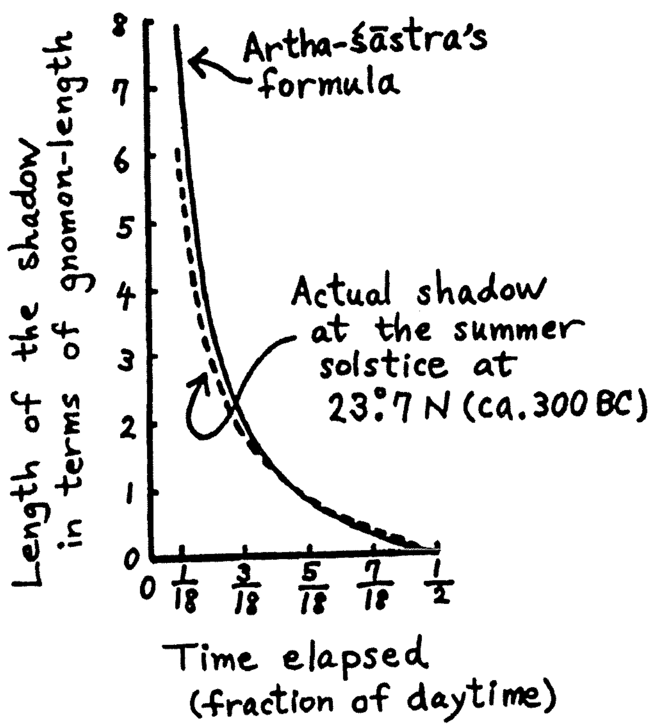

Figure 3 *Arthaśāstra*'s diurnal variation of the gnomon-shadow

one is included in the *bsTan-'gyur*, a collection of the Tibetan translation of the Buddhist treatises.[4]

Indian traditional cosmology, in which the earth is flat, with Mt. Meru (Sumeru) at its centre, must also have developed in this period.

The earliest extant Sanskrit text on Greek horoscopy is the *Yavanajātaka* (AD 269/270) of Sphujidhvaja (Pingree, 1978a). It is based on a work written in the 2nd century AD, which shows that Greek horoscopy must have been introduced into India at this time. During this period, Greek horoscopy, which later developed into the *horā* branch of the Hindu astronomical sciences, spread widely in India. In this system, the zodiacal signs and the positions of the planets are used for astrological purposes. However, Greek mathematical astronomy was not fully introduced at this stage, and the Vedāṅga calendar seems to have been used for some time. Vedāṅga astronomy incorporating Greek influence is found in some Sanskrit texts such as the *Vāsiṣṭhasamāsasiddhānta* (quoted in the *Pañcasiddhāntikā* II. 8–13) and a Chinese Buddhist text *Dafangdeng-dajijing, Ricangfen.*

There is no direct source material illustrating the process of the introduction of Greek mathematical astronomy into India, and the process can only be indirectly surmised from quotations in Varāhamihira's *Pañcasiddhāntikā.* In this period, ca. 4th century AD, a Greek geometrical model of geocentric mathematical astronomy was introduced into India. This kind of model was well accepted and became one of the foundations of Hindu classical astronomy.

At the end of the 5th century AD, Hindu classical astronomy was established.[5] The whole system of Hindu classical astronomical sciences in a wide sense is called *jyotiḥśāstra* and consists of three branches:

1. *Siddhānta* (or *Gaṇita*), mathematical astronomy,
2. *Horā* (or *Jātaka*), horoscopic astrology which is of Greek origin, and
3. *Saṁhitā*, Indian knowledge of natural phenomena and astrology.

Among them, the *siddhānta* section is considered Hindu classical astronomy in a narrow sense. The Sanskrit texts of Hindu classical mathematical astronomy are divided into three types: *siddhānta*, *tantra*, and *karaṇa.* A siddhānta is a fundamental treatise, while a karaṇa is a handy practical work. The main texts from this period are shown in Table 1.

A siddhānta usually consists of two parts, the *grahagaṇitādhyāya* (calculation of the position of planets) and the *golādhyāya* (spherics). The *grahagaṇitādhyāya* further consists of several chapters on such topics as mean motion, true motion, three problems (direction, place, and time), and lunar and solar eclipses, lunar phases, heliacal rising and setting, conjunction of the planets and stars, and planetary nodes. In the calculation of the true planets, the eccentric model and epicyclic model are used.

There were four main schools of Hindu classical astronomy in this period:

1. Ārdharātrika school, whose texts are the *Sūryasiddhānta* (now lost) quoted in the *Pañcasiddhāntikā* and the *Khaṇḍakhādyaka*;
2. Ārya school, whose fundamental text is the *Āryabhaṭīya*;
3. Brāhma school, whose fundamental text is the *Brāhmasphuṭasiddhānta*; and
4. Saura school, whose fundamental text is the *Sūryasiddhānta* (this extant work is sometimes called 'modern *Sūryasiddhānta*', and is different from the lost *Sūryasiddhānta* of the Ārdharātrika school).

Table 1 Main texts of Hindu classical astronomy

Title	Author	Date	Type
Āryabhaṭasiddhānta (fragment)	Āryabhaṭa	5th–6th century AD	
Āryabhaṭīya	Āryabhaṭa	AD 499	unique type
Pañcasiddhāntikā	Varāhamihira	6th century	compilation
Mahābhāskarīya	Bhāskara I	7th century	tantra
Laghubhāskarīya	Bhāskara I	7th century	tantra
Brāhmasphuṭasiddhānta	Brahmagupta	AD 628	siddhānta
Khaṇḍakhādyaka	Brahmagupta	AD 665	karaṇa
Śiṣyadhīvṛddhidatantra	Lalla	ca. 8th century	tantra
Vaṭeśvarasiddhānta	Vaṭeśvara	AD 904	siddhānta
Laghumānasa	Mañjula	AD 932	karaṇa
Sūryasiddhānta	anonymous	ca. 10th–11th century	siddhānta
Siddhāntaśekhara	Śrīpati	11th century	siddhānta
Karaṇaprakāśa	Brahmadeva	AD 1092	karaṇa
Bhāsvatī	Śatānanda	AD 1099	karaṇa
Siddhāntaśiromaṇi	Bhāskara II	AD 1150	siddhānta
Karaṇakutūhala	Bhāskara II	AD 1183	karaṇa

These schools use more or less similar astronomical systems, but their astronomical constants are slightly different.

Kālacakra astronomy, which was introduced into Tibet, is based on Hindu classical astronomy. I shall later show that it is close to the Ārdharātrika school.

From the 13/14th century AD to the 18/19th century AD, Hindu astronomy and Islamic astronomy coexisted. Information about the Hijra is found in the *Kālacakratantra* (11th century AD), which we shall discuss below, but Islamic mathematical astronomy is not found there. Therefore, it should be considered to be in the scope of Hindu classical astronomy and not in the coexistent period of Hindu and Islamic astronomy.

After the establishment of the Delhi Sultanate dynasties in North India, Islamic astronomy was introduced into India systematically. In AD 1370, Mahendra Sūri composed the *Yantrarāja*. This is the first Sanskrit work on the astrolabe and the first Sanskrit work based on Islamic astronomy.[6] At this time, some Sanskrit works on the Hindu astronomical sciences were also translated into Persian by the order of the Sultan of the Tughluq dynasty Fīrūz Shāh (r. AD 1351–1388). These events mark the real beginning of the coexistent period of Hindu and Islamic astronomy.

From the 18th and 19th centuries to the present, modern astronomy has of course been studied in India. At the same time, traditional astronomy is also used for the compilation of the traditional calendar and other uses. The modern period can be said to be the coexistent period of modern and traditional astronomy.

KĀLACAKRA ASTRONOMY

The history of Indian Buddhism is divided into several stages, and Esoteric Buddhism is its last stage. The history of Esoteric Buddhism is further divided into several stages, and the *Kālacakratantra* belongs to its last stage. The *Kālacakratantra* is an Esoteric Buddhist work originally written in Sanskrit[7] and translated into Tibetan and included in the *bKa'-'gyur*.[8] It was also translated into Mongolian, but was not introduced into East Asia (China, Korea, and Japan) in the pre-modern period.

The *Kālacakratantra* consists of five chapters, and the first chapter, 'Lokadhātupaṭala' (chapter of the parts of the world) contains a detailed description of mathematical astronomy. This description has become the basis of Tibetan traditional astronomy. There is an authentic commentary, *Vimalaprabhā*, on the *Kālacakratantra*. It was also originally written in Sanskrit[9] and translated into Tibetan and included in the *bsTan-'gyur*.[10] This is also a very important source for Kālacakra astronomy. According to this text, there was the 'root text' (*Mūlatantra*), where the siddhānta system of astronomy was explained, and the 'abridged text' (*Laghutantra*), in which the karaṇa system of astronomy was explained. The extant *Kālacakratantra* is the abridged text. The root text is not extant, and we only know it from some of its fragments quoted in the *Vimalaprabhā* and other works. It is difficult to say whether the root text really existed as a whole or not. At any rate, both the

siddhānta system (*grub-rtsis* in Tibetan) and the karaṇa system (*byed-rtsis* in Tibetan) were transmitted to Tibet. They are basically similar, and only the length of a year and a month are different.

There are some other related Sanskrit texts on Kālacakra astronomy by Indian authors, such as the *Kālacakrāvatāra* of Abhayākara Gupta (11–12th century AD). Its Tibetan translation is included in the *bsTan-'gyur*.[11]

The origin of Kālacakra astronomy is controversial, and it is the main theme of this essay.

The *Kālacakratantra* (I.27) reads:

> The year elapsed from the year prabhava added to 403 is the 'mleccha's year'. The 'mleccha's lord's year' is diminished by 182, multiplied by 12, and added to the months elapsed from the month Caitra. [This amount is put down at two places,] the value written below is multiplied by 4 and divided by 130, and the result is added to the value written above. This is, oh king, the exact sum of the months. (Translated from the Sanskrit text in Banerjee, 1985: 7).

This calculation can be expressed:

y = years elapsed from prabhava (AD 1027)
m = months elapsed from Caitra
M = sum of the months (counted from Caitra, AD 806)
$M = [(Y + 403 - 182) \times 12 + m] \times (1 + \frac{4}{130})$

The prabhava is the first year of the Indian 60-year cycle. In Tibet, the prabhava in this text is considered to be the first year of the first 60-year cycle of the Tibetan calendar, AD 1027. If so, the initial 'mleccha's year' becomes AD 624 (AD 1027–403). This must have been meant to be the Hijra (AD 622), the first year of the Islamic calendar, because the Sanskrit word 'mleccha' (*kla-klo* in Tibetan, which literally means 'barbarian') in this context meant Muslim. However, there was an error of two years in this text. We shall discuss this later.

There is a Tibetan legend that the *Kālacakratantra* was introduced into Tibet in AD 1027, and had been introduced into India (from a legendary land called Śambhala) 60 years before that, i.e., AD 967. This legend was already mentioned by Csoma de Körös, the pioneer of Tibetology, and is well known (Csoma de Körös, 1834: 183–84, 192). (However, Csoma de Körös wrote in one place that the first prabhava was AD 1025 and in another that it was 1026.) There are some differing speculations by Tibetologists regarding the year of the initial prabhava and the 'mleccha's year'. The traditional Tibetan interpretation is that the initial prabhava was 1027 and the initial 'mleccha's year' AD 624.

Csoma de Körös's determination was that the initial prabhava was 1025 or 1026, and the initial 'mleccha's year' was 622. According to Zuihō Yamaguchi, the 'mleccha's year 403' is 1024, and the initial 'mleccha's year' is 622. In this interpretation, the 'mleccha's year 403' is the 403rd year since the initial 'mleccha's year' and not 403 years after that. AD 1024 coincides with the first year '*jiazi*' of the Chinese 60-year cycle (Yamaguchi, 1992: 882). John Newman suggests that the 'mleccha's year 403' was originally meant to be 403 A.H. which corresponds to AD 1012–1013 (Newman, 1987a: 100). Giacomella Orofino stated that the time from the end of 622 to the beginning of 1026 (the last year of the 60-year cycle which precedes 1027) is a span of 403 years (Orofino, 1994: 15–16).

These are some interpretations from Tibetologists, but we should note that they only considered the chronological aspect and did not take into account other astronomical aspects, such as the motion of planets.

Let us find out the year of the real initial prabhava from an astronomical point of view. By comparing the position of planets in Kālacakra astronomy and modern astronomy, we can prove that the initial prabhava is definitely AD 1027. Figure 4 shows the mean position of the sun, Mars, Jupiter, and Saturn according to Kālacakra astronomy, and Figures 5 and 6 show their positions according to modern astronomy. (This is their mean position, not the true position. As regards their position according to modern astronomy, I connected the position of opposition by straight lines.)[12]

It is clear just by looking that the initial prabhava must be AD 1027. There is no other year which agrees with Kālacakra astronomy's position of the planets. And other astronomical aspects are also harmonious with this identification. Therefore we can conclude definitely that the original author of the *Kālacakratantra* meant that the initial prabhava was 1027.

According to the *Kālacakratantra* (I.27), 403 is 'added (*vimiśra*)' to the years elapsed since the initial prabhava. So there is mathematically no room to doubt that the initial 'mleccha's year' was (AD 1027–403) = AD 624, two years later than the Hijra. According to the calculation of intercalary months in this verse,

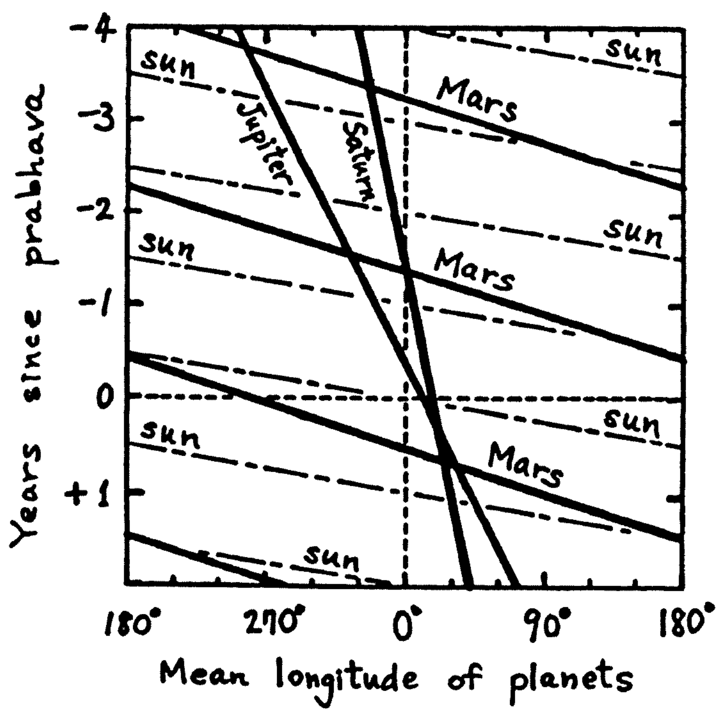

Figure 4 Kālacakra astronomy's mean longitude of the planets

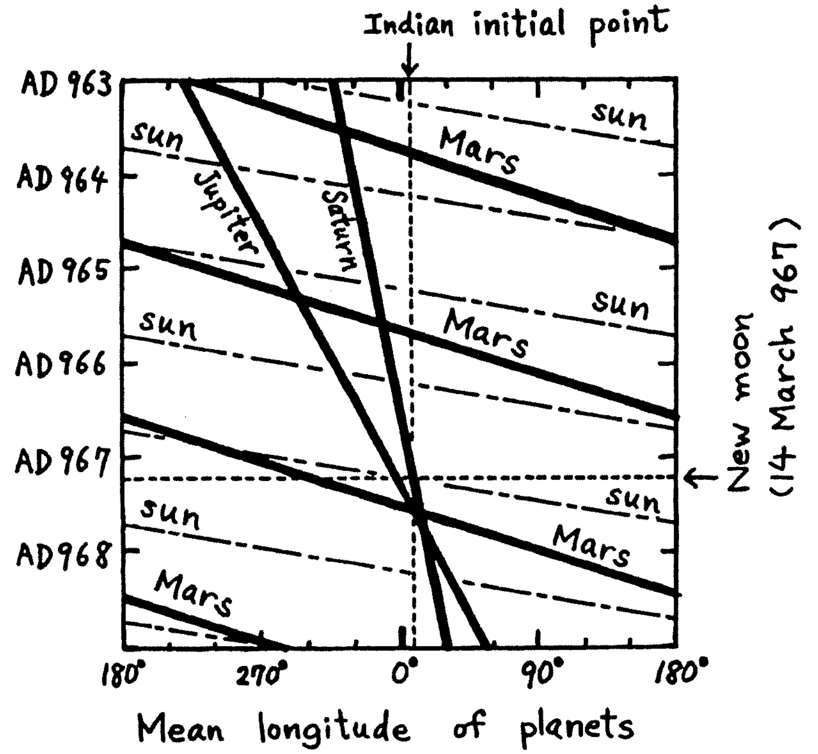

Figure 5 Modern astronomy's mean longitude of the planets (i)

it is clear that 403 years are counted by a luni-solar year, and not by the Islamic lunar year.

The year 806 (= 1027 − 403 + 182), which is mentioned in the *Kālacakratantra* (I.27), may be the epoch of the original astronomical work which was used by the compiler of the *Kālacakratantra*. As far as I know, there is no extant Sanskrit work whose epoch is 806. It may be noted here that a Chinese Buddhist astronomical work, *Qiyaorangzaijue*,[13] compiled by a West Indian Brahman Jinjuzha, uses 806 as the epoch for the calculation of the position of Rāhu (lunar ascending node) and Ketu. According to Michio Yano (1986: 31), Ketu in this text is the lunar apogee. This is a peculiar definition, which is not found in Indian texts. Usually, Ketu is considered to be the lunar descending node in some texts and to be comets in some others. Now, let us compare the orbital period of Rāhu in the *Kālacakratantra* and the *Qiyaorangzaijue*. According to the Kālacakratantra, it is 230 synodic months (= 6792.02 days). In the *Qiyaorangzaijue*, Rāhu revolves 5 times in 93 years, which means that its orbital period is about 6793.65 days. (The *Qiyaorangzaijue* also describes Rāhu's motion in some other ways which show that its orbital period is 6791.43 days, 6900.39 days, 6939.75 days, etc.) As these two texts do not agree exactly, we cannot say that Kālacakra astronomy is related to the *Qiyaorangzaijue*.

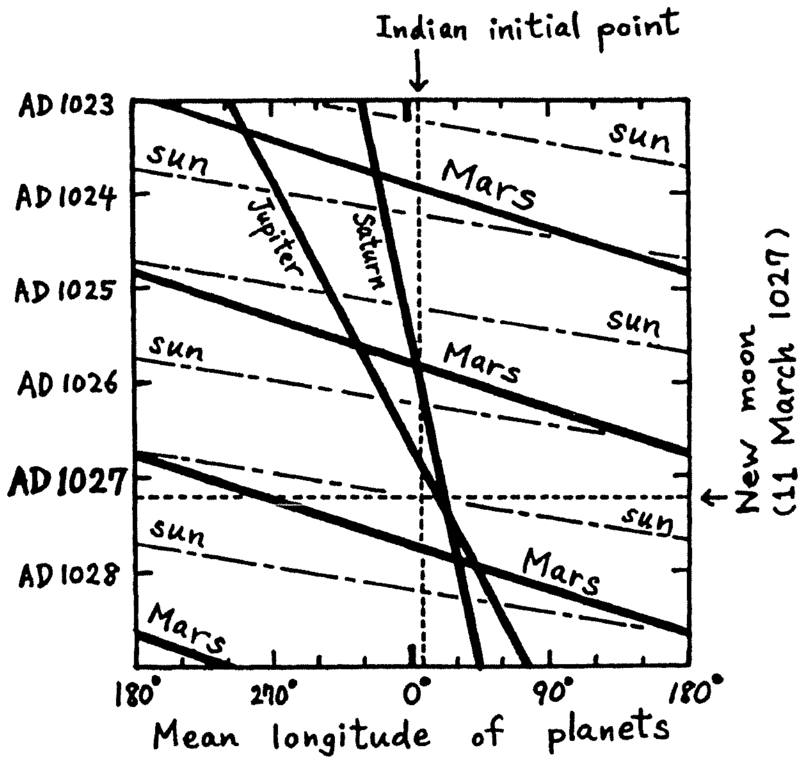

Figure 6 Modern astronomy's mean longitude of the planets (ii)

We already have seen that the initial 'mleccha's year' in the *Kālacakratantra* is 624, two years later than the Hijra. Let us discuss the reason for this two years' error. Let us start from the examination of the 60-year cycle, because the prabhava is the first year of the Indian 60-year cycle. There are two different origins of the 60-year cycle in the East, one Chinese and the other Indian. The Chinese cycle is counted successively regardless of the actual motion of the planet Jupiter, while there are two major systems of the Indian 60-year cycle. One is the North Indian reckoning, where the omission of every 86th year has been maintained in order to keep pace with the motion of Jupiter, whose orbital period is not exactly 12 years but about 11.86 years. The other is the South Indian reckoning, where years are counted successively regardless of the actual motion of Jupiter.[14] The beginnings of the cycle around 1027 according to these systems are:

Chinese:	AD 1024.
North Indian:	AD 1025.
South Indian:	AD 1027.

It is clear that Kālacakra astronomy follows the South Indian reckoning, because the initial prabhava in Kālacakra astronomy is 1027 and the cycle is counted successively there. However, we should also note that 1025, the year

prabhava according to North Indian reckoning, is exactly 403 years after 622, i.e. the year of the Hijra. From these facts, I suspect that Kālacakra astronomy itself is based on South Indian reckoning, but the information for the year of the Hijra is based on North Indian reckoning and was mistakenly incorporated into Kālacakra astronomy. I shall show in the next section that Kālacakra astronomy is similar to the Ārdharātrika school of Hindu classical astronomy which was popular in East India. It is not strange that certain North Indian elements are found there.

From the above considerations, we can suppose that Kālacakra astronomy is a mixture of South and North Indian astronomy. It may be noted here that Kālacakra astronomy originally had nothing to do with the Chinese 60-year cycle system.

The traditional story of the origin of the *Kālacakratantra* is that the *Kālacakra-mūla-tantra* (the Kālacakra root text, which is not extant, but whose fragments are quoted in some texts) was taught by Buddha to Sucandra at Dhānyakaṭaka and was brought to Śambhala (which is said to be in the north of the Śītā river) by Sucandra. It was abridged by Yaśas (the first Kalkin of Śambhala) as the *Kālacakra-laghutantra* (Kālacakra 'abridged' text, which is now extant, and usually simply called *Kālacakratantra*), and its commentary *Vimalaprabhā* (also extant) was composed by Puṇḍarīka (Yaśas's son, and the second Kalkin of Śambhala).

This story, which basically originated in the *Vimalaprabhā* itself and was repeated in Tibetan works, is a kind of religious legend and cannot be accepted as historical fact. Here, we should note that Buddha is considered to be the 9th incarnation of Viṣṇu in Vaishnavism, Kalkin is the 10th and last incarnation of Viṣṇu, and Śambhala is a fabulous place where Kalkin is supposed to appear, according to Vaishnavism. The *Kālacakratantra* itself is a Buddhist text, but we should keep in mind that there is a certain influence of Vaishnavism.

We shall not go into the religious aspect here, but instead we will discuss the original place of Kālacakra from an astronomical point of view. Before starting our discussion, let us look at some previous interpretations of the origin of the *Kālacakratantra*.

1. Csoma de Körös says: 'The Kála Chakra doctrine of Adibuddha was delivered by Shákya, in his 80th year, at Shrí Dhanya kataka, (Cuttak in Orissa, ...)' (1834: 192). 'The peculiar religious system entitled the Kála Chakra is stated, generally, to have been delivered from Shambhala – a fabulous country in the north – situated between about 45° and 50° north latitude, beyond the Sita or Jaxartes, where the increase of the days from the vernal equinox till the summer solstice amounted to 12 Indian hours, or 4 hours, 48 minutes, European reckoning.' (1833: 57)
2. Alexander Cunningham located Dhanakataka, mentioned by the famous Chinese traveller Xuanzang, in Dhâranikotta, or Amaravati in Andhra in South India (1871: 447–459). Referring to this identification, Shōun Toganō wrote that the *Sarvatathāgata-tattvasaṁgraha*, a famous Esoteric Buddhist text, was actually preserved in Amaravati, and that this fact developed into

the legend that Kālacakra was taught in Dhānyakaṭaka, or Amaravati (1925).

3. Benoytosh Bhattacharyya wrote that Uḍḍiyāna, which is the place where Tantric Buddhism first developed, and is divided into two kingdoms, Śambhala and Laṅkāpuri, would have to be located in Assam, probably in the western part (1932: 45). Referring to this identification, Shōun Toganō wrote in his pioneering study of Kālacakra Buddhism that Kālacakra Buddhism arose in Śambhala in East India.[15]
4. Hakuyū Hadano wrote that Kālacakra is a product of Indian Tantric Buddhism, and that the story of Śambhala is just a fiction borrowed from Vaishnavism.

It is now generally accepted that Dhānyakaṭaka is Amaravati,[16] but the location of Śambhala is still controversial.

If certain astronomical data, such as the length of the midday shadow on a certain day of the year, or the annual variation of the length of daytime, are given, it is possible to estimate the latitude of the place where these data were observed. Csoma de Körös' estimation, which we already have seen, is of this kind. We should keep in mind here that we should not rely too much on this kind of estimation, because the original data may not be so exact.

Now, let us examine the original text of the *Kālacakratantra*, and check Csoma de Körös' estimation.

The *Kālacakratantra* (I.38 b–d) reads:

According to the movement of the sun, there are decreases and increases of day and night during six months. There are increases and decreases of 3 liptās and 4 prāṇas every day during a half year. There is an increase of nighttime in the southern course [of the sun], and of daytime in the northern course at Himagiri [mountain]. (Translated from the Sanskrit text in Banerjee, 1985: 10)

The units of time used here are:

One day = 60 *nāḍīs* (or *nāḍikās*, *ghaṭīs*, *ghaṭīkās*, etc.)
1 *nāḍī* = 60 *liptās* (or *vināḍīs*, etc.)
1 *liptā* = 6 *prāṇas*

According to the above text, the length of daytime changes by $((3 + \frac{4}{6}) \times 182.6) = 669.5$ liptās or 11.16 nāḍīs during a half year. Therefore, it changes by about 5.6 nāḍīs during a quarter year (from the equinox to the solstice). This text does not agree with the statement by Csoma de Körös. Let us see some other verses of the *Kālacakratantra*. Verse I.54 d reads:

For a day and night of 60 nāḍīs, there are decreases and increases by one sixth [of 60 nāḍīs during a half year] on account of the sun and moon. (Translated from the Sanskrit text in Banerjee, 1985: 14)

One sixth of 60 nāḍīs is 10 nāḍīs. That this is for a half year and not for a quarter year is attested by the subsequent verse (I.62 c–d) which reads:

[When the sun is at] the middle of the celestial sphere (i.e. equinox), the day and night are 30 nāḍīs [respectively]. [When the sun is at] the southern and northern hemispheres, there are

decreases and increases of 5 [nāḍīs during a quarter year], and they are 3 liptās and 2 *śvāsas* (= prāṇas) per day. (Translated from the Sanskrit text in Banerjee, 1985: 16)

From the fact that $((3 + 2/6) \times 91.3) = 304.3$ liptās or about 5 nāḍīs, it is clear that 5 nāḍīs in this verse are for a quarter year, and that 10 nāḍīs in the above verse are for a half year.

There is an interesting commentary in the *Vimalaprabhā* on the above verses. It comments of the *Kālacakratantra* (I.54 d).

... Here is the rule of shadow (i.e. solar motion) in the area of Kailāsa, which is not in the country of Ārya (India). In the country of Ārya, there are decreases and increases of one tenth [of a day] from the winter solstice to the summer solstice and from the summer solstice to the winter solstice, on account of the rule of shadow. Similarly, it should be shown that in such an area as Bhoṭa (Tibet), Līca, and Cīna (China), there are decrease[s] and increase[s] by one ninth, one eighth, and one seventh, on account of the rule of shadow, as far as the country of Śambhala. (Translated from the Sanskrit text in Upadhyaya, 1986: 101)

The *Vimalaprabhā* also comments on the *Kālacakratantra* (I.62 c–d). This is in agreement with the above commentary. '... This measure is for the area of Kailāsa, and not for the country of Ārya. It should be known that in the country of Ārya, there are increases and decreases of 2 *liptās* per day' (p. 107).

As regards the relationship between Kailāsa and Śambhala, the *Kālacakratantra* (I.151) tells that Śambhala is in the southern half of Kailāsa. The statement in the *Vimalaprabhā* can be tabulated as in Table 2.

This statement from the *Vimalaprabhā* is very strange. The value given in the *Kālacakratantra* (I.54 d and 62 c–d) already fits the Indian latitude but does not fit the northern latitude. On the contrary, the value for the 'country of Ārya' given in the *Vimalaprabhā* is too small for an Indian latitude. In order to make this more intelligible, I have graphed the value of the *Kālacakratantra* (I.54d), that of the *Kālacakratantra* (I.62 c–d), and the value for the 'country of Ārya' given in the *Vimalaprabhā* together with the actual seasonal variation of the length of daytime at the latitude 27°N in Figure 7. It is clear just by looking that the value given in the *Kālacakratantra* is more or less similar to the ancient Indian zig-zag function shown in Figure 1. Therefore, it is most probable that Kālacakra astronomy originated in India, and the statement in the *Vimalaprabhā* is just a fiction in order to make it somewhat mysterious. In any case, the conjecture of Csoma de Körös that the Śambhala is actually situated between 45°N and 50°N is not supported by the original text.

Now, let us discuss the place of the formation of Kālacakra astronomy from

Table 2 The change of the length of daytime according to the *Vimalaprabhā*

	Change in a half year	Change in a quarter year
Kailāsa	$\frac{1}{6}$ of a day (10 nāḍīs)	5 nāḍīs
Cīna (China)	$\frac{1}{7}$ of a day (8.57 nāḍīs)	4.29 nāḍīs
Līca	$\frac{1}{8}$ of a day (7.5 nāḍīs)	3.75 nāḍīs
Bhoṭa (Tibet)	$\frac{1}{9}$ of a day (6.67 nāḍīs)	3.33 nāḍīs
Country of Ārya (India)	$\frac{1}{10}$ of a day (6 nāḍīs)	3 nāḍīs

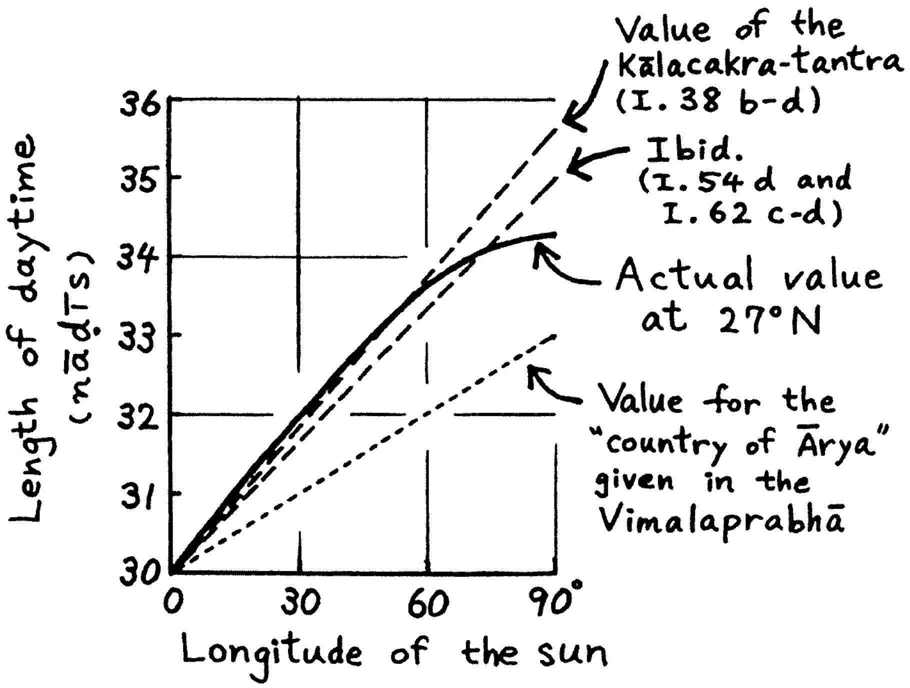

Figure 7 *Kālacakratantra* and *Vimalaprabhā*'s seasonal variation of the length of daytime

another point of view, which will also suggest that it is in India. Kālacakra astronomy is based on the eccentric and epicyclic model. Therefore, it is clear that it is not based on Chinese mathematical astronomy which did not use a geometrical model but a kind of arithmetical method which was probably empirical. We have also seen that the original Kālacakra astronomy has nothing to do with the Chinese 60-year cycle, which is now used in Tibet and some other countries. From these facts, we can say that there is no influence of Chinese mathematical astronomy upon Kālacakra astronomy.

As for the possibility of the influence of Islamic mathematical astronomy, we have seen that information about the Hijra is included in Kālacakra astronomy, although there is an error of two years. However, this cannot be adequate proof of the influence of Islamic astronomy or its calendar, because information about the Hijra can be communicated without exact knowledge of astronomy or the calendar. In order to discuss the possibility, we have to compare these astronomical systems. The Kālacakra, Islamic, and Hindu classical astronomies use the eccentric and epicyclic model, but astronomical constants are slightly different among them. Let us compare the orbital period of five planets. Although there are some differing schools of Hindu classical astronomy, the difference of the orbital periods of five planets is very small, and we can consider for this purpose that they are approximately the same. The orbital periods of five planets in terms of days, according to Ptolemaic-Islamic, Hindu Classical, Kālacakra, and modern astronomies are shown in Table 3.

We cannot rely too much on the comparison of astronomical constants,

Table 3 The orbital periods of five planets in terms of days

	Ptolemaic-Islamic	Hindu Classical	Kālacakra	Modern
Mercury	87.97	87.97	87.97	87.97
Venus	224.7	224.7	224.7	224.7
Mars	686.9	687.0	687	687.0
Jupiter	4331.0	4332.3	4332	4332.6
Saturn	10750	10766	10766	10759

because people are looking at the same celestial phenomena, and it is possible that the same constants are obtained independently. However, if the same inexact value is used at different places, it suggests a kind of relationship. In the above comparison, the orbital period of Saturn strongly suggests that Kālacakra astronomy is only based on Hindu Classical astronomy, and that there is no explicit influence of Islamic mathematical astronomy. Kālacakra astronomy's relationship with Hindu Classical astronomy will become clearer by comparing other astronomical constants.

There are four main schools of Hindu Classical astronomy. Their astronomical systems are more or less similar, but they use slightly different astronomical constants. Let us compare the astronomical constants of Kālacakra astronomy with those of the schools of Hindu Classical astronomy and also with Ptolemaic-Islamic astronomy. Table 4 compares of the maximum equation of centre of five planets, Table 5 the longitude of their apogee, and Table 6 their maximum epicyclic correction.

Table 4 Comparison of the maximum equation of centre

		Hindu Classical astronomy				
	Ptolemaic	Ārdharātrika	Ārya	Brāhma	Saura	Kālacakra
Mercury	3° 2′	4° 28′	3° 55′	6° 3′ 33″	4° 28′	4° 26′ 40″
Venus	2° 24′	2° 14′	1° 26′	1° 45′ 3″	1° 45′	2° 13′ 20″
Mars	11° 25′	11° 13′	13° 7′	11° 12′ 41″	11° 32′	11° 6′ 40″
Jupiter	5° 15′	5° 6′	5° 43′	5° 15′ 35″	5° 6′	5° 6′ 40″
Saturn	6° 31′	9° 36′ 55″	9° 32′	4° 46′ 47″	7° 40′	9° 33′ 20″

Table 5 Comparison of the longitude of apogee

		Hindu Classical astronomy				
	Ptolemaic	Ārdharātrika	Ārya	Brāhma	Saura	Kālacakra
Mercury	190°	220°	210°	224° 54′	220° 27′	220°
Venus	55°	80°	90°	81° 15′	79° 50′	80°
Mars	115° 30′	110°	118°	128° 24′	130° 2′	126° 40′
Jupiter	161°	160°	180°	172° 32′	171° 18′	160°
Saturn	233°	240°	236°	260° 55′	236° 37′	240°

Table 6 Comparison of the maximum epicyclic correction

		Hindu Classical astronomy				
	Ptolemaic	Ārdharātrika	Ārya	Brāhma	Saura	Kālacakra
Mercury	21° 2′	21° 30′	21° 57′	21° 31′ 30″	21° 31′	21° 33′ 20″
Venus	45° 57′	46° 15′	53° 37′	46° 22′ 54″	46° 24′	46° 13′ 20″
Mars	41° 9′	40° 30′	44° 53′	42° 37′ 39″	40° 16′	40° 26′ 40″
Jupiter	11° 3′	11° 30′	10° 53′	10° 53′ 19″	11° 31′	11° 33′ 20″
Saturn	6° 13′	6° 20′	5° 44′	5° 34′ 46″	6° 22′	6° 13′ 20″

There are some variations in Islamic astronomy, but I only quote Ptolemy's original value which is enough for this purpose.[17] The original sources of the schools of Hindu Classical astronomy are the *Sūryasiddhānta*, quoted in the *Pañcasiddhāntikā*, for the Ārdharātrika school, the *Āryabhaṭīya* for the Ārya school, the *Brāhmasphuṭasiddhānta* for the Brāhma school, and the modern *Sūryasiddhānta* for the Saura school.[18]

From the above comparison, it is clear that the astronomical constants of Kālacakra astronomy are very close to those of the Ārdharātrika school of Hindu Classical astronomy. Especially, the longitude of the apogee of five planets is exactly the same except for that of Mars. It is also clear that these astronomical constants of Kālacakra astronomy are different from those of Ptolemaic-Islamic astronomy. Therefore, we again can conclude that Kālacakra astronomy was not influenced by Islamic mathematical astronomy.

As regards the relationship between the Ārdharātrika school and Buddhist astronomy, we have some other examples. In the 17th century, an astronomical work was procured from Siam (now Thailand) by a French ambassador, Simon de la Loubère, and was studied by J. D. Cassini (de la Loubère, 1693: 186–227). S. B. Dikshit (1981: 378) pointed out that the length of the year in this work was the same as that of the original *Sūryasiddhānta* (quoted in the *Pañcasiddhāntikā*) and the *Khaṇḍa-khādyaka*, and that the work may have followed either the original *Sūryasiddhānta* or some karaṇa work by Āryabhaṭa I, based on the *Sūryasiddhānta*, which is now lost. Also, in the Tang dynasty of China, an unofficial calendar, *Jiuzhi-li* (AD 718) was composed by Qutan Xida (probably a Chinese transliteration of *Gotama-siddha*) and was included in his *Kaiyuanzhanjing*. Kiyosi Yabuuti pointed out that this calendar is related to the Ārdharātrika school (Yabuuti, 1979; Yano, 1979). From these facts, we can suppose that the astronomical system of the Ārdharātrika school was quite popular among Indian Buddhists for a certain period.

The main Sanskrit texts of the Ārdharātrika school are the *Āryabhaṭasiddhānta* (only fragments are extant) of Āryabhaṭa, the *Sūryasiddhānta* quoted in the *Pañcasiddhāntikā* of Varāhamihira, the *Khaṇḍakhādyaka* (AD 665) of Brahmagupta, and the *Bhāsvatī* (AD 1099) of Śatānanda, among others. Brahmagupta was a resident of Bhillamāla in the southern border of Rajasthan. According to an eminent historian of Indian astronomy, S. B. Dikshit (1853–1898), the *Khaṇḍakhādyaka* was still in use in Kashmir (1981: 89). David

Pingree, who is surveying Sanskrit manuscripts of astronomical works, says the *Khaṇḍakhādyaka* remained the standard karaṇa in Kashmir, Nepal, and Assam till modern times, but in the medieval period it was popular throughout North and West India (1981: 33). Śatānanda is said to have been a resident of Puruṣottamapurī (Puri in Orissa). According to Dikshit, the *Bhāsvatī* was well known in North India (1981: 112); according to Pingree, it was popular in North and Northeast India and in Nepal (1981: 35). We can therefore conclude that the astronomical system of the Ārdharātrika school was well known in North India.

Kālacakra Buddhism was also popular in East India at one time. The *History of Buddhism in India* (AD 1608), written by a famous Tibetan scholar, Tāranātha, reads: 'Piṭo ācārya brought the Kālacakra Tantra during the latter half of the life of Mahīpāla, but he spread it during the period of this king (Mahāpāla).' (Translated by Lama Chimpa and Alaka Chattopadhyaya, 1980: 289)

Mahīpāla and Mahāpāla were the kings of the Pāla dynasty of East India. As the astronomical system of the Ārdharātrika school, which must be the basis of Kālacakra astronomy, was popular in East India also, the most probable place of the formation of Kālacakra astronomy must be East India.

In sum, Kālacakra astronomy must have been based on the Ārdharātrika school of Hindu Classical astronomy without the influence of Islamic and Chinese mathematical astronomy. It must have been formed in India, most probably in East India, where both the Ārdharātrika school and Kālacakra Buddhism are known to have been popular. We should also recall that the South Indian reckoning of the 60-year cycle is used in Kālacakra astronomy along with the North Indian reckoning of the 60-year cycle. Therefore, Kālacakra astronomy must be a mixture of East and South Indian astronomy. It must have been formed in the 11th century AD, because it used 1027 as the initial year of the 60-year cycle.

A BRIEF HISTORY OF THE DEVELOPMENT OF THE KĀLACAKRA ASTRONOMY IN TIBET

There are several Tibetan texts on sKar-rtsis or Tibetan astronomy based on Kālacakra astronomy, and we hope future scholars will investigate these texts fully. I will only give a brief history of its development.[19]

The Kālacakra calendar has been used in Tibet from about the 12th century AD. In the early years, Tibetan astronomical works were written by two scholars of the Sa-skya sect of Tibetan Buddhism, Grags-pa-rgyal-mtshan (1147–1216) and 'Phags-pa (1235–1280).[20]

In the 14th century, Bu-ston Rin-chen-grub (1290–1364) composed a comprehensive treatise on Kālacakra astronomy entitled *mKhas-pa-dga'-byed* (1326) (Ōhashi, 1984, 1986 and 1997c). Also in the 14th century, Raṅ-byuṅ-rdo-rje (1284–1339) composed the *rTsis kyi bstan-bcos kun las btus-pa'i rtogs-pa* (1318) (Schuh, 1973: 34–36). I regret that I have not seen this text.

In the 15th century, lHun-grub-rgya-mtsho composed the *Pad-dkar-žal-luṅ* (1447),[21] and his system was developed as the Phug school. The most famous work of the Phug school is the *Vaiḍūrya-dkar-po* (1683) of Saṅs-rgyas-rgya-mtsho, who was the regent of the fifth Dalai Lama. Another famous work of this school is the *Ñin-byed-snaṅ-ba* (1714) of Dharmaśrī (= Chos-dpal). In 1827, Phyag-mdzod-gsuṅ-rab wrote the *Rigs-ldan-sñiṅ-thig*, which is based on the *Vaiḍūrya-dkar-po* and the *Ñin-byed-snaṅ-ba*. In 1916, mKhyen-rab-nor-bu built a college called sMan-rtsis-khaṅ (House of Medicine and Astronomy) in Lhasa under the thirteenth Dalai Lama (Rechung, 1973: 22–25). mKhyen-rab-nor-bu reedited the *Rigs-ldan-sñiṅ-thig* in 1927, and it is the basis of the calendar edited in the sMan-rtsis-khaṅ.[22] In 1987, a Chinese Tibetologist, Huang Mingxin and a Chinese historian of astronomy, Chen Jiujin, published the original Tibetan text of the *Rigs-ldan-sñiṅ-thig* with a Chinese translation and detailed astronomical commentary. This is a very good introduction to Tibetan astronomy.

There is another school called mTshur-phu, whose most influential text is the *Ṅer-mkho-bum-bzan* (1732) of Karma Ṅes-legs-bstan-'dzin. It is said that the astronomical system of the mTshur-phu school originated in the work of Raṅ-byuṅ-rdo-rje. The Phug school and the mTshur-phu school are the main schools of Tibetan astronomy in modern Tibet.

Kālacakra astronomy is also followed in Mongolia. Traditional Mongolian astronomy basically follows the *dGe-ldan-rtsis-gsar* (1747) of Sum-pa-mkhan-po (1702–1774), a famous Tibetan Buddhist scholar. At present, the Mongolian traditional almanac is edited by Dr. Terbish Lhasran of Mongolian State University and published yearly.[23]

Kālacakra astronomy is also followed in Bhutan. According to Yoshiro Imaeda, the calendrical system on which the official calendar of Bhutan is based was established by Padma-dkar-po (1527–1592) of the 'Brug-pa sect of Tibetan Buddhism and followed by his disciple, lHa-dbaṅ-blo-gros (1549/50–1632). It was introduced into Bhutan by Źabs-druṅ Ṅag-dbaṅ-rnam-rgyal (1594–1651) (Imaeda, 1984). I have not seen the work of Padma-dkar-po,[24] but have seen the works of lHa-dbaṅ-blo-gros (Sureśamati) and Lo-chen Ṅag-dbaṅ-dpal-ldan-bzaṅ-po published by the National Library of Bhutan. It appears that the most fundamental text of this system is the *gDan-dus-thun-moṅ gi rtsis-gźi* of lHa-dbaṅ-blo-gros. At present, the Bhutanese traditional almanac is published yearly by the Council for Ecclesiastical Affairs, Punakha and Thimphu, Bhutan.[25]

Lastly, mention may be made of the *rGya-rtsis*, which is not based on Kālacakra astronomy but on the Chinese Shixian calendar. The Shixian calendar is the last luni-solar calendar in China, and was officially used from 1645. This is a luni-solar calendar in Chinese traditional style, but certain elements of European astronomy based on the system of Tycho Brahe, introduced by Jesuits such as Adam Schall, have been adopted. The Chinese treatise on this astronomical system *Xiyang xinfa suanshu* (1669) was first translated into Mongolian and published in 1711, and then translated into Tibetan from the Mongolian version and published in 1715. After this, some more practical works on this system were written in Tibetan, and the method to predict solar and lunar eclipses according to this system is now used in the Tibetan traditional almanac of sMan-rtsis-khaṅ, although the almanac of sMan-rtsis-khaṅ basically follows Kālacakra astronomy. This *rGya-rtsis* was studied by Huang Mingxin and Chen Jiujin in detail in 1987.

Tibetan traditional astronomers are still active. A comprehensive treatise on Tibetan astronomy in five large volumes was recently edited by Byams-pa-'phrin-las (1998).

AN OUTLINE OF TIBETAN KĀLACAKRA ASTRONOMY

I would like to present a rough outline of Tibetan Kālacakra astronomy and the astronomical meaning of Tibetan astronomical terms.

The Tibetan Kālacakra calendar is a luni-solar calendar, with two intercalary months for 65 ordinary months. One year is a sidereal year just like in the Hindu traditional luni-solar calendar. One month is a synodic month which ends with the new moon.

There are three kinds of days. A *ñin-źag* is a civil day measured from sunrise to sunrise. A *tshes-źag*, which corresponds to the Sanskrit *tithi*, is a 30th part of a synodic month, during which the longitudinal distance between the sun and moon changes by 12°. A *khyim-źag* is a 360th part of a sidereal year. One day is divided into 60 *chu-tshod*, one *chu-tshod* is divided into 60 *chu-sraṅ*, and one *chu-sraṅ* is divided into 6 *dbugs*.

The ecliptic is divided into 12 *khyim* or zodiacal signs, and also into 27 *rgyu-skar* or lunar mansions. One *rgyu-skar* is divided into 60 *chu-tshod*, one *chu-tshod* into 60 *chu-sraṅ*, and one *chu-sraṅ* into 6 *dbugs*. As Kālacakra astronomy is based on Hindu Classical astronomy, the ecliptical coordinates are fixed at their position on the celestial sphere in the 6th century AD or so and do not shift by precession. Therefore, one revolution of the sun on these coordinates is the same as one sidereal year. This method is the same as that of the Hindu traditional calendar.

The orbital period of a heavenly body is called *dkyil-'khor*, and its mean daily motion is called *rtag-pa'i-loṅs-spyod*.

As regards the position of planets, their mean motion is calculated first, then the *dal-ba'i-las* or operation of the equation of centre and the *myur-ba'i-las* or operation of the epicyclic correction are applied. In the case of the sun, only the *dal-ba'i-las* is applied; in the case of the moon, only the *myur-ba'i-las*. It may be that as the movement of the lunar apogee is rapid, the correction of the lunar inequality was considered to be the epicyclic correction rather than the equation of centre or the eccentric correction. An anomalistic month is roughly considered to be 28 *tshes-źag* first, and a correction is applied to the length of each *tshes-źag*. Then a special correction is applied so as to diminish the period of the anomalistic month, because the period of 28 *tshes-źag* is a little longer than the actual anomalistic month.

In the case of the *dal-ba'i-las*, the amount of the correction is given by *dal-rkaṅ* (slow step) for each *khyim*. The *dal-rkaṅ* is the difference between the mean motion and true motion of the sun or planet during one *khyim*'s movement of the mean sun or planet in terms of *chu-tshod*. The ecliptic is divided into two parts: the *rim-pa*, from the apogee to the perigee, and the *rim-min*, from the perigee to the apogee. Each half is subdivided into the *sṅa-rkaṅ* and the *phyi-rkaṅ*.

In the case of five planets, the *myur-ba'i-las* is further applied. The five planets are divided into two groups: the *źi-ba'i-gza'*, which correspond to the inner planets, and the *drag-gza'*, which correspond to the outer planets. The *źi-ba'i-gza'* includes lhag-pa (Mercury) and pa-saṅs (Venus). The *drag-gza* includes mig-dmar (Mars), phur-bu (Jupiter), and spen-pa (Saturn). The epicyclic model of planets is shown in Figure 8. In the figure, A is the earth, B the centre of the epicycle which is on the deferent, C the planet on the epicycle, and ϒ the direction of the first point of Aries.

In the case of the inner planets, B in Figure 8 is the mean sun corrected by the planet's equation of centre, and D the direction of the true sun. The motion of the planet on the epicycle is called *myur-rkaṅ-'dzin* (parameter of fast step), and the amount of its motion corresponds to ∠CBϒ in the figure. The daily motion of the *myur-rkaṅ-'dzin* minus the true daily motion of the sun is the daily motion of the *rkaṅ-'dzin* (parameter of step), and the amount of its motion corresponds to ∠CBD in the figure. The *rkaṅ-'dzin* is used to count the steps of epicyclic correction. The period of 60 *chu-tshod*'s change of the *rkaṅ-'dzin* is considered to be one step, and there are 27 steps (or more exactly two sets of 13 steps and a half step) in one cycle. The amount of the epicyclic correction

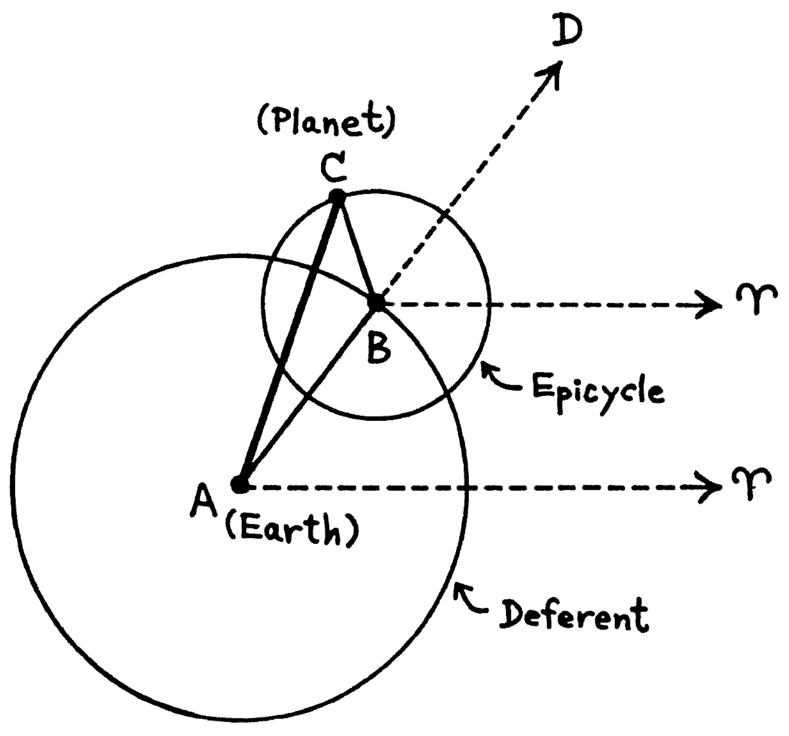

Figure 8 Epicyclic model of the planets

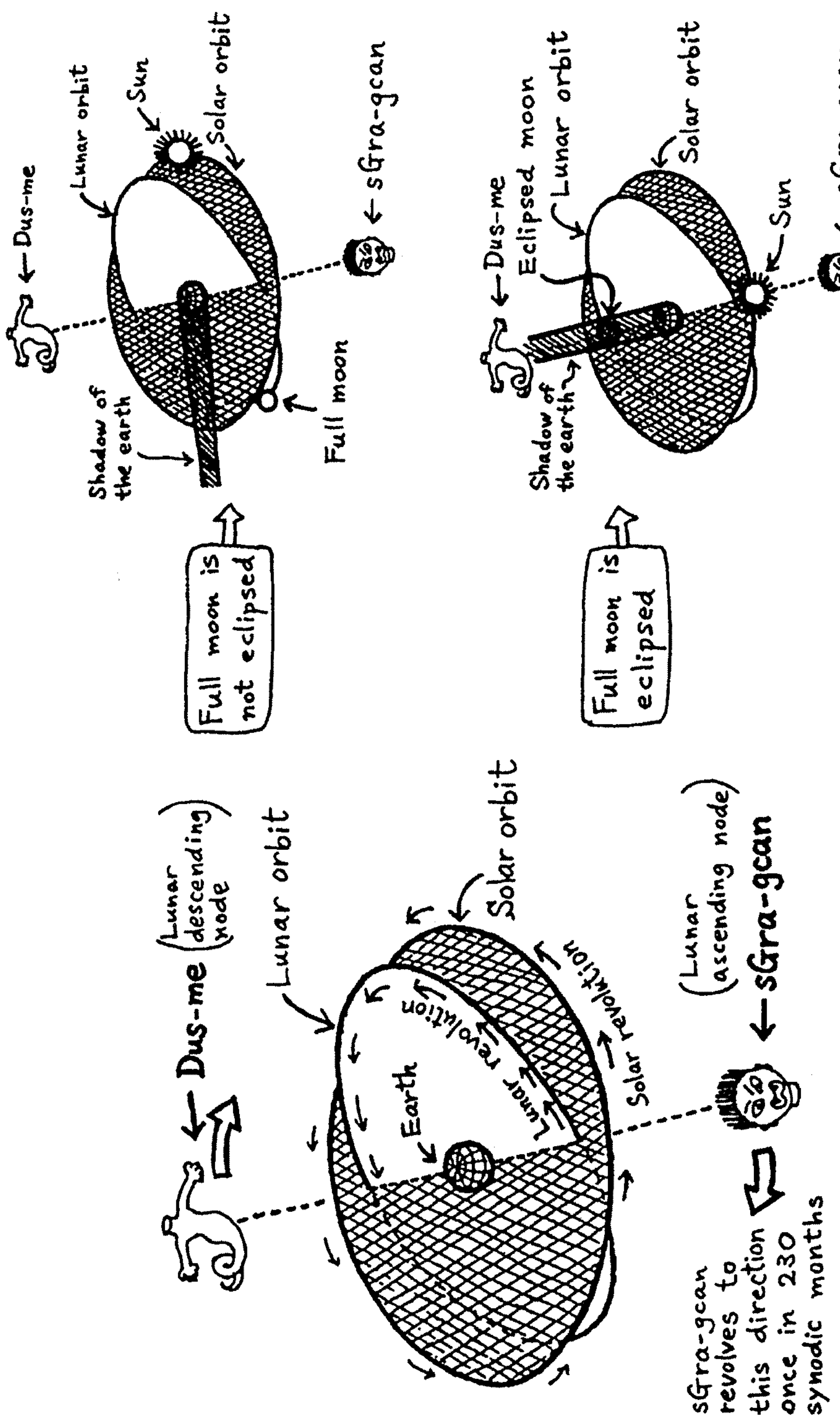

Figure 9 sGra-gcan and Dus-me

is given by the *myur-rkaṅ* (fast step) for each step. The *myur-rkaṅ* is the amount of the epicyclic correction during one step in terms of *chu-tshod*. This correction is applied to the movement of the mean planet (= mean sun) corrected by the planet's equation of centre (point B in the figure), and the movement of the true planet (C in the figure) is obtained.

In the case of the outer planets, B in Figure 8 is the mean planet corrected by its equation of centre, and D is the same direction. The angle CBY corresponds to the amount of the mean motion of the sun. Here, the mean daily motion of the sun minus the daily motion of the mean planet corrected by its equation of centre is the daily motion of the *rkaṅ-'dzin*, and the amount of its motion corresponds to ∠CBD in the figure. For each step, the *myur-rkaṅ* is given, and the epicyclic correction is calculated. This correction is applied to the movement of the mean planet corrected by its equation of centre (B in the figure), and the movement of the true planet (C in the figure) is obtained.

As regards the solar and lunar eclipses, which are called *gza'-'dzin*, the *sgra-gcan*, which corresponds to *rāhu* in Sanskrit, and the *dus-me* are used. The *sgra-gcan* and *dus-me* are the ascending and descending nodes of the lunar orbit respectively. When the new or full moon occurs around them, the eclipse is predicted. I illustrated this in Figure 9. (It should be noted that Figures 8 and 9 are my teaching drawings and are not necessarily the same as images used by Tibetan people.)

* * *

To conclude, Kālacakra astronomy began in the 11th century AD in India, most probably in East India. It must have been based on the Ārdharātrika school of Hindu Classical astronomy which was popular in East India at that time, and the epoch of the original work on which it is based might have been AD 806. The sixty-year cycle used in Kālacakra astronomy is a South Indian reckoning, and AD 1027 is used as the initial year. However, the information about the Hijra, based on North Indian reckoning, must have wrongly been incorporated there, and produced a two years' error for the year of the Hijra. Kālacakra astronomy can be said to be a mixture of East and South Indian astronomy. Kālacakra astronomy was introduced into Tibet, and followed in Tibet, Mongolia, Bhutan, etc. up to the present. There are some different schools of Kālacakra astronomy in these places.

NOTES

[1] For a brief history of Indian astronomy, see Ōhashi, 1997b and 1998. For more detailed discussion, see Ōhashi, 1993, 1994 and 1997a.

[2] One of the best publications on *Vedāṅgajyotisa* is Sastry, 1984.

[3] Peking edition, no. 1027.

[4] Peking edition, no. 5815.

[5] For some information on this period, see Ōhashi, 1994.

[6] For the introduction of the astrolabe into India see Ōhashi, 1997a.

[7] The Sanskrit text of the *Kācakratantra* was published by Raghu Vira and Lokesh Chandra (1966) and Banerjee (1985). The first includes Tibetan and Mongolian translations. For the *Kācakratantra*, also see Newman (1987 b), and Hoffmann (1964) and (1969).

[8] *Tibetan Tripitaka*, Peking ed., vol. l, no.4.

[9] The Sanskrit text of the *Vimalaprabhā* was published by Upadhyaya, 1986 and Samdhong Rinpoche, 1994.

[10] *Tibetan Tripitaka*, Peking ed., vol.46, no.2064.

[11] *Tibetan Tripitaka*, Peking ed., vol.47, no.2098.

[12] I have used the data in Tuckerman, 1964.

[13] Taisho ed., vol.21, no.1308. Also see Yano, 1986.

[14] For the Indian 60-year cycle, see Cunningham, 1883:18–25; Kielhorn, 1889; and Sewell and Dikshit, 1896:32–37.

[15] Toganō, 1989: 696. This is a posthumous publication, and the original manuscript was written in *ca.* 1944.

[16] See Hoffmann, 1973, and Newman, 1987 a: 93.

[17] For the details of Islamic astronomy, see Kennedy, 1956.

[18] For a summary of the schools of Hindu astronomy, see Pingree, 1978 b and 1981.

[19] For information on the Tibetan calendar and astronomy, see Pelliot, 1913; Laufer, 1913 and1914; von Staël-Holstein, 1935–36; Sakai, 1938; Vogel, 1964; Petri, 1968; Yamaguchi, 1973; Schuh, 1973; Tshul-khrims-chos-sbyor, 1983; Ōhashi, 1984, 1986, and 1997c; Huang and Chen, 1987; Huang, 1994; and Dagthon, 1995.

[20] The work of Grags-pa-rgyal-mtshan is in the *Sa-skya-pa'i-bka'-'bum* (1968), vol. 4, no. 130, and the works of 'Phags-pa are in *ibid.* vol. 7, nos. 284–294.

[21] I have never come across a modern publication. I have seen microfilm preserved in the collection of Tibetan works in the Department of Letters, Tokyo University.

[22] The original sMan- rtsis-khaṅ is in Lhasa. Another sMan- rtsis-khaṅ (Tibetan Medical and Astrological Institute) was built by Tibetan refugees in Dharamsala, India, under the fourteenth Dalai Lama.

[23] I am grateful to Dr. Terbish Lhasran who kindly sent his almanac to me.

[24] For the astronomical work of Padma-dkar-po, see Schuh, 1973: 36–37.

[25] I am grateful to Mr. Masahito Nishiwaki, who provided me with a partial copy of a Bhutanese traditional almanac for the year 1992.

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